A Unified Approach to Power Control in Large Energy-Constrained CDMA Systems

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***Abstract*—A unified approach to power control is proposed for maximizing utility in terms of energy efficiency in code-division multiple access (CDMA) networks. The approach is applicable to a large family of multiuser receivers including the matched filter, the decorrelator, the linear minimum mean-square error (MMSE) receiver, and the (nonlinear) optimal detectors. It exploits the linear relationship between the transmit power and the output signal-to-interference-plus-noise ratio (SIR) for each user in the large-system limit. Suppose that each user seeks to selfishly maximize its own energy efficiency, a unique Nash equilibrium is shown to exist and be SIR-balanced, thus extending a previous result on linear receivers. A unified power control algorithm for reaching the Nash equilibrium is proposed, which adjusts transmit powers iteratively by computing the large-system multiuser efficiency, which is independent of instantaneous spreading sequences. The convergence of the algorithm is proved for linear receivers, and is demonstrated via simulation for the multiuser maximum likelihood detector. Moreover, the performance of the algorithm in finite-size systems is studied and compared with that of a conventional power control scheme, in which user powers depend on the instantaneous spreading sequences.**

***Index Terms*—Code-division multiple access (CDMA), energy efficiency, game theory, large systems, multiuser detection, multiuser efficiency, Nash equilibrium, power control.**

I. Introduction

Power control is used for interference management and resource allocation in both uplink and downlink transmission in code-division multiple access (CDMA) networks [1]–[8]. For example, in the uplink, each user transmits just enough power to achieve the required quality of service (QoS) without causing excessive interference to other users. Power control algorithms for multiuser receivers such as the linear minimum mean-square error (MMSE) receiver and successive interference cancellation receivers have been proposed in [4] and [7], respectively. In such schemes, the output signal-to-interference-plus-noise ratio (SIR) of each user is measured and then the user's transmit power is adjusted linearly to achieve the desired SIR.

Power control is often modeled as an optimization problem under some QoS constraints. A practically appealing scheme is to minimize the total transmit power under the constraint that the SIR of each user is above some threshold. The power of one user affects the performance of other users in the network through multiple-access interference. Under reasonable assumptions, it is shown that the total transmit power is minimized when all the SIR requirements are met with equality [1]. Another scheme is to choose the transmit powers in such a way as to maximize the spectral efficiency (in bits/s/Hz) of the network, where the optimal strategy is essentially a water-filling scheme [6].

More recently, power control has been modeled as a non-cooperative game in which each user selfishly maximizes its utility (e.g., [9]–[16]). Reference [15] applies game theory to the cross-layer design of power control and multiuser detection. The utility function therein measures the number of bits transmitted per joule of energy consumed, which is particularly suitable for energy-constrained networks. Assuming linear multiuser detection, a Nash equilibrium is shown to exist, which refers to a configuration of user strategies under which no user can unilaterally change its strategy to improve its own utility [17]. Moreover, the users are SIR-balanced at equilibrium (i.e., they have the same output SIR).

This work extends the results in [15] to a large family of linear and nonlinear receivers in the so-called *large-system* regime, where the number of users and the spreading factor are large with a given ratio. This is due to results in [18], [19] where a linear relationship between the input power and the output SIR is shown to exist for generic multiuser detection in the large-system limit. Members of this family include well-known receivers such as the matched filter (MF), the decorrelator (DEC), the linear MMSE receiver, as well as the individually optimal (IO) and jointly optimal (JO) multiuser detectors.[1] By exploiting the linear relationship, which is characterized by the multiuser efficiency, we propose a unified power control (UPC) algorithm for reaching the Nash equilibrium. The convergence of the proposed algorithm is proved for linear receivers and is demonstrated by means of simulation for the ML detector.

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[1]The individually optimal detector minimizes the error probability of detecting the symbol of an individual user whereas the jointly optimal detector minimizes the probability of error for simultaneously detecting the symbols of all users [20]. The jointly optimal detector is often referred to as the (multiuser) maximum likelihood (ML) detector.

Because of its large-system nature, the UPC algorithm does not depend on instantaneous spreading sequences. Therefore, the actual SIR achieved by the UPC algorithm fluctuates around the target SIR as the spreading sequences change, which results in a loss in the utility. The performance of UPC in finite-size systems is studied and it is shown that if the spreading factor is reasonably large, the SIR achieved by UPC stays close to the target SIR most of the time, hence the utility loss is insignificant.

The rest of the paper is organized as follows. Section II provides the system model and relevant results in multiuser detection. Section III discusses the game-theoretic approach to power control. The UPC algorithm for reaching Nash equilibrium is proposed and studied in Section IV. The performance of UPC in finite-size systems is studied in Section V. Simulation results are presented in Section VI and conclusions are given in Section VII.

II. Multiuser Detection and Power Control

A. CDMA and Multiuser Detection

Consider the uplink of a synchronous DS-CDMA system with K users and spreading factor N. Let p_k, h_k, and σ^2 represent the transmit power, channel gain and the variance of background noise and co-channel interference respectively, for user k. The received signal-to-noise ratio (SNR) for user k is then $\Gamma_k = p_k h_k / \sigma^2$. The received signal in one symbol duration (after chip-matched filtering) can be represented as

$$\mathbf{Y} = \mathbf{S}\mathbf{X} + \mathbf{W} = \sum_{k=1}^{K} \sqrt{\Gamma_k} X_k \mathbf{s}_k + \mathbf{W} \tag{1}$$

where X_k denotes the transmitted symbol of user k, $\mathbf{s}_k$ denotes its $K \times 1$ unit-norm spreading sequence, and $\mathbf{W} \sim \mathcal{N}(0, \mathbf{I})$ denotes the noise vector consisting of independent and identically distributed (i.i.d.) standard Gaussian entries. Random spreading sequence is assumed for all users, and the input symbols X_k are assumed to be i.i.d. with probability mass function p_X which is of unit variance.[2]

