Utility-Based Resource Allocation for Wireless Networks with Mixed Voice and Data Services

Chi Zhou[a], Michael L. Honig[a], Scott Jordan[b], Randall Berry[a]

[a]Dept. of Electrical Engineering
Northwestern University
2145 Sheridan Road, Evanston, IL 60208
Phone: 847-467-5169, Fax: 847-467-3550
{czhou,mh,rberry}@ece.nwu.edu

[b]Dept. of Electrical Engineering
University of California, Irvine
544D Engg. Tower, Irvine, CA 92697
Phone: 949-824-2177, Fax: 949-824-3203
sjordan@uci.edu

***Abstract*—Power allocation across users in two adjacent cells is studied for a wireless Code Division Multiple Access (CDMA) network with mixed voice and data services. We assume that each user has a utility function that measures the user's satisfaction, or utility, as a function of the received Signal-to-Interference-plus-Noise-Ratio (SINR). Each particular service (voice or data) is associated with a different utility function. We consider the forward link. Our objective is to allocate transmitted power to maximize the total utility summed over all active users subject to rate and power constraints. We show that the maximum utility can be achieved with a pricing scheme. We characterize the solution to a one-cell utility maximization problem with fixed interference from the other cell. For two-cell utility maximization, the two cells must cooperate to achieve the maximum utility.**

I. Introduction

An objective for the next generation of wireless networks is to support a wide range of services, including voice and data. Although some work has been done in the area of resource allocation for either data or voice services alone, there has been little research done on multimedia wireless networks. In this paper, we set up a framework to allocate resources for a two-cell model with mixed voice and data services. The approach we take is based on maximization of the total utility. That is, we assume that each user has a utility function that measures the user's satisfaction, or utility, as a function of the received Quality of Service (QoS), represented in terms of received Signal-to-Interference-plus-Noise-Ratio (SINR) in wireless CDMA networks. Each service is mapped into a different utility function, e.g., a step function for voice service and an increasing concave function for data service.

Prior work on utility-based resource allocation in wireless networks has been concerned with a voice service only , as in [4], [6], [5], or a data service only, as in [1], [2], [3], [7]. The authors in [1], [2], [3] model a distributed power control scheme as a non-cooperative game, and show that pricing improves the outcome. References [4], [6], [5] show that pricing can be used to achieve the maximum utility. The most closely related work to the work presented here is [6], [5], [7], in which utility-based resource allocation is analyzed for the forward link of a two-cell wireless CDMA network.

We consider the forward link and model the cells as one-dimensional with uniformly distributed voice and data users throughout the cells and orthogonal signatures within each cell. The resource of interest is transmitted power, which determines the received data rate. We constrain the total data rate as well as power. Our objective is to find a power allocation for both voice users and data users, which maximizes the total utility per code over the two cells subject to rate and power constraints in each cell.

We show that the optimal power allocation for voice service is specified by a radius in each cell, as shown in [5], [6]. That is, there exists a radius of active users with respect to the desired base station, such that all voice users within this radius are active, and all voice users outside this radius are inactive. We further show that the maximum utility can be achieved via a pricing scheme, in which each base station announces a price per unit transmitted power and a price per unit data rate (corresponding to the received SINR); each user responds by requesting an amount of transmitted power which maximizes the user's surplus (utility minus cost),

To maximize one-cell utility with fixed interference from the other cell, the cell must exhaust the available rate or power, or both. Numerical results show how to partition resources between voice and data requests, and how the partition changes when varying the relative priorities for each type of service. For two-cell utility maximization, the numerical results suggest that generally the cells must cooperate to achieve the maximum utility.

II. System Model

We consider the forward link for two adjacent cells, which are interfering with each other. The resource of interest is transmitted power, which determines the assigned data rate.

