6.14. A random signal has the autocorrelation function

$$R(\tau) = 9 + 3\Lambda(\tau/5)$$

where $\Lambda(x)$ is the unit-area triangular function defined in Chapter 2. Determine the following:

a. The AC power.

b. The DC power.

c. The total power.

d. The power spectral density. Sketch it and label carefully.

6.15. A random process is defined as $Y(t) = X(t) + X(t-T)$, where $X(t)$ is a wide-sense stationary random process with autocorrelation function $R_X(\tau)$ and power spectral density $S_X(f)$.

a. Show that $R_Y(\tau) = 2R_X(\tau) + R_X(\tau + T) + R_X(\tau - T)$.

b. Show that $S_Y(f) = 4S_X(f)\cos^2(\pi f T)$.

c. If $X(t)$ has autocorrelation function $R_X(\tau) = 5\Lambda(\tau)$, where $\Lambda(\tau)$ is the unit-area triangular function, and $T = 0.5$, find and sketch the power spectral density of $Y(t)$ as defined in the problem statement.

6.16. The power spectral density of a wide-sense stationary random process is given by

$$S_X(f) = 10\delta(f) + 25\,\mathrm{sinc}^2(5f) + 5\delta(f-10) + 5\delta(f+10)$$

a. Sketch and fully dimension this power spectral density function.

b. Find the power in the DC component of the random process.

c. Find the total power.

d. Given that the area under the main lobe of the sinc-squared function is approximately 0.9 of the total area, which is unity if it has unity amplitude, find the fraction of the total power contained in this process for frequencies between 0 and 0.2 Hz.

6.17. Given the following functions of τ,

$$R_{X_1}(\tau) = 4\exp(-\alpha|\tau|)\cos 2\pi\tau$$
$$R_{X_2}(\tau) = 2\exp(-\alpha|\tau|) + 4\cos 2\pi b\tau$$
$$R_{X_3}(f) = 5\exp(-4\tau^2)$$

a. Sketch each function and fully dimension.

b. Find the Fourier transforms of each and sketch. With the information of part (a) and the Fourier transforms justify that each is suitable for an autocorrelation function.

c. Determine the value of the DC power, if any, for each one.

d. Determine the total power for each.

e. Determine the frequency of the periodic component, if any, for each.

Section 6.4

6.18. A stationary random process $n(t)$ has a power spectral density of 10^{-6} W/Hz, $-\infty < f < \infty$. It is passed through an ideal lowpass filter with frequency-response function $H(f) = \Pi(f/500\,\text{kHz})$, where $\Pi(x)$ is the unit-area pulse function defined in Chapter 2.

a. Find and sketch the power spectral density of the output?

b. Obtain sketch the autocorrelation function of the output.

c. What is the power of the output process? Find it two different ways.

6.19. An ideal finite-time integrator is characterized by the input-output relationship

$$Y(t) = \frac{1}{T}\int_{t-T}^{t} X(\alpha)\,d\alpha$$

a. Justify that its impulse response is $h(t) = \frac{1}{T}[u(t) - u(t-T)]$.

b. Obtain its frequency response function. Sketch it.

c. The input is white noise with two-sided power spectral density $N_0/2$. Find the power spectral density of the output of the filter.

d. Show that the autocorrelation function of the output is

$$R_0(\tau) = \frac{N_0}{2T}\Lambda(\tau/T)$$

where $\Lambda(x)$ is the unit-area triangular function defined in Chapter 2.

e. What is the equivalent noise bandwidth of the integrator?

f. Show that the result for the output noise power obtained using the equivalent noise bandwidth found in part (e) coincides with the result found from the autocorrelation function of the output found in part (d).