Suppose the receiver is a linear filter $\mathbf{c}_k$, the detection output can be represented as a sum of three independent components:

$$\hat{X}_k = \mathbf{c}_k^T \mathbf{Y} = \sqrt{\Gamma_k}\left(\mathbf{c}_k^T \mathbf{s}_k\right) X_k + \mathsf{MAI}_k + V_k \tag{2}$$

i.e., the desired signal, the MAI and Gaussian background noise. The quality of the output is effectively measured in terms of the output SIR,

$$\gamma_k = \frac{\Gamma_k (\mathbf{c}_k^T \mathbf{s}_k)^2}{\mathbf{c}_k^T \mathbf{c}_k + \sum_{j \neq k} \Gamma_j (\mathbf{c}_k^T \mathbf{s}_j)^2} = \eta_k \Gamma_k \tag{3}$$

because the multiple-access interference (MAI_k) is asymptotically Gaussian [21]. Clearly, $\gamma_k \leq \Gamma_k$, i.e., the SIR is no greater than the SNR due to the MAI. The degradation factor η_k is known as the *multiuser efficiency*. In fact, (2) can be regarded as an equivalent single-user Gaussian channel for user k as depicted in Fig. 1(b), in which $X = X_k$ and Z is proportional to $\hat{X}_k$.

[2]The results in the paper also apply to continuous input distributions such as Gaussian input where p_X denotes the probability density function (see [19] for a general discussion).

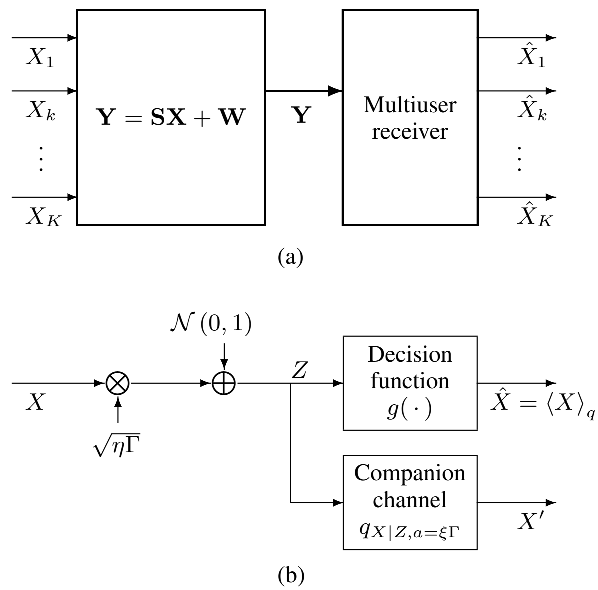

Fig. 1. (a) The DS-CDMA system with multiuser detection. (b) The equivalent scalar Gaussian channel for user k and the companion channel.

Note that for nonlinear detection, it is in general not possible to directly decompose the detection output into a sum of desired signal and interference and noise as in (2); neither is the output asymptotically Gaussian. We defer the treatment of nonlinear detection to Section III.

B. Power Control

Power control can be modeled as a non-cooperative game in which each user adapts its transmit power to selfishly maximize its own utility (see e.g., [9]–[16]). We follow [9] to define the utility of user k as

$$u_k = T_k / p_k \ \text{(bits/joule)} \tag{4}$$

where T_k is the net number of information bits delivered correctly per unit time for user k, referred to as the *goodput*). This utility captures the tradeoff between the throughput and energy consumption and is particularly suitable for applications in which energy efficiency is critical.

The throughput for user k can be quantified as

$$T_k = f(\gamma_k) RL/M \tag{5}$$

where R is the transmission rate, L and M are the number of information bits and the total number of bits in a packet, respectively (i.e., $L - M$ bits of overhead), all independent of γ_k, and $f(\gamma_k)$ is the efficiency function representing the packet success rate at SIR equal to γ_k. The underlying assumption is that packets in error are retransmitted. The utility of user k is thus given as a function of the transmit power by

$$u_k(p_k) = \frac{RL}{M} \frac{f(\gamma_k)}{p_k} \,. \tag{6}$$

The efficiency function, $f(\cdot)$, is assumed to be increasing and sigmoidal (S-shaped), i.e., there is a point above which the function is strictly concave, and below which the function is strictly convex. We also require that $f(\infty) = 1$ and $f(0) = 0$

to ensure that $u_k = 0$ when $p_k = 0$. These assumptions are valid in many practical systems (see [15] for further details). It can be shown that the utility function in (6) is quasiconcave; that is, there exists a point below which the function is non-decreasing, and above which the function is non-increasing.

The non-cooperative power control game is described as

$$\max_{0 \le p_k \le P_{\max}} u_k \quad \text{for} \quad k = 1, \cdots, K \tag{7}$$

where $P_{\max}$ is the maximum allowed power. For such a non-cooperative game, a Nash equilibrium is a set of transmit powers $(p_1, \ldots, p_K)$ for which no user can unilaterally improve its own utility. It has been shown in [15] that the following proposition holds for all linear receivers.

Proposition 1: The utility-maximizing strategy for user k is given by $p_k = \min(p_k^*, P_{max})$, where p_k^* is such that the resulting output SIR $\gamma_k = \gamma^*$, which is the unique solution to

$$f(\gamma) = \gamma f'(\gamma). \tag{8}$$

Furthermore, the power control game has a unique Nash equilibrium.