Figure 1 illustrates the one-dimensional, two-cell model considered here. The cell radius is normalized to one, and d_0 is a close-in reference point in the far field region of the transmitter antenna. We make the following assumptions:

- The channel for each user is a scalar attenuation based on distance from the base station.
- All codes within a cell are orthogonal. A matched filter receiver is assumed, and codes from the adjacent cell are treated as random with i.i.d. elements.
- The system is static, i.e., the number of users in the system is fixed.

This work was supported by the Motorola-Northwestern Center for Telecommunications, and by NSF under grant CCR 9903055.

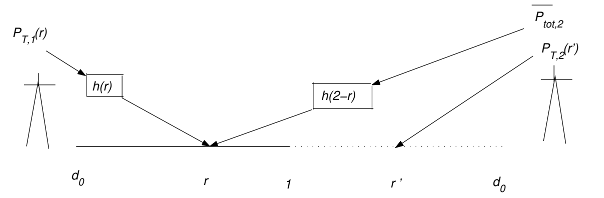


Fig. 1. One-dimensional two-cell system.

- There is a fixed transmission rate per code. A single user may be assigned multiple codes or may be assigned a fraction of a code (and therefore a fraction of the rate).

We also assume that the two cells are identical, and that both voice users and data users are uniformly distributed across the cells. These assumptions can be relaxed, and are made primarily to simplify our discussion. To facilitate the analysis, we evaluate the large system performance by letting the number of voice users K_v, the number of data users K_d, and the number of available codes per cell M each increase to infinity, with fixed offered load for voice service $L_v = K_v/M$, and fixed offered load for data service $L_d = K_d/M$.

Referring to Figure 1, consider a user in cell 1 located at distance r from base station 1. The received SINR, denoted as ξ, for this user is:

$$\xi = \frac{P_{T,1}(r)h(r)}{\sigma^2 + I} \tag{1}$$

where $P_{T,1}(r)$ is the transmitted power from base station 1 for the user at distance r, $h(r)$ is the attenuation modeled as a decreasing function of the separation distance r between the base station and the user, σ^2 is the noise level, and I is the received interference from the other cell. The intracell interference is zero since all codes in the cell are orthogonal.

We assume that the processing gain is equal to the number of codes, M. Let $P_{s,i}(r)$ $(s = v, d)$ be the power allocated to a voice or data user at distance r in cell i, respectively. The total transmitted power summed over all voice users or data users in cell i is

$$\overline{P}_{s,i} = \frac{L_s}{1-d_0}\int_{d_0}^{1} P_{s,i}(r)\,dr \qquad s = v, d\ . \tag{2}$$

The average transmitted power per code from cell 2 is $\overline{P}_{tot,2} = \overline{P}_{v,2} + \overline{P}_{d,2}$. The interference from cell 2 to a user at distance r from base station 1 is therefore

$$I(r, \overline{P}_{tot,2}) = \overline{P}_{tot,2}h(2-r)\ . \tag{3}$$

As r increases, both the attenuation of the desired signal and the interference increase, so that more transmitted power is required to achieve a specific SINR.

The utility function for voice traffic $U_v(\xi)$ can be characterized as a step function, rising from zero utility when the SINR is below some threshold to a fixed positive utility when the SINR meets or exceeds the threshold. For a data service, QoS typically depends on the data transmission rate. We assume that rate is proportional to the received SINR and the proportional factor is one so that the utility function $U_d(\xi)$ is increasing and concave with received SINR. That is, the utility increases with rate (SINR), and the increase in utility for one unit increase in rate decreases with rate. To simplify the discussion, all users within each service type (i.e., voice and data) are assumed to have the same utility function.

