Communication over Fading Channels with Finite Buffer Constraints - Single User and Multiple Access Cases

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Abstract — **We consider communication over a fading channel where arriving data is stored in a buffer of size L until it is transmitted. Any data arriving when the buffer is full is discarded. The transmitter utilizes a power and rate control policy that can depend on both the buffer occupancy as well as the channel state. The trade-off between minimizing transmission power and buffer overflow probability is analyzed in asymptotic regime as $L \to \infty$. Both single user and multi-access models are considered.**

I. Introduction

Fading channels in which the transmitter and receiver have some degree of Channel State Information (CSI) are common models for wireless communication systems. The transmitter can use the CSI to allocate communication resources over time, *e.g.* via the use of power and/or rate control. Such techniques can impact not only physical layer performance, but also higher layer issues such as buffer overflow and delay. In this work we discuss simple models which take into account both physical and higher layer performance issues.

Consider a single user model in which information bits arrive and are placed into a buffer of size L. Periodically bits are removed from this buffer, encoded and transmitted over a fading channel. We model the channel as a block fading, additive white Gaussian noise channel. Let H_m be the channel gain during the mth block of N channel uses. For the simplicity, assume that each codeword is sent over a single block but the code rate, *i.e.* the number of information symbols per block can vary. Let A_m be the number of bits that arrive at the start of the mth block, and let U_m be the number of bits encoded and transmitted over the channel during the mth block. Any information bits that arrive when the buffer is full are lost. Thus the buffer occupancy, S_m evolves according to:

$$S_{m+1} = \min\{S_n + A_{n+1} - U_n, L\}.$$

Assume that both $\{A_n\}$ and $\{H_n\}$ are independent sequences of i.i.d. random variables.

Let $P(h,u)$ be the transmission power required during a block when $H_m = h$ and $U_m = u$. We assume that $P(h,u)$ is the minimum power required so that the mutual information rate over the N channel uses is equal to u/N. When N is large, this choice will give a reasonable indication of the power needed to reliably transmit at rate u/N. Let $\mu : \mathcal{S} \times \mathcal{H} \mapsto \mathbb{R}^+$ be a stationary policy which specifies U_m as a function of S_m and H_m for all m. Under a policy μ, let $\bar{P}^\mu$ be the average transmission power, *i.e.*

$$\bar{P}^\mu = \lim_{n\to\infty} \frac{1}{n}\mathbb{E}\left(\sum_{m=0}^{n} P(H_m, \mu(S_m, H_m))\right).$$

Likewise let p^μ_{of} be the probability of buffer overflow under policy μ. Consider the weighted combination $\bar{P}^\mu + \beta p^\mu_{of}$; the policy that minimizes this combination can be found computationally via dynamic programming techniques.

We study the asymptotic behavior of the above cost as $L \to \infty$ using techniques similar to those in [1]. Let $\mathcal{P}(\bar{A})$ denote the minimum average power required to transmit at average rate $\bar{A} = \mathbb{E}A_m$, without any buffering constraints. This is achieved by using a "water-filling" power allocation [2]. Furthermore if β is chosen "large enough" then it can be shown that $\mathcal{P}(\bar{A})$ is a lower bound on $\bar{P}^\mu + \beta p^{\mu_L}_{of}$ for all L (if β is not large enough, then the optimal policy will be to simply transmit nothing). As $L \to \infty$, this bound can be approached. The next proposition bounds the rate at which this limit is approached. It holds for any "admissible" sequence of policies, which satisfy some mild technical conditions.

Prop. 1: *For any admissible sequence of policies, $\{\mu_L\}$, the rate of convergence of the average power required and the buffer overflow probability have the following relationship: if $p^{\mu_L}_{of} = o(1/L^2)$, then $\bar{P}^{\mu_L} - \mathcal{P}(\bar{A}) = \Omega(1/L^2)$.*

Thus as $L \to \infty$, for a "large enough" choice of β, the total cost cannot converge faster than $1/L^2$. We show that this rate of convergence can nearly be attained by a sequence of "simple policies", where a simple policy is policy in which the power allocation only weakly depends on the buffer occupancy, and strongly on the channel state. Furthermore, this rate of convergence can not be obtained by a policy which only depends on the channel state.

The previous results can be extended to the case where there are M transmitters communicating to a common receiver over a block fading, Gaussian multiple-access channel. At each transmitter, independent data arrives and is placed into a buffer. With multiple transmitters, an issue to be addressed is how much knowledge each transmitter has about the others. We consider the case where there is a centralized controller which has access to each user's channel state and buffer occupancy. Again we look at the asymptotic behavior as the buffer size of each user gets large. The techniques used in the single user case do not directly generalize to this multiuser setting. Nevertheless, we show that analogous results hold.

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References

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