The key to the proof of Proposition 1 is that, for the linear receivers of interest, there is a linear relationship between the output SIR and transmit power of each user. Without loss of generality, let the relationship be described as $\gamma_k = \phi_k \, p_k$ where ϕ_k is not dependent on p_k. Taking the partial derivative of the utility function in (6) with respect to the transmit power and equating it to zero, we have

$$\frac{\partial u_k(p_k)}{\partial p_k} = \frac{RL}{M} \frac{\partial}{\partial p_k} \left(\frac{f(\gamma_k)}{p_k} \right) = 0 \tag{9}$$

which is equivalent to

$$\frac{\partial}{\partial \gamma_k} \left(\frac{f(\gamma_k)}{\gamma_k} \right) = \frac{f'(\gamma_k)\gamma_k - f(\gamma_k)}{\gamma_k^2} = 0. \tag{10}$$

Therefore, $\gamma_k = \gamma^*$ which is the solution to (8) maximizes the user's utility as long as the corresponding p_k is feasible; otherwise $p_k = P_{\max}$.

The existence of a Nash equilibrium is guaranteed by the quasiconcavity of the utility function. The uniqueness of the equilibrium is because of the uniqueness of γ^* and the one-to-one relationship between the transmit power and output SIR. Interestingly, Proposition 1 implies that, at Nash equilibrium, all users have the same output SIR (i.e., the Nash equilibrium is SIR-balanced).

III. Large Multiuser Systems and Power Control

This paper studies power control in *large* CDMA systems. Mathematically, we consider the *large-system limit*, where both the number of users and the spreading factor tend to infinity but with their ratio converging to a positive number, i.e., $K/N \to \alpha$. Let us also assume that the received SNRs Γ_k are i.i.d. with distribution P_Γ at a given time. Moreover, we assume that P_Γ varies slowly over time (i.e., slow fading channel).

In general, the multiuser efficiency depends on the received SNRs, the spreading sequences as well as the type of detector. However, in the large-system regime, the dependence on the spreading sequences vanishes and the received SNRs affect the efficiency only through their distribution. In particular, the multiuser efficiency of the matched filter and the decorrelator are obtained as

$$\eta^{mf} = \frac{1}{1 + \alpha \, \mathbb{E}\{\Gamma\}} \tag{11}$$

$$\eta^{dec} = 1 - \alpha \quad \text{for} \quad \alpha < 1 \tag{12}$$

while the efficiency of the (linear) MMSE receiver, η^{mmse}, is the unique solution to the following fixed-point equation

$$\frac{1}{\eta} = 1 + \alpha \, \mathbb{E} \left\{ \frac{\Gamma}{1 + \eta \, \Gamma} \right\} \tag{13}$$

where the expectation is over P_Γ.

As pointed out in Section II-A, Gaussian characterization of detection output in the analysis of the SIR using (3) does not directly apply to nonlinear receivers. Remarkably, reference [18], [19] finds that, under mild assumptions, the output of a generic nonlinear receiver converges in the large-system limit to a simple monotone function of a "hidden" Gaussian statistic conditioned on the input, i.e.,

$$\hat{X}_k \to g\big(\sqrt{\eta \Gamma_k} \, X_k + U_k\big) \tag{14}$$

where $U_k \sim \mathcal{N}(0,1)$ is independent of X_k. By applying an inverse of function $g(\cdot)$ to the detection output $\hat{X}_k$, an equivalent conditionally Gaussian statistic $Z_k = \sqrt{\eta \Gamma_k} \, X_k + U_k$ is recovered. Each symbol X_k traverses an equivalent single-user Gaussian channel, so that the output SIR (defined for the equivalent Gaussian statistic Z_k) completely characterizes the system performance. This result is referred to as the "decoupling principle." The equivalent channel is illustrated in Fig. 1(b). Indeed, as far as the posterior probability $P_{X_k | \hat{X}_k}$ is concerned, the multiuser model (Fig. 1(a)) and the single-user model (Fig. 1(b)) are asymptotically indistinguishable.

A. Posterior Mean Estimators

The decoupling principle holds for a broad family of multiuser receivers, called the *posterior mean estimators* (PME) [18], [19]. Given the observation $\mathbf{Y}$ and the spreading matrix $\mathbf{S}$, a PME computes the mean value of some posterior probability distribution $q_{\mathbf{X}|\mathbf{Y},\mathbf{S}}$, which is conveniently denoted as

$$\langle \mathbf{X} \rangle_q = \mathbb{E}_q \left\{ \mathbf{X} \mid \mathbf{Y}, \mathbf{S} \right\}. \tag{15}$$

In this work, the posterior $q_{\mathbf{X}|\mathbf{Y},\mathbf{S}}$ supplied to the PME is induced from the following postulated CDMA system,

$$\mathbf{Y} = \mathbf{S}\mathbf{X}' + \varrho \mathbf{W} \tag{16}$$

which differs from the actual channel (1) by only the input and the noise variance. In particular, the components of $\mathbf{X}'$ are i.i.d. with distribution q_X, and the postulated noise level ϱ serves as a control parameter. The posterior $q_{\mathbf{X}|\mathbf{Y},\mathbf{S}}$ is determined by q_X and $q_{\mathbf{Y}|\mathbf{X},\mathbf{S}}$ according to Bayes' formula

$$q_{\mathbf{X}|\mathbf{Y},\mathbf{S}}(\mathbf{x}|\mathbf{y},\mathbf{S}) = \frac{q_{\mathbf{X}}(\mathbf{x}) q_{\mathbf{Y}|\mathbf{X},\mathbf{S}}(\mathbf{y}|\mathbf{x},\mathbf{S})}{\sum_{\mathbf{x}} q_{\mathbf{X}}(\mathbf{x}) q_{\mathbf{Y}|\mathbf{X},\mathbf{S}}(\mathbf{y}|\mathbf{x},\mathbf{S})}. \tag{17}$$

Indeed, the PME so defined is parameterized by (q_X, ϱ) and can be regarded as the optimal detector for the postulated multiuser system (16). In case the postulated posterior $q_{\mathbf{X}|\mathbf{Y},\mathbf{S}}$ is identical to $p_{\mathbf{X}|\mathbf{Y},\mathbf{S}}$, the PME is a soft version of the individually optimal detector. The postulated posterior, however, can also be chosen such that the PME becomes one of many other detectors, including but not limited to the MF, DEC, linear MMSE, as well as the JO detectors. Thus the concept of PME is generic and versatile.

