Aspects on Linear Parallel Interference Cancellation in CDMA

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Abstract — **In this paper we devise a matrix-algebraic approach to analysing linear parallel interference cancellation. We show that linear parallel interference cancellation schemes correspond to linear matrix filtering.**

I. Introduction

A significant improvement to parallel interference cancellation (PIC) was suggested by Divsalar and Simon in [1] where they proposed a weighted cancellation scheme. Here, we show that linear weighted PIC schemes correspond to linear matrix filtering. It is then possible to get an analytical expression for the bit error rate and it is possible to derive necessary conditions on the eigenvalues of the correlation matrix and the potential weighting factors for ensuring convergence.

II. Algebraic Description of PIC

User k in a K-user communication system transmits an M-ary PSK information symbol d_k by multiplying the symbol with a spreading code $\mathbf{s}_k$ and then transmitting over an AWGN channel. The spreading codes are assumed to be symbol-synchronous. The output of a chip-matched filter is then expressed as a linear combination of spreading codes, $\mathbf{r} = \mathbf{Ad} + \mathbf{n} \in \mathbb{C}^N$, where $\mathbf{A} = (\mathbf{s}_1, \mathbf{s}_2, \cdots, \mathbf{s}_K)$, $\mathbf{d} = (d_1, d_2, \cdots, d_K)^\top$ and $\mathbf{n}$ is a noise vector where each sample is independently, circularly complex Gaussian distributed with zero mean and variance N_0.

Inspired by the PIC structure in [1], we suggest a modified linear scheme. The decision statistic of this scheme is described by

$$y_{m+1,k} = \mu \mathbf{s}_k^\top \left(\mathbf{r} - \sum_{i=1}^{K} \mathbf{s}_i y_{m,i} \right) + (1 - \mu\alpha) y_{m,k},$$

where $y_{1,k} = \mathbf{s}_k^\top \mathbf{r}$ and μ and α are appropriate weighting factors. This structure is identical to conventional PIC if $\mu = 1$ and $\alpha = 0$ and equal to the weighted PIC in [1] if $\alpha = 0$. In matrix notation, the scheme is described by

$$\mathbf{y}_{m+1} = \mu \mathbf{y} + (\mathbf{I} - \mu(\mathbf{R} + \alpha\mathbf{I}))\mathbf{y}_m,$$

where $\mathbf{y}_1 = \mu \mathbf{A}^\top \mathbf{r}$. Using this recursion, we can express $\mathbf{y}_m$ as

$$\mathbf{y}_m = \mu \sum_{i=0}^{m-1} (\mathbf{I} - \mu(\mathbf{R} + \alpha\mathbf{I}))^i \mathbf{A}^\top \mathbf{r} = \mathbf{G}_m^\top \mathbf{r},$$

where $\mathbf{G}_m^\top$ can be re-written as

$$\mathbf{G}_m^\top = (\mathbf{I} - (\mathbf{I} - \mu(\mathbf{R} + \alpha\mathbf{I}))^m)(\mathbf{R} + \alpha\mathbf{I})^{-1} \mathbf{A}^\top. \tag{1}$$

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Since $\mathbf{G}_m$ is a linear filter, the BER for user k at stage m can be calculated using the same techniques as for the conventional detector. Based on (1), it is clear that the PIC scheme converges to $(\mathbf{R} + \alpha\mathbf{I})^{-1}\mathbf{A}^\top$ if $(\mathbf{I} - \mu(\mathbf{R} + \alpha\mathbf{I}))^m \to 0$ for $m \to \infty$. If $\alpha = 0$, the steady-state solution is the decorrelating filter, while for $\alpha = \sigma^2$, we get the MMSE filter. Through a similarity transformation, $(\mathbf{I} - \mu(\mathbf{R} + \alpha\mathbf{I}))^m$ can be expressed as

$$(\mathbf{I} - \mu(\mathbf{R} + \alpha\mathbf{I}))^m = \mathbf{U}(I - \mu(\mathbf{\Lambda} + \alpha\mathbf{I}))^m \mathbf{U}^\top,$$

where $\mathbf{U}$ is a matrix of eigenvectors of $\mathbf{R}$ and $\mathbf{\Lambda}$ is a diagonal matrix of the corresponding eigenvalues. We can therefore guarantee convergence when

$$-1 < 1 - \mu(\lambda_k + \alpha) < 1 \quad \Rightarrow \quad 0 < \mu(\lambda_k + \alpha) < 2 \quad \text{for all } k.$$

The conventional PIC will therefore only converge if $\lambda_{\max} < 2$, which accounts for the somewhat unstable behaviour of the PIC. We can ensure that the PIC will always converge by selecting $\mu < 2/(\lambda_{\max} + \alpha)$.

It is possible to select the weighting factor to minimise the mean squared error (MSE) for an arbitrary number of stages. Based on (1), the MSE is determined by

$$\begin{aligned} J(\mu) &= \sum_{k=1}^{K} \frac{\sigma^2}{\lambda_k + \sigma^2} + \sum_{k=1}^{K} \frac{\lambda_k(\lambda_k + \sigma^2)}{(\lambda_k + \alpha)^2} \\ &\quad \times \left[\frac{\sigma^2 - \alpha}{\lambda_k + \sigma^2} - (1 - \mu(\lambda_k + \alpha))^m \right]^2 \\ &= J_{\mathrm{MMSE}} + J_{\mathrm{ex}}(\mu), \end{aligned}$$

where J_{MMSE} denotes the MMSE and J_{ex} is the excess MSE due to the limited number of stages in the PIC. To find the global minimum, we equate the derivative with respect to μ to zero,

$$\begin{aligned} \frac{\mathrm{d}J_\mu(\mu)}{\mathrm{d}\mu} &= \sum_{k=1}^{K} 2m \frac{\lambda_k(\lambda_k + \sigma^2)}{(\lambda_k + \alpha)} \left[\frac{\sigma^2 - \alpha}{\lambda_k + \sigma^2} \right. \\ &\quad \left. - (1 - \mu(\lambda_k + \alpha))^m \right] (1 - \mu(\lambda_k + \alpha))^{(m-1)} = 0. \end{aligned} \tag{2}$$

This equation has at least one real solution. For $\alpha = \sigma^2$, it has exactly one real solution which can be shown by considering the second derivative,

$$\begin{aligned} \frac{\mathrm{d}^2 J_\mu(\mu)}{\mathrm{d}\mu^2} &= \sum_{k=1}^{K} 2m(2m-1)\lambda_k(\lambda_k + \sigma^2) \\ &\quad \times (1 - \mu(\lambda_k + \sigma^2))^{2(m-1)} > 0. \end{aligned}$$

We observe that for $\alpha = \sigma^2$, the second derivative is always positive and therefore, Eqn. (2) has only one real solution.

References

[1] D. Divsalar, M. Simon and D. Raphaeli, "Improved parallel interference cancellation for CDMA," *IEEE Trans. Commun.*, vol. 46, pp. 258–268, Feb. 1998.