III. Problem Formulation and Pricing

We wish to find a power allocation for both voice users and data users, which maximizes the total utility per code over the two cells subject to rate and power constraints in each cell. Let $\overline{C}_{s,i}$ and $U_{s,i}$ $(s = v, d)$ be the total data rate and total utility per code summed over all voice or data users in cell i, respectively. We then have

$$\overline{C}_{s,i} = \frac{L_s}{1-d_0}\int_{d_0}^{1} \frac{P_{s,i}(r)}{A_i(r)}\,dr \qquad s = v, d \tag{4}$$

$$U_{s,i} = \frac{L_s}{1-d_0}\int_{d_0}^{1} U_s\left(\frac{P_{s,i}(r)}{A_i(r)}\right)dr \qquad s = v, d \tag{5}$$

where

$$A_i(r) = \frac{\sigma^2 + \overline{P}_{tot,j}h(2-r)}{h(r)} \qquad i \neq j \tag{6}$$

To minimize power consumption, each voice user operates at either zero SINR or the threshold SINR, $\xi = \xi_0$. That is, the voice user is either inactive with zero power allocated, or active with the amount of power required to achieve $\xi = \xi_0$. Let $P^*_{T,i}(r, \overline{P}_{tot,j})$ be the transmitted power needed to achieve the target SINR for a voice user in cell i at distance r, i.e.,

$$P^*_{T,i}(r, \overline{P}_{tot,j}) = \frac{\xi_0[\sigma^2 + \overline{P}_{tot,j}h(2-r)]}{h(r)} \qquad i \neq j \tag{7}$$

Therefore, each active voice user receives a constant data rate, corresponding to a target SINR, and generates utility u_v. Each inactive voice user receives zero data rate, and generates zero utility. It is shown in [5] that the optimal power allocation for voice users is defined by the radius of active users r^*_i, $i = 1, 2$. That is, users at distances $d_0 \le r \le r^*_i$ are active, and users at distances $r^*_i < r \le 1$ are inactive. Correspondingly, from (7), the transmitted power allocated to voice users at distance r increases with r for $d_0 \le r \le r^*_i$, and is zero for $r^*_i < r \le 1$.

The optimization problem is therefore to find a power allocation for data users and a radius of active users for voice users, which maximizes the total utility per code over the two cells subject to rate and power constraints in each cell,

$$\max_{\{P_{d,1}(r), r^*_1, P_{d,2}(r), r^*_2\}} U_{tot} = U_{tot,1} + U_{tot,2} \tag{8}$$

$$\text{subject to:} \quad \overline{C}_{tot,i} \le \mathcal{C} \quad i = 1, 2 \tag{9}$$

$$\overline{P}_{tot,i} \le \mathcal{P} \quad i = 1, 2 \tag{10}$$

where U_{tot} is the total utility per code summed over the two cells, $\mathcal{C}$ is the total available rate per cell, $\mathcal{P}$ is the total available power, and $U_{tot,i}$ and $\overline{C}_{tot,i}$ are total utility per code and total rate per code summed over all users (voice and data) in cell i, i.e., $U_{tot,i} = U_{v,i} + U_{d,i}$ and $\overline{C}_{tot,i} = \overline{C}_{v,i} + \overline{C}_{d,i}$.

The maximum utility can be achieved via the following pricing scheme:

- Base station i announces a price per unit transmitted power, $\alpha_{p,i}$, and a price per unit data rate (corresponding to the received SINR) $\alpha_{r,i}$, $i = 1, 2$.
- Each voice or data user in cell i requests the transmitted power which maximizes individual surplus (utility minus cost), $U(\xi) - \alpha_{p,i} P_{T,i}(r) - \alpha_{r,i} P_{T,i}(r)/A_i(r)$.

Theorem 1: There exist prices, $\alpha_{r,i}$ and $\alpha_{p,i}$, such that the preceding pricing scheme achieves the maximum utility.

The proof is based on the constrained optimization theory [9], and is omitted here. The prices are the shadow costs associated with the consumptions of resources. Therefore, the preceding optimization problem is equivalent to setting the prices $\alpha_{r,i}$ and $\alpha_{p,i}$ to maximize the utility subject to rate and power constraints in each cell. When the power constraint satisfies the equality in cell i, the optimal prices are $\alpha_{r,i} = 0$ and $\alpha_{p,i} > 0$. Correspondingly, the data rate received by a data user at distance r from the base station decreases with r. When the equality holds in the rate constraint in cell i , the optimal prices satisfy $\alpha_{r,1} > 0$ and $\alpha_{p,1} = 0$, which implies that every user in each service type (i.e., voice or data) receives the same data rate, and the power requested by users in each service type at distance r from the base station increases with r. When both the power and the rate constraints satisfy the equality, we have $\alpha_{r,1} > 0$ and $\alpha_{p,1} > 0$, and again data users farther away from the base station receive lower data rates.