B. Decoupling Principle for PME Receivers

Let the input–output relationship of a general scalar Gaussian channel be denoted as

$$p_{Z|X;a}(z|x;a) = \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(z - \sqrt{a}\,x\right)^2\right] \qquad (18)$$

where a is a parameter describing the quality of the channel. Similar to that in the multiuser setting, by postulating the input distribution to be q_X, a posterior probability distribution $q_{X|Z;a}$ is induced from q_X and $p_{Z|X;a}$ using Bayes' formula (cf. (17)). In particular, we have a single-user companion channel defined by $q_{X|Z;a}(\cdot|\cdot;\xi\Gamma)$, which outputs a random variable X' given the channel output Z (Fig. 1(b)). A single-user PME is defined naturally as:

$$\langle X \rangle_q = \mathbb{E}_q\left\{ X \mid Z; \xi\Gamma \right\} = \sum_x x\, q_{X|Z;a}(x|Z;\xi\Gamma). \qquad (19)$$

The probability law of the composite system depicted by Fig. 1(b) is determined by Γ, η and ξ.

Proposition 2 ([18], [19]): Consider the multiuser channel (1) with PME receiver (15). Fix $(\alpha, P_\Gamma, p_X, q_X, \varrho)$. The joint probability distribution of $(X_k, \langle X_k \rangle_q)$ converges in the large-system limit to the joint probability distribution of $\left(X, g\left(\sqrt{\eta\Gamma_k}\,X + U\right)\right)$ where $U \sim \mathcal{N}(0,1)$ and

$$g(z) = \mathbb{E}_q\left\{ X \mid Z = z; \xi\Gamma \right\} \qquad (20)$$

where the multiuser efficiency η satisfies together with ξ a pair of coupled equations:

$$\eta^{-1} = 1 + \alpha\,\mathbb{E}\left\{\Gamma\left(X - g(Z)\right)^2\right\} \qquad (21a)$$

$$\xi^{-1} = \varrho^2 + \alpha\,\mathbb{E}\left\{\Gamma\left(X' - g(Z)\right)^2\right\} \qquad (21b)$$

where the expectations are taken over the following joint distribution of (X, Z, X', Γ):

$$p_X(x)\, p_{Z|X;\eta\Gamma}(z|x;\eta\Gamma)\, q_{X|Z;\xi\Gamma}(x'|z;\xi\Gamma)\, P_\Gamma(\Gamma). \qquad (22)$$

In case of multiple solutions to (21), (η, ξ) is chosen to minimize the so-called free energy (see [18]).

Proposition 2 reveals that, from an individual user's viewpoint, the input–output relationship of the multiuser channel concatenated with the multiuser receiver is asymptotically identical to that of the scalar Gaussian channel with a (nonlinear) decision function. In fact, the joint distribution described by (22) is nothing but that of (X, Z, X', Γ) in the equivalent scalar system depicted by Fig. 1(b). Note that even though (24) is a large-system result, it is a good approximation for most finite-size systems of practical interest.

As shown in [18], [19], by choosing appropriate parameterization (q_X, ϱ), the PME can be made to represent the MF, the DEC, the linear MMSE as well as the IO and JO detectors. In particular, the multiuser efficiencies of the linear receivers are given as (11)–(13). The multiuser efficiency of the IO detector with BPSK inputs is found to satisfy the fixed-point equation

$$\frac{1}{\eta} = 1 + \alpha\mathbb{E}\left\{\Gamma - \Gamma\int_{-\infty}^{+\infty} \frac{e^{-\frac{z^2}{2}}}{\sqrt{2\pi}} \tanh\left(\eta\Gamma - z\sqrt{\eta\Gamma}\right) \mathrm{d}z\right\}. \quad (23)$$

Although there is usually no known analytical solution to the fixed-point equations (21) or (23), the multiuser efficiency is in general easy to compute by solving such equations numerically. In principle, the multiuser efficiency may be different for different users with different types of receivers. In the uplink, it is natural to assume the same type of receiver for all users and hence all users have the same multiuser efficiency.

C. Power Control with PME Receivers

Of particular importance to the power control problem is the linear relationship between the transmit power and the output SIR of each user implied by the decoupling principle, i.e.,

$$\gamma_k = \eta\,\Gamma_k \qquad (24)$$

where the multiuser efficiency η depends on the SNR distribution P_Γ rather than the individual SNRs Γ_k. This is mainly due to the fact that in a large system, as one user's transmit power varies, the interference seen by the user essentially stays the same as long as the empirical distribution of the received powers remains the same.

Note that the key to the proof of Proposition 1, which is the linear relationship between the transmit power and received SIR, also holds for the PME receivers in the large-system limit. This implies that, in large systems with PME receivers, a unique Nash equilibrium exists for the power control game (7). Moreover, at equilibrium, each user transmits at a power level that achieves an output SIR equal to γ^*, the solution of $f(\gamma) = \gamma f'(\gamma)$. To the best of our knowledge, this work is the first to establish the existence and uniqueness of Nash equilibrium for multiuser systems with nonlinear receivers. It is interesting to note that, due to the specific choice of utility function (6), the Nash equilibrium SIR is independent of the type of receiver and depends only on physical-layer configuration such as modulation, coding and packet size.

IV. Unified Power Control Algorithm

A. The Algorithm

In this section, a unified power control algorithm for reaching the Nash equilibrium is proposed for uplink CDMA with PME receivers. Similar to some of the other power control algorithms, the UPC algorithm iteratively adjusts the transmit powers in order to reach the target output SIR of γ^*.