6.20. White noise with two-sided power spectral density $N_0/2$ drives a second-order Butterworth filter with frequency-response function magnitude

$$|H_{2bu}(f)| = \frac{1}{\sqrt{1+(f/f_3)^4}}$$

where f_3 is its 3-dB cutoff frequency.

a. What is the power spectral density of the filter's output?

b. Show that the autocorrelation function of the output is

$$R_0(r) = \frac{\pi f_3 N_0}{2} \exp\left(-\sqrt{2}\pi f_3|\tau|\right)\cos\left(\sqrt{2}\pi f_3|\tau| - \pi/4\right)$$

Plot as a function of $f_3\tau$. *Hint*: Use the integral given below:

$$\int_0^{\infty} \frac{\cos(ax)}{b^4+x^4}dx = \frac{\sqrt{2}\pi}{4b^3}\exp\left(-ab/\sqrt{2}\right)\times$$

$$\left[\cos\left(ab/\sqrt{2}\right)+\sin\left(ab/\sqrt{2}\right)\right], \quad a,b>0$$

c. Does the output power obtained by taking $\lim_{\tau\to 0} R_0(\tau)$ check with that calculated using the equivalent noise bandwidth for a Butterworth filter as given by (6.115)?

6.21. A power spectral density given by

$$S_Y(f) = \frac{f^2}{f^4+100}$$

is desired. A white-noise source of two-sided power spectral density 1 W/Hz is available. What is the frequency response function of the filter to be placed at the noise-source output to produce the desired power spectral density?

6.22. Obtain the autocorrelation functions and power spectral densities of the outputs of the following systems with the input autocorrelation functions or power spectral densities given.

a.

Transfer function

$$H(f) = \Pi(f/2B)$$

Autocorrelation function of input

$$R_X(\tau) = \frac{N_0}{2}\delta(\tau)$$

N_0 and B are positive constants.

b.

Impulse response

$$h(t) = A\exp(-\alpha t)u(t)$$

Power spectral density of input :

$$S_X(f) = \frac{B}{1+(2\pi\beta f)^2}$$

$A, \alpha, B,$ and β are positive constants.

6.23. The input to a lowpass filter with impulse response

$$h(t) = \exp(-10t)u(t)$$

is white, Gaussian noise with single-sided power spectral density of 2 W/Hz. Obtain the following:

a. The mean of the output

b. The power spectral density of the output

c. The autocorrelation function of the output

d. The probability density function of the output at an arbitrary time t_1

e. The joint probability density function of the output at times t_1 and $t_1 + 0.03$ s

6.24. Find the noise-equivalent bandwidths for the following first-and second-order lowpass filters in terms of their 3-dB bandwidths. Refer to Chapter 2 to determine the magnitudes of their transfer functions.

a. Chebyshev

b. Butterworth

6.25. A second-order Butterworth filter, has 3-dB bandwidth of 500 Hz. Determine the unit impulse response of the filter, and use it to compute the noise-equivalent bandwidth of the filter. Check your result against the appropriate special case of Example 6.9.

6.26. Determine the noise-equivalent bandwidths for the filters having transfer functions given below:

a. $H_a(f) = \Pi(f/4) + \Pi(f/2)$.

b. $H_b(f) = 2\Lambda(f/50)$.

c. $H_c(f) = 10/(10+j2\pi f)$.

d. $H_d(f) = \Pi(f/10) + \Lambda(f/5)$.

6.27. A filter has frequency-response function

$$H(f) = H_0(f-500) + H_0(f+500)$$

where

$$H_0(f) = 2\Lambda(f/100)$$

Find the noise-equivalent bandwidth of the filter.

6.28. Determine the noise-equivalent bandwidths of the systems having the following transfer functions.
Hint: Use the time-domain approach.

a. $H_a(f) = 10/[(j2\pi f + 2)(j2\pi f + 25)]$.

b. $H_b(f) = 100/(j2\pi f + 10)^2$.

Section 6.5

6.29. Noise $n(t)$ has the power spectral density shown in Figure 6.16. We write

$$n(t) = n_c(t)\cos(2\pi f_0 t + \theta) - n_s(t)\sin(2\pi f_0 t + \theta)$$

Make plots of the power spectral densities of $n_c(t)$ and $n_s(t)$ for the following cases:

a. $f_0 = f_1$.

b. $f_0 = f_2$.

c. $f_0 = \frac{1}{2}(f_2 + f_1)$.

d. For which of these cases are $n_c(t)$ and $n_s(t)$ uncorrelated?

