Table 7.1 Noise Performance Characteristics

System	Postdetection SNR	Transmission bandwidth
Baseband	$\frac{P_T}{N_0 W}$	W
DSB with coherent demodulation	$\frac{P_T}{N_0 W}$	$2W$
SSB with coherent demodulation	$\frac{P_T}{N_0 W}$	W
AM with envelope detection (above threshold) or AM with coherent demodulation. *Note*: E is efficiency	$\frac{EP_T}{N_0 W}$	$2W$
AM with square-law detection	$2\left(\frac{a^2}{2+a^2}\right)^2 \frac{P_T/N_0 W}{1+(N_0 W/P_T)}$	$2W$
PM above threshold	$k_p^2 \overline{m_n^2} \frac{P_T}{N_0 W}$	$2(D+1)W$
FM above threshold (without preemphasis)	$3D^2 \overline{m_n^2} \frac{P_T}{N_0 W}$	$2(D+1)W$
FM above threshold (with preemphasis)	$\left(\frac{f_d}{f_3}\right)^2 \overline{m_n^2} \frac{P_T}{N_0 W}$	$2(D+1)W$

11. The use of pre-emphasis and de-emphasis can significantly improve the noise performance of an FM system. Typical values result in a better than 10-dB improvement in the SNR of the demodulated output.
12. As the input SNR of an FM system is reduced, spike noise appears. The spikes are due to origin encirclements of the total noise phasor. The area of the spikes is constant at 2π, and the power spectral density is proportional to the spike frequency. Since the predetection bandwidth must be increased as the modulation index is increased, resulting in a decreased predetection SNR, the threshold value of $P_T/N_0 W$ increases as the modulation index increases.
13. An analysis of PCM, which is a nonlinear modulation process due to quantizing, shows that, like FM, a trade-off exists between bandwidth and output SNR. PCM system performance above threshold is dominated by the wordlength or, equivalently, the quantizing error. PCM performance below threshold is dominated by channel noise.
14. A most important result for this chapter is the postdetection SNRs for various modulation methods. A summary of these results is given in Table 7.1. Given in this table is the postdetection SNR for each technique as well as the required transmission bandwidth. The trade-off between postdetection SNR and transmission bandwidth is evident for nonlinear systems.

Further Reading

All the books cited at the end of Chapter 3 contain material about noise effects in the systems studied in this chapter. The books by Lathi (1998) and Haykin (2000) are especially recommended for their completeness. The book by Taub and Schilling (1986), although an older book, contains excellent sections on both PCM systems and threshold effects in FM systems. The book by Tranter et al. (2004) discusses quantizing in some depth.

Problems

Section 7.1

7.1. In discussing thermal noise at the beginning of this chapter, we stated that at standard temperature (290 K) the white noise assumption is valid to bandwidths exceeding 1000 GHz. If the temperature is reduced to 3 K, the variance of the noise is reduced, but the bandwidth over which the white noise assumption is valid is reduced to approximately 10 GHz. Express both of these reference temperatures (3 and 290 K) in degrees fahrenheit.

7.2. The waveform at the input of a baseband system has signal power P_T and white noise with single-sided power spectral density N_0. The signal bandwidth is W. In order to pass the signal without significant distortion, we assume that the input waveform is bandlimited to a bandwidth $B = 3W$ using a Butterworth filter with order n. Compute the SNR at the filter output for $n = 1, 3, 5$, and 10 as a function of $P_T/N_0 W$. Also compute the SNR for the case in which $n \to \infty$. Discuss the results.

7.3. Derive the equation for $y_D(t)$ for an SSB system assuming that the noise is expanded about the frequency $f_x = f_c \pm \frac{1}{2}W$. Derive the detection gain and $(\text{SNR})_D$. Determine and plot $S_{n_c}(f)$ and $S_{n_s}(f)$.

7.4. Derive an expression for the detection gain of a DSB system for the case in which the bandwidth of the bandpass predetection filter is B_T and the bandwidth of the lowpass postdetection filter is B_D. Let $B_T > 2W$ and let $B_D > W$ simultaneously, where W is the bandwidth of the modulation. (There are two reasonable cases to consider.) Repeat for an AM signal.

