Forward-Link Resource Allocation for a Two-Cell Voice Network with Multiple Service Classes

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***Abstract*— Resource allocation is studied for a forward-link two-cell Code-Division Multiple-Access (CDMA) voice network with multiple service classes. System resources are transmitted power and codes. The service classes are specified by different user utility functions that relate utility to received Signal-to-Interference-plus-Noise-Ratio (SINR). The objective of the resource allocation is to maximize total utility over the two cells. The optimal power allocation is characterized by a set of distances, or radii, from the desired base station. Each radius corresponds to the set of active users in a particular service class, and can be enforced through a pricing scheme. We also consider setting prices to maximize revenue. In general, the prices and power allocation that maximize revenue differ from those that maximize utility.**

I. INTRODUCTION

Resource allocation in a wireless network based on the maximization of total utility has been studied in [1]-[6]. Utility maximization can naturally account for different user preferences, priorities, service classes, and different types of services (e.g., voice and data). The key assumption behind utility-based resource allocation is that each user has a utility function, which specifies received utility vs. a quality measure, such as Signal-to-Interference-Plus-Noise-Ratio (SINR).

In this paper we study resource allocation for a voice service where the utility function is assumed to be a step. Namely, the utility is zero below a threshold SINR, and is a positive constant above the threshold. The service classes are distinguished by the height of the step. Our objective is to assign resources, namely, power and codes, to maximize either the total utility summed over all users, or total revenue, subject to constraints on the total number of codes and power available at the base station.

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Related work on utility-based resource allocation for cellular systems has been presented in [1]-[6]. Reverse-link power allocation is studied in [1], [2], while [3]-[6] focus on the forward-link. This work is a continuation of [5]. However, in contrast to [5] in which only a *single* service class is considered, this paper considers a voice service with *multiple* service classes. In addition to utility maximization, we also consider two-cell revenue maximization, which has not been considered in [5]. Generally, revenue maximization gives a different power allocation than utility maximization. (See also [4], [8], which consider single-cell revenue maximization.) In other related work [6], two-cell resource allocation is analyzed for a data type of service.

We show that the optimal power allocation, with respect to either utility or revenue maximization, is specified by a set of distances or radii for the different service classes. That is, associated with each service class is a radius of active users with respect to the desired base station. For a given service class, all users within this radius are active, and all users outside this radius are inactive. (This assumes deterministic, distance-based channel attenuation.) For utility maximization, we show that if the power constraint is sufficiently small, then both cells use all available power. An example with a relatively large power constraint is given for which utility is maximized when only *one* cell uses its available power. Of course, given sufficient codes and power, the utility is maximized by activating all users, assuming that they can meet the SINR constraint. We also show that utility maximization can be achieved by a pricing scheme, in which each base station announces prices for each resource, and each user responds by requesting the amount of resources to maximize its surplus (utility minus cost).

For revenue maximization, we show that the number

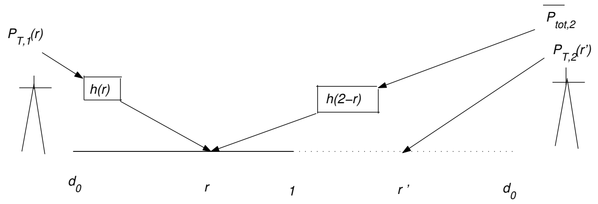

Fig. 1. One-dimensional two-cell system.

of active users is generally less than that obtained with utility maximization. Furthermore, for a single class of users, the optimal price per power is arbitrarily small, but must be positive in order to realize the appropriate radius of active users. All users are then essentially charged the price per code. This is also true with multiple service classes when requests for service exceed the supply of codes.

II. System Model

We consider the one-dimensional, two-cell model, which is illustrated in Figure 1. The model may seem simplistic, but it sets up the framework, and the interesting results shown here were previously unknown. The cell radius is normalized to one, and d_0 is a close-in reference point in the far field region of the transmitter antenna. We make the following assumptions:

- The channel for each user is a scalar attenuation based on distance from the base station.
- All codes within a cell are orthogonal. A matched filter receiver is assumed, and codes from the adjacent cell are treated as random with i.i.d. elements.
- The system is static, i.e., the number of users in the system is fixed.
- There are N service classes numbered $n = 1, \ldots, N$.