IV. One-Cell Utility Maximization

Solutions to the preceding optimization problem can be classified according to which constraints are binding. Namely, with an optimal allocation of power, we identify the following three scenarios,

- Cell i is *rate-limited* when the rate constraint is satisfied with equality,
- Cell i is *power-limited* when all available transmitted power is allocated,
- Cell i is *rate-power-limited* when both the power and rate constraints are satisfied with equality.

One-cell utility maximization with fixed interference from the other cell is considered first. We can identify a feasible price region, that is, when the prices are chosen within this region, neither the rate nor power constraint is violated. As prices decrease, the demand for resources increases, and so does the received utility. Therefore, the maximum utility occurs on the boundary curve, along which at least one constraint is tight.

Theorem 2: The power allocation which maximizes utility for one cell in the presence of fixed interference, results in the cell being either rate-limited, power-limited, or rate-power-limited.

For the numerical results which follow, we assume that each data user has the exponential utility function, $U(\xi) = u_0\,[1 - \exp(-\xi/\lambda)]$, where u_0 is a constant, and λ determines the rate of increase. Figure 2 shows the feasible price region, total transmitted power, total data rate and corresponding total utility along the feasible price boundary. For voice users, the target SINR is $\xi_0 = 5\ dB$, the utility received for the target SINR is

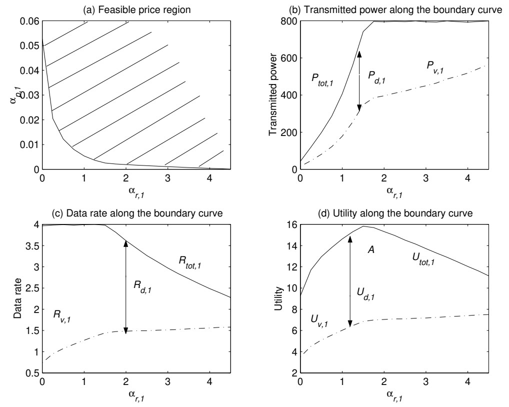


Fig. 2. Behavior of a rate-power-limited cell when utility is maximized

$u_v = 15$, and the load is $L_v = 0.5$. For data users, the exponential utility function is assumed with $u_0 = 30$ and $\lambda = 5$, and the load is $L_d = 0.5$. The pathloss $h(r) = (d_0/r)^4$, the reference point $d_0 = 0.1$, the noise level $\sigma^2 = 0.1$, the interference $\overline{P}_{tot,2} = 800$, the total available power $\mathcal{P} = 800$, and the total available rate $\mathcal{C} = 4$. The feasible price region is shown as the shaded area in Figure 2 (a). Along the boundary curve of the feasible price region, the price per transmitted power $\alpha_{p,1}$ decreases with $\alpha_{r,1}$. In Figure 2(b), (c), and (d), the solid curve represents the sum of voice and data services, and the dashed curve represents the voice service. The difference between the two curves therefore represents the data service. From Figure 2(b), the cell is power-limited along the boundary curve when $\alpha_{r,1} > 1.5$. Figure 2(c) shows that the cell is rate-limited along the boundary curve when $\alpha_{r,1} < 1.5$. The cell is rate-power-limited when $\alpha_{r,1} = 1.5$. From Figure 2 (d), we notice that the total utility first increases and then decreases along the boundary curve as $\alpha_{r,1}$ increases, and the maximum utility occurs at point A where the cell is rate-power-limited. We have $\alpha_{r,1} > 0$ and $\alpha_{p,1} > 0$ at the optimum, which is consistent with Theorem 1.