The UPC Algorithm carries out the following iteration:

1) Set $n = 0$ and use arbitrary powers $p_1(0), \cdots, p_K(0)$;
2) Use the power profile to compute the multiuser efficiency $\eta(n)$ by solving (21).
3) For user k, update the powers according to

$$p_k(n+1) = \frac{1}{\eta(n)} \frac{\gamma^*\,\sigma^2}{h_k}\ ; \qquad (25)$$

4) n=n+1, stop if convergence; otherwise, go to Step 2.

Assuming that the empirical distribution of user SNRs is available, the receiver can compute $\eta(n)$ in step 2. The transmit power for achieving SIR γ^* can then be computed using (25) based on estimates of the channel gain h_k and the noise variance σ^2.

The UPC algorithm applies to a large family of linear and nonlinear receivers, including the MF, DEC, linear MMSE, and IO and JO detectors. In particular, for the matched filter, the UPC algorithm becomes the same as the bilinear power control algorithm proposed in [22] for minimizing the SIR error. In fact, one may also extend Proposition 2 to the case that different users use different type of receivers so that the UPC algorithm remains applicable by replacing η by individual efficiencies η_k.

It is an apparent paradox that the success of the power control scheme depends on the assumption that the empirical distribution of SNR is fixed, while the purpose is to adjust transmit powers in order to affect the SNRs. This, however, can be resolved naturally in practice, where the number of users is finite, by continued iteration and by replacing the expectations in (21) with an average over all users' received SNRs (or their estimates). For example, for the linear MMSE receiver, (13) can be expressed as

$$\frac{1}{\eta} = 1 + \frac{\alpha}{K}\sum_{k=1}^{K}\frac{\Gamma_k}{1+\eta\Gamma_k}. \qquad (26)$$

B. Convergence

In the following, we establish the convergence of the UPC algorithm for the MF, DEC, and linear MMSE receiver. The technique, which requires a monotone relationship between the transmit powers and the multiuser efficiency, does not apply to the optimal detectors. For the ML receiver, the convergence is instead verified through simulation in Section VI.

Let $\mathbf{\Gamma} = [\Gamma_1, \cdots, \Gamma_K]$ and define an interference function,

$$I(\mathbf{\Gamma}) = \frac{\gamma^*}{\eta(\mathbf{\Gamma})} \qquad (27)$$

in which the dependence of the efficiency on $\mathbf{\Gamma}$ is explicit. By (25) and (27), the UPC algorithm is equivalent to

$$\Gamma_k(n+1) = I\left(\mathbf{\Gamma}(n)\right), \quad k = 1, \ldots, K. \qquad (28)$$

Proposition 3: For the matched filter, the decorrelator, and the MMSE receiver, if there exists a $\hat{\mathbf{\Gamma}}$ such that $\hat{\Gamma}_k \geq I(\hat{\mathbf{\Gamma}})$, $k = 1, \ldots, K$, then for every initial vector $\mathbf{\Gamma}(0)$, the recursion (28) converges to the unique fixed point solution of $\Gamma_k^* = I(\mathbf{\Gamma}^*)$, $k = 1, \ldots, K$. Furthermore, for any feasible $\hat{\mathbf{\Gamma}}$ (i.e., $\hat{\Gamma}_k \geq I(\hat{\mathbf{\Gamma}})$ for all k), $\Gamma_k^* \leq \hat{\Gamma}_k$ for all k.

Proof: The existence of a $\hat{\mathbf{\Gamma}}$ implies that a feasible SNR vector exists for achieving γ^*. It suffices then to show that $I(\mathbf{\Gamma})$ is a standard interference function [3], i.e., for all $\mathbf{\Gamma}$ with $\Gamma_k \geq 0$ for all k, the following three properties are satisfied: 1) Positivity: $I(\mathbf{\Gamma}) > 0$; 2) Monotonicity: If ${\Gamma'}_k \geq \Gamma_k$ for all k, then $I(\mathbf{\Gamma}') \geq I(\mathbf{\Gamma})$; 3) Scalability: For all $\theta > 1$, $\theta I(\mathbf{\Gamma}) > I(\theta\mathbf{\Gamma})$. Evidently, it is equivalent to showing the three properties for $\hat{I}(\mathbf{\Gamma}) = 1/\eta(\mathbf{\Gamma})$.

Positivity of $\hat{I}(\mathbf{\Gamma})$ is trivial for all receivers since $\eta \in [0, 1]$.

Consider first the matched filter. The multiuser efficiency is given by (11), where $\mathbb{E}\{\Gamma\} = \sum_{k=1}^{K}\Gamma_k/K$. If $\mathbf{\Gamma}' \geq \mathbf{\Gamma}$, then $\mathbb{E}\{\Gamma'\} \geq \mathbb{E}\{\Gamma\}$ and hence $\hat{I}(\mathbf{\Gamma}') \geq \hat{I}(\mathbf{\Gamma})$. To prove the third property, note that, for $\theta > 1$, $\hat{I}(\theta\mathbf{\Gamma}) = 1 + \alpha\mathbb{E}\{\theta\Gamma\} < \theta + \alpha\theta\mathbb{E}\{\Gamma\} = \theta\hat{I}(\mathbf{\Gamma})$.

Consider now the decorrelator, the multiuser efficiency of which is constant $\eta = 1 - \alpha > 0$ for $\alpha < 1$. Properties 2 and 3 are trivial.