7.5. A message signal is defined by

$$m(t) = A\cos(2\pi f_1 t + \theta_1) + B\cos(2\pi f_2 t + \theta_2)$$

where A and B are constants, $f_1 \neq f_2$, and θ_1 and θ_2 are random phases uniformly distributed in $[0, 2\pi)$. Compute $\widehat{m}(t)$ and show that the power in $m(t)$ and $\widehat{m}(t)$ are equal. Compute $E[m(t)\widehat{m}(t)]$, where $E[\cdot]$ denotes statistical expectation. Comment on the results.

7.6. In Section 7.1.3 we expanded the noise component about f_c. We observed, however, that the noise components for SSB could be expanded about $f_c \pm \frac{1}{2}W$, depending on the choice of sidebands. Plot the power spectral density for each of these two cases and for each case write the expressions corresponding to (7.16) and (7.17).

7.7. A message signal has the Fourier transform

$$M(f) = \begin{cases} A, & f_1 \leq |f| \leq f_2 \\ 0, & \text{otherwise} \end{cases}$$

Determine $m(t)$ and $\widehat{m}(t)$. Plot $m(t)$ and $\widehat{m}(t)$, and for f_2 fixed and $f_1 = 0, f_1 = -f_2/2$ and $f_1 = -f_2$. Comment on the results.

7.8. Assume that an AM system operates with an index of 0.6 and that the message signal is $12\cos(8\pi)$. Compute the efficiency, the detection gain in dB, and the output SNR in decibels relative to the baseband performance $P_T/N_0 W$. Determine the improvement (in decibels) in the output SNR that results if the modulation index is increased from 0.6 to 0.9.

7.9. An AM system has a message signal that has a zero-mean Gaussian amplitude distribution. The peak value of $m(t)$ is taken as that value that $|m(t)|$ exceeds 0.5% of the time. If the index is 0.7, what is the detection gain?

7.10. The threshold level for an envelope detector is sometimes defined as that value of $(\text{SNR})_T$ for which $A_c > r_n$ with probability 0.99. Assuming that $a^2\overline{m_n^2} \cong 1$, derive the SNR at threshold, expressed in decibels.

7.11. An envelope detector operates above threshold. The modulating signal is a sinusoid. Plot $(\text{SNR})_D$ in decibels as a function of $P_T/N_0 W$ for the modulation index equal to 0.4, 0.5, 0.7, and 0.9.

7.12. A square-law demodulator for AM is illustrated in Figure 7.20. Assuming that $x_c(t) = A_c[1 + am_n(t)]\cos(2\pi f_c t)$ and $m(t) = \cos(2\pi f_m t) + \cos(4\pi f_m t)$, sketch the spectrum of each term that appears in $y_D(t)$. Do not neglect the noise that is assumed to be bandlimited white noise with bandwidth $2W$. In the spectral plot identify the desired component, the signal-induced distortion, and the noise.

7.13. Verify the correctness of (7.59).

7.14. Starting with (7.63) derive an expression for $(\text{SNR})_D$ assuming that the message is the sinusoid $m(t) = A\sin(2\pi f_m t)$. From this result verify the correct-

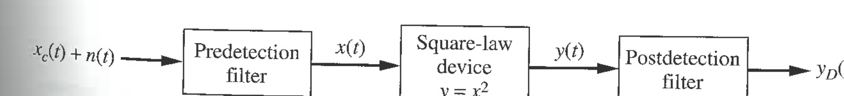

Figure 7.20

ASK:

$$s_1(t) = 0, \quad nt_0 \le t \le nt_0 + T$$
$$s_2(t) = A\cos(\omega_c t), \quad nt_0 \le t \le nt_0 + T$$

FSK:

$$s_1(t) = A\cos(\omega_c t), \quad nt_0 \le t \le nt_0 + T$$
$$s_2(t) = A\cos(\omega_c + \Delta\omega)t, \quad nt_0 \le t \le nt_0 + T$$

If $\Delta\omega = 2\pi l/T$ for FSK, where l is an integer, it is an example of an orthogonal signaling technique. If $m = 0$ for PSK, it is an example of an antipodal signaling scheme. A value of E_b/N_0 of approximately 10.53 dB is required to achieve an error probability of 10^{-6} for PSK with $m = 0$; 3 dB more than this is required to achieve the same error probability for ASK and FSK.