We can change the relative priorities of the voice and data services by varying the utility u_v voice users receive from the target SINR, while keeping all other parameters fixed. Figure 3 shows the amount of transmitted power, data rate and total utility received by voice and data users vs. u_v when the utility in cell 1 is maximized. The solid curve represents the voice service, and the dashed curve represents the data service. The system parameters are the same as in Figure 2. This shows that cell 1 is rate-power-limited when the power allocation maximizes the total utility. Therefore, for any u_v, all available resources (power and rate) are allocated to voice and data users. As u_v increases, voice users have more priority over data users, therefore voice users are allocated more resources (power and rate), and more voice users become active. Correspondingly, the amount of power and rate allocated to data users decreases with u_v. When $u_v = 40$, all voice users are active, therefore the allocated power and rate for voice users reach the maximum value, and stay constant for $u_v > 40$. The corresponding total utility summed over voice users increases monotonically with u_v. When $u_v < 40$, the increase in utility is due to the increase in the number of active voice users, and the increase in the utility each active voice user receives. When $u_v > 40$, the increase in utility is only due to the increase in u_v. The total utility summed over data users monotonically decreases with u_v for $u_v < 40$, and stays constant for $u_v > 40$, since in that case the resources allocated to data users do not change with u_v.

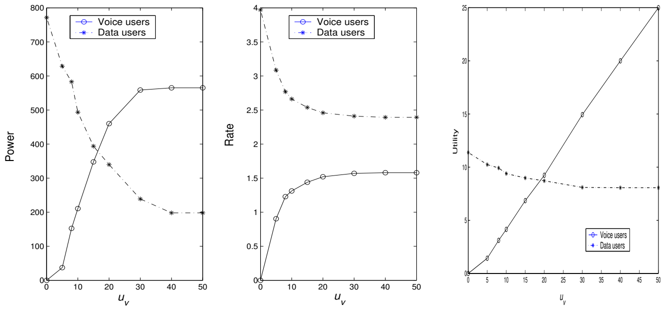


Fig. 3. Total transmitted power, data rate, and total utility over voice or data users vs. u_v.

V. Two-Cell Solution

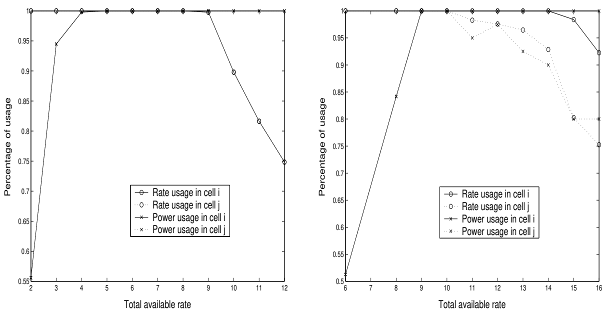
In this section we state some properties of the optimal two-cell power allocation. We first start with *distributed optimization*, which refers to the situation where each cell independently maximizes its own utility without coordinating with the other cell. The corresponding solution is the fixed point, or *Nash Equilibrium (NE)*, reached by the two cells [8]. Namely, at the NE cell 1 cannot increase its utility (or revenue) given the fixed interference from cell 2, and vice versa. From Theorem 2, at the NE when both cells are either rate-limited, power-limited, or rate-power-limited.

Theorem 3: If at the NE both cells are rate-limited, then the NE corresponds to the global optimum.

