EECS 395/495: Algorithmic Mechanism Design	Final Exam
Assigned: 12/05/11	**Due: 12/07/11, 4pm**

This is a take-home exam and is open-book and open-notes: you may use your textbook and your notes from class; you may not consult any other sources, including the Internet, the library, or your colleagues. *Answer three of the following four questions.*

1. Consider surplus maximization in a single-item environment with two agents. Agent 1 has $v_1 \sim U[0,1]$ and Agent 2 has $v_2 \sim U[0,2]$.

   (a) Show that the first-price auction does not maximize social surplus.

   (b) Give a surplus-maximizing mechanism with first-price semantics, i.e., (a) agents submit bids, (b) the auction selects a winner, and (c) the winner pays her bid. In particular, specify explicitly how to determine the winner from the bids in step (b). Prove your mechanism is correct.

   (c) Generalize your mechanism above to the case where there are n agents and agent i's value is $U[0,i]$.

2. Consider a special case of the n agent m item single-minded combinatorial auction where each agent i has private value v_i for (publicly known) pair of items, i.e., $S_i \subset \{1,\ldots,m\}$ with $|S_i| = 2$ (She must receive both items in S_i to obtain value v_i). The agents' values are drawn from product distribution $\mathbf{F} = (F_1 \times \cdots \times F_n)$. Consider the design of posted pricing mechanisms for maximizing social surplus.

   (a) Show that there exists agent-specific prices $\mathbf{p} = (p_1,\ldots,p_n)$ such that when agents arrive in any arbitrary order and are made a take-it-or-leave-it while-supplies-last offer, the expected social surplus obtained is a constant approximation to the optimal social surplus. I.e., at the point agent i arrives, if both the items in her demand set S_i are still available, then she is permitted to buy them at price p_i; of course, she buys if and only if $v_i \geq p_i$. Explicitly state the constant your approximation obtains.

   (b) Computing the prices described above may be challenging. Suppose you have an ad hoc algorithm for finding a feasible subset of agents and when values are drawn from the given distribution the algorithm performs pretty well. Give a method for converting this algorithm into a posted pricing that approximately preserves the algorithms performance. I.e., there exists a constant β such that your posted pricing's social surplus is at least a β-fraction of the expected social surplus of the ad hoc algorithm.

3. Consider an exchange between a buyer and seller where the buyer's value and seller's cost are drawn from distributions F_b and F_s, respectively. Consider a mechanism that is brokering the trade between the two agents. The objective of the mechanism, as the broker, is to maximize revenue, i.e., the expected difference between the payment made by the buyer and the payment received by the seller.

   (a) For $F_b = F_s = U[0, 1]$, describe the optimal dominant-strategy incentive-compatible (DSIC) mechanism. What is its expected revenue?

   (b) The DSIC mechanism is rarely implemented in practice. The following protocol is more prevalent: The broker publishes a fee structure $B : \mathbb{R}_+ \to \mathbb{R}_+$, the seller post a price p, the buyer accepts the price (and buys the item and pays the seller the asked price) or not, and the seller pays the broker the commission $B(p)$ if the buyer buys. For instance, in real estate markets $B(p) = .06p$ (i.e., 6%).

   For $F_b = F_s = U[0, 1]$, find the optimal commission structure. What is your mechanism's expected revenue?

   (c) Suppose that the buyer's distribution is monotone hazard rate and the seller's distribution is arbitrary. Either give a counter example or show that a constant commission structure $B(p) = B_0$ gives a constant approximation to the optimal DSIC mechanism.

4. Consider digital goods with externalities. E.g., cultural goods such as books, movies, music, and TV have positive externalities, the more people who consume the good the more valuable its consumption is. E.g., collector art such as photography has a negative externality, the more copies of the piece of art in production the less valuable each copy is. One model of externalities that is consistent with our single-dimensional agent model is the following. The externality is described by public weights $\mathbf{w} = (w_1, \ldots, w_n)$. Agent i has a private value v_i. If k copies of the good are sold then agent i's value for a copy is $v_i w_k$.

   (a) Propose a reasonable benchmark against which to evaluate the performance of prior-free auctions for digital goods with externalities. Give a formal justification for your benchmark.

   (b) Negative externalities correspond to decreasing weights $w_k \geq w_{k+1}$. Give a prior-free auction that approximates your benchmark under negative externalities.

   (c) Positive externalities correspond to increasing weights $w_k \leq w_{k+1}$. A natural assumption for positive externalities is "diminishing returns," i.e., for all k, $w_k - w_{k-1} \geq w_{k+1} - w_k$. Give a prior-free auction that approximates your benchmark under positive externalities with diminishing returns.