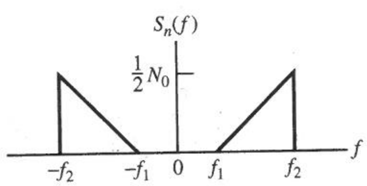

Figure 6.16

6.30.

a. If $S_n(f) = \alpha^2/(\alpha^2 + 4\pi^2 f^2)$, show that $R_n(\tau) = Ke^{-\alpha|\tau|}$. Find K.

b. Find $R_n(\tau)$ if

$$S_n(f) = \frac{\frac{1}{2}\alpha^2}{\alpha^2 + 4\pi^2(f - f_0)^2} + \frac{\frac{1}{2}\alpha^2}{\alpha^2 + 4\pi^2(f + f_0)^2}$$

c. if $n(t) = n_c(t)\cos(2\pi f_0 t + \theta) - n_s(t)\sin(2\pi f_0 t + \theta)$, find $S_{n_c}(f)$, and $S_{n_c n_s}(f)$, where $S_n(f)$ is as given in part (b). Sketch each spectral density.

6.31. The double-sided power spectral density of noise $n(t)$ is shown in Figure 6.17. If $n(t) = n_c(t)\cos(2\pi f_0 t + \theta) - n_s(t)\sin(2\pi f_0 t + \theta)$, find and plot $S_{n_c}(f)$, $S_{n_s}(f)$, and $S_{n_c n_s}(f)$ for the following cases:

a. $f_0 = \frac{1}{2}(f_1 + f_2)$.

b. $f_0 = f_1$.

c. $f_0 = f_2$.

d. Find $R_{n_c n_s}(\tau)$ for each case where $S_{n_c n_s}(f)$ is not zero. Plot.

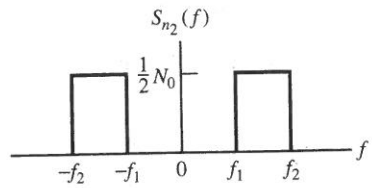

Figure 6.17

6.32. A noise waveform $n_1(t)$ has the bandlimited power spectral density shown in Figure 6.18. Find and plot the power spectral density of $n_2(t) = n_1(t)\cos(\omega_0 t + \theta) - n_1(t)\sin(\omega_0 t + \theta)$, where θ is a uniformly distributed random variable in $[0, 2\pi)$.

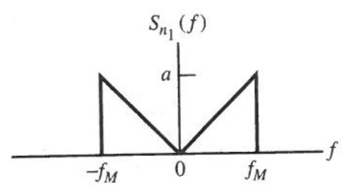

Figure 6.18

Section 6.5
Problems Extending Text Material

6.33. Consider a signal-plus-noise process of the form

$$z(t) = A\cos[2\pi(f_0 + f_d)t] + n(t)$$

with

$$n(t) = n_c(t)\cos(2\pi f_0 t) - n_s(t)\sin(2\pi f_0 t)$$

an ideal bandlimited Gaussian white-noise process with double-sided power spectral density equal to $N_0/2$ for $f_0 - B/2 \le |f| \le f_0 + B/2$ and zero otherwise. Write $z(t)$ as

$$z(t) = A\cos[2\pi(f_0 + f_d)t] + n_c'(t)\cos[2\pi(f_0 + f_d)t] - n_s'(t)\sin[2\pi(f_0 + f_d)t]$$

a. Express $n_c'(t)$ and $n_s'(t)$ in terms of $n_c(t)$ and $n_s(t)$. Using the techniques developed in Section 6.5, find the power spectral densities of $n_c'(t)$ and $n_s'(t)$, $S_{n_c'}(t)$ and $S_{n_s'}(f)$, respectively.

7.21. An FM demodulator operates above threshold, and therefore the output SNR is defined by (7.118). Using Carson's rule, write this expression in terms of B_T/W, as was done in (7.119). Plot $(\text{SNR})_T$ in decibels as a function of B_T/W with P_T/N_0W fixed at 30 dB. Determine the value of B_T/W that yields a value of $(\text{SNR})_T$ that is within 0.5 dB of the asymptotic value defined by (7.119).