Consider the MMSE receiver. The multiuser efficiency is the solution to (13), or equivalently, the unique solution of

$$\eta + \alpha\mathbb{E}\left\{\left(\frac{1}{\eta\Gamma}+1\right)^{-1}\right\} = 1. \qquad (29)$$

Note that the left-hand side of (29) increases if both η and Γ increase. Thus if $\mathbf{\Gamma}' \geq \mathbf{\Gamma}$, we must have $\eta(\mathbf{\Gamma}') \leq \eta(\mathbf{\Gamma})$ to maintain the equality. Hence, $\hat{I}(\mathbf{\Gamma}') \geq \hat{I}(\mathbf{\Gamma})$. To prove the third property, let us define $\eta' = \eta(\theta\mathbf{\Gamma})$ and $\eta'' = \theta\eta'$, where $\theta > 1$. Therefore,

$$\eta'' + \alpha\theta\mathbb{E}\left\{\left(\frac{1}{\eta''\Gamma}+1\right)^{-1}\right\} = \theta. \qquad (30)$$

Showing $\theta\hat{I}(\mathbf{\Gamma}) > \hat{I}(\theta\mathbf{\Gamma})$ is equivalent to showing $\eta'' > \eta$. Since the left-hand side of (29) is increasing in η, and

$$\eta'' + \alpha\mathbb{E}\left\{\left(\frac{1}{\eta''\Gamma}+1\right)^{-1}\right\} = 1 + \left(1-\frac{1}{\theta}\right)\eta'' > 1, \qquad (31)$$

we must have $\eta'' > \eta$. Therefore, $\theta\hat{I}(\mathbf{\Gamma}) > \hat{I}(\theta\mathbf{\Gamma})$. ■

V. Performance Evaluation and Discussion

The UPC takes a large-system approach and is hence independent of the spreading sequences. Therefore, after convergence, the transmit powers need not be updated as long as the channel gains remain static. However, the actual SIRs depend on the spreading sequences in finite-size systems (see (3) as an example). As the spreading sequence changes, the output SIRs achieved by the UPC algorithm fluctuate around the target SIR, which results in a loss in energy efficiency.

The question is how close will the SIRs be to the target if the UPC algorithm is applied. In the following, we study the performance of the UPC algorithm for DEC and linear MMSE receiver and compare the performance with that of an SIR-based algorithm, which adjusts the transmit power to compensate for the mismatch between the received SIR and the target.

A. The Decorrelator

For the decorrelator, it is sensible to assume $\alpha < 1$. The large-system multiuser efficiency is given by $\eta^{dec} = 1 - \alpha$. Hence the SNR dictated by the UPC algorithm is

$$\Gamma_k^* = \Gamma_{dec}^* = \gamma^*/(1-\alpha), \quad k = 1, \cdots, K, \qquad (32)$$

which should lead to an SIR of γ^* in the large-system limit. The actual output SIR for a finite-size system is given by

$$\gamma_k = \left(\frac{\gamma^*}{1-\alpha}\right) \Big/ \left[\left(\tilde{\mathbf{S}}^T\tilde{\mathbf{S}}\right)^{-1}\right]_{kk}, \qquad (33)$$

where $\tilde{\mathbf{S}} = [\mathbf{s}_1, \mathbf{s}_2, \cdots, \mathbf{s}_K]$ and $[\cdot]_{kk}$ extracts the kth diagonal entry of a matrix.

It has been shown that in large systems, the distribution of $1/\big[(\tilde{\mathbf{S}}^T\tilde{\mathbf{S}})^{-1}\big]_{kk}$ can be approximated by a beta distribution with parameters $(N-K+1, K-1)$ [23]. As a result, the probability density function of γ_k is given approximately by

$$f_{\gamma_{dec}}(z) = \left(\frac{1}{\Gamma^*_{dec}}\right)^{N-1} \frac{z^{N-K}(\Gamma^*_{dec} - z)^{K-2}}{B(N-K+1, K-1)} \tag{34}$$

where $z \le \Gamma^*_{dec}$ and $B(a,b) = \int_0^1 t^{a-1}(1-t)^{b-1}\mathrm{d}t$. Therefore, as the spreading sequences change from symbol to symbol, the probability that γ_k stays within Δ dB of γ^* is given by

$$P_{\Delta,dec} = \Pr\{|\gamma_{dec}(\mathrm{dB}) - \gamma^*(\mathrm{dB})| \le \Delta\} \tag{35}$$

$$= \int_{\gamma_L}^{\gamma_H} f_{\gamma_{dec}}(z)\mathrm{d}z \ , \tag{36}$$

where $\gamma_L = 10^{-\frac{\Delta}{10}}\gamma^*$ and $\gamma_H = 10^{\frac{\Delta}{10}}\gamma^*$.

Alternatively, the fluctuation of the actual SIR around γ^* can also be approximated less accurately by a Gaussian distribution with variance [24]

$$\zeta^2 = \frac{2\gamma^{*2}\alpha}{(1-\alpha)N} \ , \tag{37}$$

i.e., $\gamma_{dec} \sim \mathcal{N}\big(\gamma^*, \zeta^2\big)$. Therefore, the probability that γ_k stays within Δ dB of γ^* is approximately given by

$$P^{\text{norm}}_{\Delta,dec} = \Phi\left(\frac{\gamma_H - \gamma^*}{\zeta}\right) - \Phi\left(\frac{\gamma_L - \gamma^*}{\zeta}\right), \tag{38}$$

where $\Phi(\cdot)$ is the cumulative distribution function of the standard Gaussian distribution.

B. The MMSE Receiver

If the linear MMSE receiver is used and all users have the same target SIR, γ^*, the steady-state SNRs will be identical to $\Gamma^* = \gamma^*/\eta$ after the UPC algorithm converges, where the multiuser efficiency is given by

$$\eta = \frac{1-\alpha}{2} - \frac{1}{2\Gamma} + \frac{1}{2}\sqrt{(1-\alpha)^2 + \frac{2(1+\alpha)}{\Gamma} + \frac{1}{\Gamma^2}}. \tag{39}$$

It can be shown that the fluctuation of the true SIR around γ^* is approximately Gaussian with variance [24], [25]:

$$\zeta^2 = \frac{1}{N}\frac{2\gamma^{*2}}{1-\alpha\big(\frac{\gamma^*}{1+\gamma^*}\big)^2} \ . \tag{40}$$

The probability that γ_k stays within Δ dB of γ^* admits a similar expression to (38) using the function $\Phi(\cdot)$.

