Linear Parallel Interference Cancellation Using Fixed Weighting Factors for Long-Code CDMA

Dongning Guo
Dept. of Electrical Engineering
Princeton University
Princeton, NJ 08544, USA
email: dGuo@Princeton.EDU

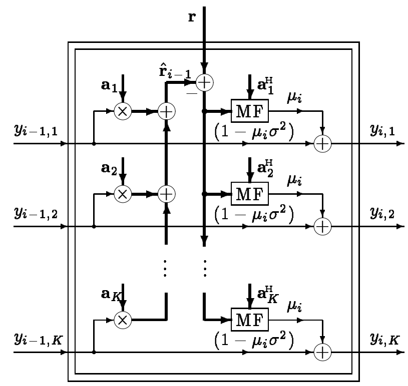
Lars K. Rasmussen
Dept. of Computer Engineering
Chalmers University of Technology
SE-412 96 Göteborg, Sweden
email: larsr@ce.chalmers.se

Abstract — **Linear weighted multistage parallel interference cancellation (PIC) implements exactly the family of polynomial expansion detectors. For long-code CDMA, a set of optimal weights is found which minimizes the ensemble averaged mean squared error (MSE) over random codes. The weights are dependent on moments of the eigenvalues of the correlation matrix, where exact expressions are derived. The loss incurred by averaging rather than using the optimal, time-varying weights is practically negligible.**

I. Introduction

Consider a K-user symbol-synchronous CDMA system with processing gain N. The received signal vector is $\underline{r} = \underline{A}\underline{d} + \underline{n}$, where $\underline{A} = (\underline{a}_1, \underline{a}_2, \cdots, \underline{a}_K)$ is the matrix containing all users' spreading codes, $\underline{d} = (d_1, d_2, \cdots, d_K)^{\mathsf{T}}$ the data vector and $\underline{n}$ the AWGN with variance σ^2.

The detailed structure of the ith PIC stage with weight μ_i is depicted here where MF denotes matched filtering. A multistage PIC is a simple cascade of m PIC stages, whose output is



$$\underline{y}_m = \left[\underline{I} - \prod_{i=1}^{m}(\underline{I} - \mu_i(\underline{R} + \sigma^2\underline{I}))\right] \left(\underline{R} + \sigma^2\underline{I}\right)^{-1} \underline{A}^{\mathrm{H}}\underline{r} \qquad (1)$$

where $\underline{R} = \underline{A}^{\mathrm{H}}\underline{A}$ is the correlation matrix. By choosing an appropriate set of weights, the PIC can implement exactly any detector of the form of a polynomial in $\underline{R}$ applied to the code matched-filtered output $\underline{A}^{\mathrm{H}}\underline{r}$.

It has previously been shown that the PIC is a realization of the steepest descent algorithm used to minimize the MSE. Following this interpretation, a unique set of weights, dependent on the eigenvalues of $\underline{R}$, was found to lead to the minimum achievable MSE for a given number of stages in a short-code system [1]. This approach is too complex for long-code systems. Instead, we consider using a set of code-invariant weights designed to give the minimum ensemble averaged MSE over random codes.

The ensemble average of the excess MSE, as compared to the MMSE, is expressed as a function of $\underline{u} = (\mu_1, \mu_2, \cdots, \mu_m)$

$$J^{(m)}(\underline{u}) = \mathrm{E}\left\{\sum_{k=1}^{K} \frac{\lambda_k}{\lambda_k + \sigma^2} \prod_{i=1}^{m} \left|1 - \mu_i(\lambda_k + \sigma^2)\right|^2\right\} \qquad (2)$$

where the λ_k's are the eigenvalues of $\underline{R}$ and the expectation is taken over random codes. By an elementary symmetric polynomial transform in $\underline{u}$ we can rewrite (2) as a quadratic function in a vector $\underline{x}$, which is a function of $\underline{u}$. A unique minimum is then obtained where the corresponding μ_k's are found as the inverse polynomial transform.

For an m-stage PIC, the weights depend on the first $2m$ moments of the eigenvalues, defined as $M_r = \mathrm{E}\{\lambda^r\}$, $r = 1, 2, \cdots, 2m$ where λ is an arbitrary eigenvalue of $\underline{R}$. Moreover,

$$\begin{aligned} M_r &= \frac{1}{K}\mathrm{E}\left\{\mathrm{tr}\left\{\underline{R}^r\right\}\right\} \\ &= \frac{1}{K}\sum_{k_1=1}^{K}\sum_{k_2=1}^{K}\cdots\sum_{k_r=1}^{K} \mathrm{E}\left\{R_{k_1k_2}R_{k_2k_3}\cdots R_{k_{r-1}k_r}R_{k_rk_1}\right\} \\ &= \frac{1}{K}\sum_{k_1=1}^{K}\sum_{k_2=1}^{K}\cdots\sum_{k_r=1}^{K}\sum_{n_1=1}^{N}\sum_{n_2=1}^{N}\cdots\sum_{n_r=1}^{N} \\ &\qquad \mathrm{E}\left\{A^*_{n_1k_1}A_{n_1k_2}A^*_{n_2k_2}A_{n_2k_3}\cdots A^*_{n_rk_r}A_{n_rk_1}\right\}. \qquad (3)\end{aligned}$$

Since A_{nk} are all independent random variables, only terms containing all complex conjugate pairs are relevant. M_r is then obtained through evaluation of the summation over all combinations of indices. As the expectation is taken over all code-sets, M_r only depends on N and K, but not on specific codes. In fact it is a polynomial in N and K [2].

With the exact expressions derived, the moments can be evaluated easily and the optimal weights computed. The computational complexity is minor for a moderate number of stages and hence can be implemented on-line. Simulation results show that the penalty of averaging rather than using the optimal weights dependent on the instantaneous spreading codes is negligible in most cases of interest.

References

[1] D. Guo, L. K. Rasmussen, S. Sun, and T. J. Lim, "A matrix-algebraic approach to linear parallel interference cancellation in CDMA," *IEEE Trans. Commun.*, vol. 48, Jan. 2000.

[2] D. Guo, L. K. Rasmussen, and T. J. Lim, "Linear parallel interference cancellation in long-code CDMA multiuser detection," *IEEE J. Selected Areas Commun.*, vol. 17, Dec. 1999.

This work was supported in part by the Centre for Wireless Communications, National University of Singapore and Oki Techno Centre (S'pore) Pte Ltd.