7.22. The process of stereophonic broadcasting was illustrated in Chapter 3. By comparing the noise power in the $l(t)-r(t)$ channel to the noise power in the $l(t)+r(t)$ channel, explain why stereophonic broadcasting is more sensitive to noise than nonstereophonic broadcasting.

7.23. An FDM communication system uses DSB modulation to form the baseband and FM modulation for transmission of the baseband. Assume that there are eight channels and that all eight message signals have equal power P_0 and equal bandwidth W. One channel does *not* use subcarrier modulation. The other channels use subcarriers of the form

$$A_k \cos(2\pi k f_1 t), \quad 1 \le k \le 7$$

The width of the guardbands is $3W$. Sketch the power spectrum of the received *baseband* signal showing both the signal and noise components. Calculate the relationship between the values of A_k if the channels are to have equal SNRs.

7.24. Using (7.123), derive an expression for the ratio of the noise power in $y_D(t)$ with de-emphasis to the noise power in $y_D(t)$ without de-emphasis. Plot this ratio as a function of W/f_3. Evaluate the ratio for the standard values of $f_3 = 2.1$ kHz and $W = 15$ kHz, and use the result to determine the improvement, in decibels, that results through the use of de-emphasis. Compare the result with that found in Example 7.3.

7.25. White noise with two-sided power spectral density $\frac{1}{2}N_0$ is added to a signal having the power spectral density shown in Figure 7.22. The sum (signal plus noise) is filtered with an ideal lowpass filter with unity passband gain and bandwidth $B > W$. Determine the SNR at the filter output. By what factor will the SNR increase if B is reduced to W?

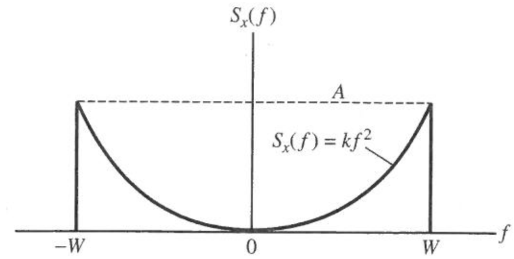

Figure 7.22

7.26. Consider the system shown in Figure 7.23. The signal $x(t)$ is defined by

$$x(t) = A\cos(2\pi f_c t)$$

The lowpass filter has unity gain in the passband and bandwidth W, where $f_c < W$. The noise $n(t)$ is white with two-sided power spectral density $\frac{1}{2}N_0$. The signal component of $y(t)$ is defined to be the component at frequency f_c. Determine the SNR of $y(t)$.

7.27. Repeat the preceding problem for the system shown in Figure 7.24.

7.28. Consider the system shown in Figure 7.25. The noise is white with two-sided power spectral density $\frac{1}{2}N_0$. The power spectral density of the signal is

$$S_x(f) = \frac{A}{1+(f/f_3)^2}, \qquad -\infty < f < \infty$$

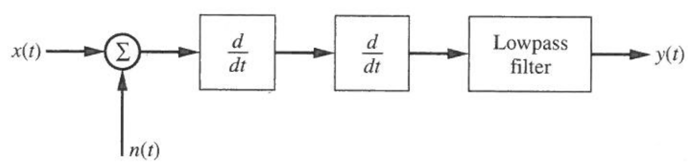

Figure 7.23

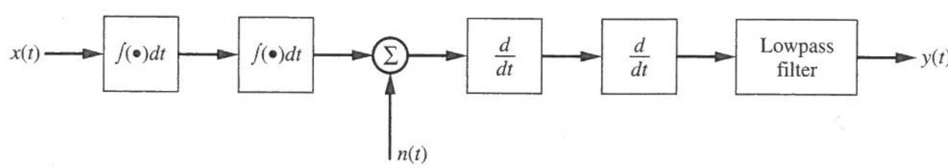

Figure 7.24