It is seen from these approximations that the variance of fluctuations of SIR decreases as $1/N$. In the following section, we demonstrate the performance of the UPC algorithm using simulations and also investigate the accuracy of the theoretical approximations discussed above.

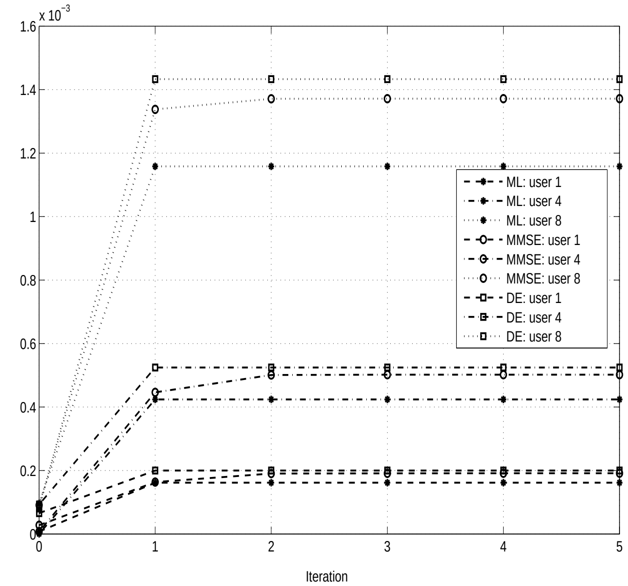

Fig. 2. Transmit powers for the ML, MMSE, and decorrelator, using the UPC algorithm ($N = 32$ and $K = 8$).

VI. Numerical Results

Consider the uplink of a randomly spread DS-CDMA system. The noise variance is assumed to be $\sigma^2 = 1.6 \times 10^{-14}$. We use $f(\gamma) = (1 - e^{-\gamma})^M$ as the efficiency function[3] with $\gamma^* = 6.4$ (=8.1 dB).

We first demonstrate the convergence of the UPC algorithm assuming $K = 8$ users and spreading factor of $N = 32$. The channel gain for user k is given by $h_k = 0.1 \times d_k^{-4}$ where d_k is the distance of user k from the uplink receiver (e.g., base station). Assume $d_k = 100 + 10k$ in meters. We implement the UPC algorithm for the decorrelator and the MMSE receiver as well as the maximum likelihood detector.

Fig. 2 plots the transmit powers for users 1, 4 and 8 at the end of each iteration. It is seen that for all three receiver types, the UPC algorithm converges quickly to steady-state values. The results are similar when the initial power values and/or K and N change. It is also observed that the steady-state transmit powers for the decorrelator and the MMSE receiver are close to those of the ML detector (the difference is less than 22% in this case). This means that in terms of energy efficiency, which is quantified by the utility achieved at Nash equilibrium, the performance of the decorrelator and the MMSE receiver is close to that of the ML detector.

We next investigate the fluctuation of the SIR and bit-error-rate (BER) achieved by the (large-system) UPC algorithm against perfect power control where the SIR is computed using instantaneous spreading sequences (labeled SIR-based in plots). Fig. 3 shows the SIR and bit-error-rate (BER) of user 1 using the MMSE detector. It is seen that the SIR-based algorithm achieves the target SIR, γ^*, at all time whereas the output SIR for the UPC algorithm fluctuates around the target

[3]This efficiency function serves as an approximation to the packet success rate that is reasonable for moderate to large packet sizes.

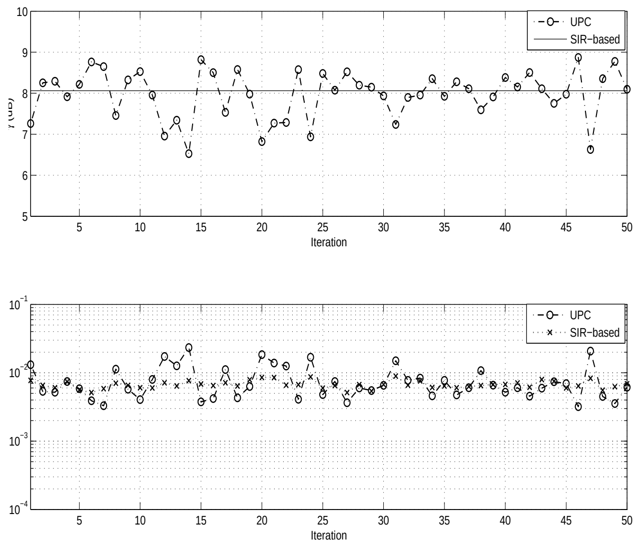

Fig. 3. User 1 output SIR and BER for the UPC algorithm and SIR-based algorithm with the MMSE receiver ($N = 32$ and $K = 8$).

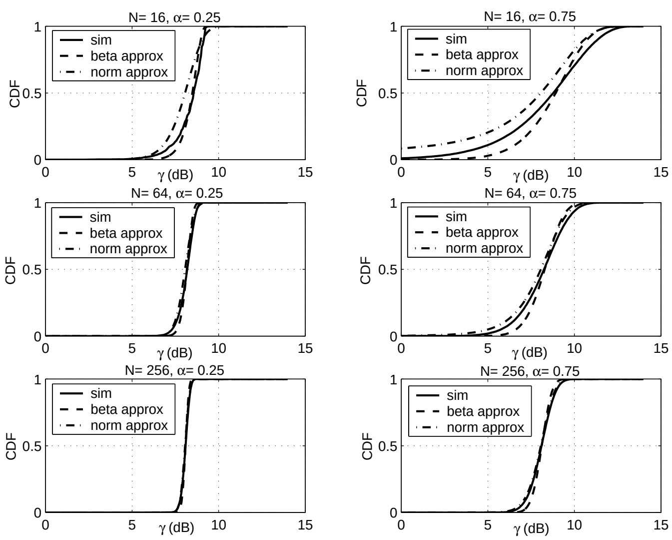

Fig. 4. CDFs of γ for the decorrelator.

