HW 10 Solutions

Problem 7.4

For DSB, the received signal and noise are given by

$$x_r(t) = A_c m(t)\cos(2\pi f_c t + \theta) + n(t)$$

At the output of the predetection filter, the signal and noise powers are given by

$$S_T = \frac{1}{2}A_c^2\overline{m^2} \qquad N_T = \overline{n^2} = N_0 B_T$$

The predetection SNR is

$$(\text{SNR})_T = \frac{A_c^2\overline{m^2}}{2N_0 B_T}$$

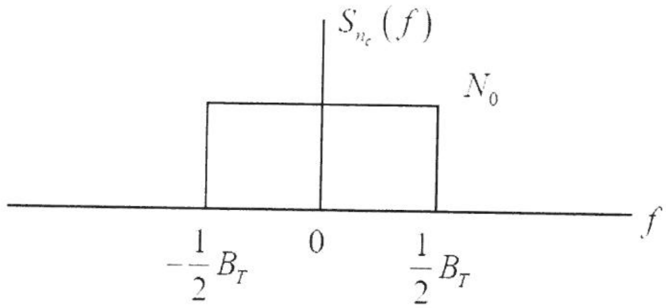

Figure 7.3: Plot of the spectrum of B_T for Problem 7.4.

If the postdetection filter passes all of the $n_c(t)$ component, $y_D(t)$ is

$$y_D(t) = A_c m(t) + n_c(t)$$

The output signal power is $A_c^2\overline{m^2}$ and the output noise PSD is shown in Figure 7.3.

Case I: $\quad B_D > \frac{1}{2}B_T$

For this case, all of the noise, $n_c(t)$, is passed by the postdetection filter, and the output noise power is

$$N_D = \int_{-\frac{1}{2}B_T}^{\frac{1}{2}B_T} N_0 df = 2N_0 B_T$$

This yields the detection gain

$$\frac{(\text{SNR})_D}{(\text{SNR})_T} = \frac{A_c^2\overline{m^2}/N_0 B_T}{A_c^2\overline{m^2}/2N_0 B_T} = 2$$

Case II: $B_D < \frac{1}{2} B_T$

For this case, the postdetection filter limits the output noise and the output noise power is

$$N_D = \int_{-B_D}^{B_D} N_0 df = 2N_0 B_D$$

This case gives the detection gain

$$\frac{(\text{SNR})_D}{(\text{SNR})_T} = \frac{A_c^2 \overline{m^2}/2N_0 B_D}{A_c^2 \overline{m^2}/2N_0 B_T} = \frac{B_T}{B_D}$$

Problem 7.8

The message signal is

$$m(t) = 12\cos(8\pi t)$$

Thus,

$$m_n(t) = \cos(8\pi t)$$

so that

$$\overline{m_n^2} = \frac{1}{2}$$

The efficiency is therefore given by

$$E_{ff} = \frac{(0.5)(0.6)^2}{1 + (0.5)(0.6)^2} = 0.1525 = 15.25\%$$

From (7.36), the detection gain is

$$\frac{(\text{SNR})_D}{(\text{SNR})_T} = 2E_{ff} = 0.305 = -5.157\,\text{dB}$$

Relative to baseband, the output SNR is

$$\frac{(\text{SNR})_D}{P_T/N_0 W} = -5.157\,\text{dB}$$

If the modulation index in increased to 0.9, the efficiency becomes

$$E_{ff} = \frac{(0.5)(0.9)^2}{1 + (0.5)(0.9)^2} = 0.2883 = 28.83\%$$

This gives a detection gain of

$$\frac{(\text{SNR})_D}{(\text{SNR})_T} = 2E_{ff} = 0.5765 = -2.39\,\text{dB}$$

which, relative to baseband, is

$$\frac{(\text{SNR})_D}{P_T/N_0 W} = -2.39\,\text{dB}$$

This represents an improvement of 2.76 dB.

Problem 7.10

The output of the predetection filter is

$$e(t) = A_c\left[1 + am_n(t)\right]\cos\left[\omega_c t + \theta\right] + r_n(t)\cos\left[\omega_c t + \theta + \phi_n(t)\right]$$

The noise function $r_n(t)$ has a Rayleigh *pdf*. Thus

$$f_{R_n}(r_n) = \frac{r}{N}e^{-r^2/2N}$$

where N is the predetection noise power. This gives

$$N = \frac{1}{2}\overline{n_c^2} + \frac{1}{2}\overline{n_s^2} = N_0 B_T$$

From the definition of threshold

$$0.99 = \int_0^{A_c}\frac{r}{N}e^{-r^2/2N}dr$$

which gives

$$0.99 = 1 - e^{-A_c^2/2N}$$

Thus,

$$-\frac{A_c^2}{2N} = \ln(0.01)$$

which gives

$$A_c^2 = 9.21\,N$$

The predetection signal power is

$$P_T = \frac{1}{2}A_c\left[1 + a^2\overline{m_n^2}\right] \approx \frac{1}{2}A_c^2\left[1+1\right] = A_c^2$$

which gives

$$\frac{P_T}{N} = \frac{A_c^2}{N} = 9.21 \approx 9.64\,dB$$

Problem 8.1
The signal-to-noise ratio is

$$z = \frac{A^2 T}{N_0} = \frac{A^2}{N_0 R}$$

Trial and error using the asymptotic expression $Q(x) \approx \exp(-x^2/2) / (\sqrt{2\pi}x)$ shows that

$$P_E = Q\left(\sqrt{2z}\right) = 10^{-5} \text{ for } z = 9.58 \text{ dB} = 9.078$$

Thus

$$\frac{A^2}{N_0 R} = 9.078$$

or

$$\begin{aligned} A &= \sqrt{9.078 N_0 R} \\ &= \sqrt{9.078 \times 10^{-7} \times 10000} \\ &= \sout{0.135\text{ V}} \\ & \quad 0.095\text{V} \end{aligned}$$