6. Examples of signaling schemes not requiring coherent carrier references at the receiver are DPSK and noncoherent FSK. Using ideal minimum-error-probability receivers, DPSK yields the error probability

$$P_E = \frac{1}{2}\exp\left(\frac{-E_b}{N_0}\right)$$

while noncoherent FSK gives the error probability

$$P_E = \frac{1}{2}\exp\left(\frac{-E_b}{2N_0}\right)$$

Noncoherent ASK is another possible signaling scheme with about the same error probability performance as noncoherent FSK.

7. In general, if a sequence of signals is transmitted through a bandlimited channel, adjacent signal pulses are smeared into each other by the transient response of the channel. Such interference between signals is termed *intersymbol interference* (ISI). By appropriately choosing transmitting and receiving filters, it is possible to signal through bandlimited channels while eliminating ISI. This signaling technique was examined by using Nyquist's pulse-shaping criterion and Schwarz's inequality. A useful family of pulse shapes for this type of signaling are those having raised cosine spectra.
8. One form of channel distortion is multipath interference. The effect of a simple two-ray multipath channel on binary data transmission was examined. Half of the time the received signal pulses interfere destructively, and the rest of the time they interfere constructively. The interference can be separated into ISI of the signaling pulses and cancelation due to the carriers of the direct and multipath components arriving out of phase.
9. Fading results from channel variations caused by propagation conditions. One of these conditions is multipath if the differential delay is short compared with the bit period but encompassing of many wavelengths. A commonly used model for a fading channel is one where the envelope of the received signal has a Rayleigh pdf. In this case, the signal power or energy has an exponential pdf, and the probability of error can be found by using the previously obtained error probability expressions for nonfading channels and averaging over the signal energy with respect to the assumed exponential pdf of the energy. Figure 8.30 compares the error probability for fading and nonfading cases for various modulation schemes. Fading results in severe degradation of the performance of a given modulation scheme. A way to combat fading is to use diversity.
10. Equalization can be used to remove a large part of the ISI introduced by channel filtering. Two techniques were briefly examined: zero-forcing and MMSE. Both can be realized by tapped delay-line filters. In the former technique, zero ISI is forced at sampling instants separated by multiples of a symbol period. If the tapped delay line is of length $(2N+1)$, then N zeros can be forced on either side of the desired pulse. In a MMSE equalizer, the tap weights are sought that give MMSE between the desired output from the equalizer and the actual output. The resulting weights for either case can be precalculated and preset, or adaptive circuitry can be implemented to automatically adjust the weights. The latter technique can make use of a training sequence periodically sent through the channel, or it can make use of the received data itself in order to carry out the minimizing adjustment.

Further Reading

A number of the books listed in Chapter 3 have chapters covering digital communications at roughly the same level as this chapter. For an authorative reference on digital communications, see Proakis (2007).

Problems

Section 8.1

8.1. A baseband digital transmission system that sends $\pm A$-valued rectangular pulses through a channel at a rate of 10,000 bps is to achieve an error probability of 10^{-6}. If the noise power spectral density is $N_0 = 10^{-7}$ W/Hz, what is the required value of A? What is a rough estimate of the bandwidth required?

8.2. Consider an antipodal baseband digital transmission system with a noise level of $N_0 = 10^{-5}$ W/Hz. The signal bandwidth is defined to be that required to pass the main lobe of the signal spectrum. Fill in the following table with the required signal power and bandwidth to achieve the error-probability and data rate combinations given.

Required Signal Powers A^2 and Bandwidth

R(bps)	$P_E = 10^{-3}$	$P_E = 10^{-4}$	$P_E = 10^{-5}$	$P_E = 10^{-6}$
1000				
10,000				
100,000				

8.3. Suppose $N_0 = 10^{-6}$ W/Hz and the baseband data bandwidth is given by $B = R = 1/T$ Hz. For the following bandwidths, find the required signal powers, A^2, to give a bit error probability of 10^{-5} along with the allowed data rates: (a) 5 kHz, (b) 10 kHz, (c) 100 kHz, (d) 1 MHz.