SIR as the spreading sequences change. Also, the fluctuations in the BER are larger when the UPC algorithm is used.

To evaluate the accuracy of the theoretical approximations given in Section V, Figs. 4 and 5 plot the cumulative distribution function (CDF) of the output SIR γ for the decorrelator and the MMSE receiver with different spreading factors and both low and high system loads. The plots show CDFs obtained from simulation (based on 100,000 realizations) as well as those predicted by the theoretical approximations given in Section V. It is seen from the figures that the theoretical approximations become more accurate as the spreading factor increases. Also, the approximations are generally more accurate when the system load is low. Note that, for the decorrelator, the approximation based on a beta distribution

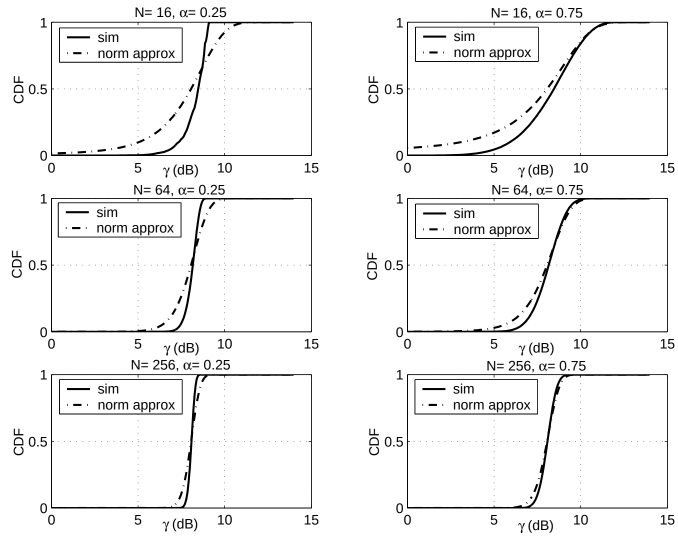

Fig. 5. CDFs of γ for the MMSE receiver.

is slightly more accurate than the one based on a Gaussian distribution, especially for small spreading factors and large system loads.

To quantify the discrepancies between the simulation results and the theoretical approximations, we compute $P_{\Delta,dec}$ and $P_{\Delta,MMSE}$ using the CDFs obtained from simulations as well as those predicted by theory (see (35) and (38)). Table I shows the results for different spreading factors and system loads for $\Delta = 1$ dB. The numbers in the table represent the probability that γ is within 1 dB of γ^*. By (6), a 1-dB increase in the output SIR results in 10% loss in the user's utility. The probabilities obtained by simulation suggest that the UPC algorithm performs better for the MMSE receiver than for the decorrelator. It is also seen from the table that when the spreading factor is small, the fluctuations in the output SIR are considerable, especially when the system load in high. The performance improves as the spreading factor increases. For example, for the MMSE receiver, when $N = 256$ and $\alpha = 0.75$, the SIR stays within 1 dB of the target SIR 98% of the time. It is also observed that the theoretical approximations for the MMSE detector are pessimistic. For the decorrelator, while approximating the SIR by a beta distribution is more accurate (see Fig. 4), the values obtained for $P_{\Delta,MMSE}$ by the Gaussian approximation are closer to the simulation results. This is because the slope of the CDF of γ is closer to the slope of the Gaussian CDF. Since $P_{\Delta,MMSE}$ heavily depends on the slope of the CDF, it is more accurately predicted by the Gaussian approximation (rather than the beta approximation).

VII. CONCLUSION

A unified approach to energy-efficient power control in large systems has been proposed, which is applicable to a large family of linear and nonlinear multiuser receivers. The approach exploits the linear relationship between the transmit power and the output SIR in large systems. Taking a non-cooperative game-theoretic approach with emphasis on energy

TABLE I
SUMMARY OF RESULTS FOR THE DECORRELATOR AND THE MMSE RECEIVER

N	$P^{sim}_{1\text{dB},dec}$ $\alpha=0.25$	$P^{beta}_{1\text{dB},dec}$ $\alpha=0.25$	$P^{norm}_{1\text{dB},dec}$ $\alpha=0.25$	$P^{sim}_{1\text{dB},dec}$ $\alpha=0.75$	$P^{beta}_{1\text{dB},dec}$ $\alpha=0.75$	$P^{norm}_{1\text{dB},dec}$ $\alpha=0.75$	$P^{sim}_{1\text{dB},MMSE}$ $\alpha=0.25$	$P^{norm}_{1\text{dB},MMSE}$ $\alpha=0.25$	$P^{sim}_{1\text{dB},MMSE}$ $\alpha=0.75$	$P^{norm}_{1\text{dB},MMSE}$ $\alpha=0.75$
16	0.77	0.87	0.74	0.28	0.19	0.30	0.93	0.46	0.41	0.33
64	0.98	1.0	0.97	0.54	0.64	0.55	0.99	0.76	0.74	0.61
256	1.0	1.0	1.0	0.87	0.96	0.87	1.0	0.98	0.98	0.91

efficiency, it is shown that the Nash equilibrium is SIR-balanced not only for linear receivers but also for some nonlinear receivers such as the individually and jointly optimal multiuser detectors. In addition, a unified power control algorithm for reaching the Nash equilibrium has been proposed and its performance of in finite-size systems has been studied. It would be straightforward to extend the unified approach to multirate and multicarrier systems based on related large-system results [26], [27].

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