Proof: If there is no externality between the two cells, the total utility over the two cells is maximized when each cell's utility is maximized. When both cells are rate-limited at the NE, there is no externality at the NE. Therefore the maximum total utility is achieved at the NE. ■

Although it is difficult to characterize the solutions for arbitrary $\mathcal{C}$ and $\mathcal{P}$, numerical results give some insight. We generate numerical examples by fixing $\mathcal{P}$ at some value and varying the available rate $\mathcal{C}$. Figure 4 shows the percentage usage of transmitted power and rate for each cell when the total utility is maximized. In Figure 4 (a), the parameters are $\mathcal{P} = 800$, $h(r) = (d_0/r)^4$, and $\xi_0 = 5$ dB. For any given $\mathcal{C}$, each cell uses the same amount of resources at the optimum. Namely, the NE corresponds to the global optimum for any $\mathcal{C}$. When $\mathcal{C} \leq 3$, both cells are rate-limited when the total utility is maximized. When $4 \leq \mathcal{C} \leq 9$, both cells are rate-power-limited at the optimum. When $\mathcal{C}$ further increases, both cells become power-limited.

Figure 4 (b) shows the case where the NE may not be the global optimum. The parameters are $\mathcal{P} = 20000$, $h(r) = (d_0/r)^2$, and $\xi_0 = 15$ dB. When $\mathcal{C} \leq 8$, the NE is the global optimum, and both cells are rate-limited. When $9 \leq \mathcal{C} \leq 10$, both cells are rate-power-limited at the optimum, and again the NE is the global optimum. However, when $11 \leq \mathcal{C} \leq 14$, only one cell is rate-power-limited, and the other cell is neither rate-limited nor power-limited. When $\mathcal{C}$ further increases, one cell becomes power-limited, and the other cell is still neither rate-limited nor power-limited. Therefore, for $\mathcal{C} \geq 11$, the NE does not correspond to the global optimum, and the two cells must need to coordinate to maximize total utility.



(a) (b)

Fig. 4. Percentage of power and rate usage for each cell vs. total available rate

VI. Conclusions

We have studied utility-based forward-link power allocation across voice and data users in two one-dimensional, adjacent CDMA cells. It is shown that utility maximization can be achieved by a pricing scheme. When the utility is maximized in a cell with fixed interference, the cell is either rate-limited, power-limited, or rate-power-limited. Numerical results showed that when we increase the priority for voice users over data users, the voice users receive more resources, and therefore more utility. In general, the cells must cooperate to maximize total utility. Our results for mixed voice and data are preliminary, and more work can be done to characterize the optimal two-cell power allocation for both utility and revenue maximization.

References

[1] D. J. Goodman and N. B. Mandayam, "Power control for wireless data",in *IEEE Personal Communications*, vol. 7, pp. 48-54, April 2000.

[2] H. Ji and D. Huang, "Non-cooperative uplink power control in cellular radio systems", in *Wireless Networks*, (4): 233-240, 1998.

[3] C. U. Saraydar, N. Mandayam, and D. J. Goodman, "Pricing and power control in a multicell wireless data network", in *IEEE Journal on Selected Areas in Communications*, vol. 19, No. 10, pp. 1883-1892, October 2001.

[4] P. Liu, M. L. Honig, and S. Jordan, "Forward-Link Resource Allocation for DS-CDMA Based on Pricing", *Proc. Wireless Commun. and Networking Conf.*, Chicago, IL, Sept. 2000.

[5] C. Zhou, M. L. Honig, S. Jordan, and R. Berry, "Forward-Link Resource Allocation for a Two-Cell Voice Network with Multiple Service Classes," *submitted to WCNC 2003*.

[6] C. Zhou, M. L. Honig, and S. Jordan, "Two-Cell Utility-Based Resource Allocation for a CDMA Voice Service," *VTC Fall 2001*, Atlantic City, NJ, Oct. 2001.

[7] C. Zhou, M. L. Honig, and S. Jordan, "Two-Cell Power Allocation for Wireless Data Based on Pricing," *39th Annual Allerton Conference*, Monticello, IL, Oct. 2001.

[8] H. R. Varian, "Microeconomics Analysis", third edition. W. W. Norton and Company, Inc., 1992.

[9] D. P. Bertsekas, "Nonlinear Programming"", pp. 416-427, Athena Scientific, Belmont, MA, 1995.