We also assume that the two cells are identical in terms of size, noise level, user density, and user utility functions, and that users in each service class are uniformly distributed between the cells. However, these assumptions can be relaxed and are made primarily to simplify our discussion. As in [5], we evaluate the large system performance by letting the number of users K and the number of available codes per cell M each increase to infinity, with fixed offered load K/M. We denote the fraction of users in class n as p_n, where $\sum_{n=1}^{N} p_n = 1$.

Referring to Figure 1, consider a class n user in cell 1 located at distance r from base station 1. The received SINR for this user is

$$\xi = \frac{P_{n,1}(r)h(r)}{\sigma^2 + I}, \tag{1}$$

where $P_{n,1}(r)$ is the transmitted power from base station 1 for a class n user at distance r, $h(r)$ is the attenuation modeled as a decreasing function of r, σ^2 is the noise level, and I is the received interference from the other cell. The intracell interference is zero since all codes in the cell are orthogonal.

The average transmitted power per code from cell 2 is

$$\overline{P}_{tot,2} = \frac{K}{M(1-d_0)} \sum_{n=1}^{N} \left(p_n \int_{d_0}^{1} P_{n,2}(r')\, dr' \right), \tag{2}$$

where $P_{n,2}(r')$ is the transmitted power from base station 2 to a class n user at distance r' in cell 2. Since a matched filter receiver is assumed, the interference from cell 2 to a user at distance r from base station 1 is

$$I(r, \overline{P}_{tot,2}) = \overline{P}_{tot,2} h(2-r). \tag{3}$$

As r increases, both the attenuation of the desired signal and the interference increase, so that more transmitted power is required to achieve a specific SINR.

The resources to be allocated are codes and transmitted power. Each active voice user is assigned one code. Users in service class n, have the utility function $U_n(\xi)$, which specifies the user's utility, or willingness to pay, vs. received SINR. For the voice service considered, $U_n(\xi) = 0$ for $\xi < \xi_0$, and $U_n(\xi) = u_n$ for $\xi > \xi_0$ where ξ_0 is the threshold SINR required for acceptable voice quality, and is the same for all users. We will assume that $u_k < u_l$ for $k < l$.

For each cell i, let $A_{n,i}$ denote the fraction of active users in class n. So, $A_{n,i} \leq p_n$ with equality only if all class n users are active in the cell. To minimize power consumption, each active user operates at the threshold SINR, $\xi = \xi_0$, hence the minimum power corresponding to an active user is $\xi_0(\sigma^2 + I)/h(d_0) > 0$. The resource allocation problem is to maximize the total utility subject to power and code constraints, i.e.,

$$\max_{\{P_{n,i}(r), n=1,\ldots,N, i=1,2\}} \quad U_{tot} = U_{tot,1} + U_{tot,2} \tag{4}$$

$$\text{subject to:} \quad C_{tot,i} \leq 1 \quad i = 1,2 \tag{5}$$

$$\overline{P}_{tot,i} \leq \mathcal{P} \quad i = 1,2 \tag{6}$$

where

$$U_{tot,i} = \frac{K}{M} \sum_{n=1}^{N} u_n A_{n,i} \tag{7}$$

$$C_{tot,i} = \frac{K}{M} \sum_{n=1}^{N} A_{n,i} \tag{8}$$

are the total utility per code in cell i and the fraction of codes used in cell i, respectively, and $\mathcal{P}$ is the average transmitted power available at each base station.

Theorem 1: *Under the power allocation which achieves the maximum utility, if a user in class n is active* *at radius r_n, then all users in class n at radius $r < r_n$ are active.*

Proof: Suppose this is not true, i.e., there is an active user k in class n at radius ρ_k and an inactive user l in class n at radius $\rho_l < \rho_k$. Activating user l and deactivating user k decreases the transmitted power while maintaining the same total utility. Since the maximum utility is a nondecreasing function of the power constraint, this leads to a contradiction. ∎

The optimal power allocation is therefore defined by the set of radii of active users $\{r^*_{n,i} : n = 1, \dots, N; i = 1, 2\}$. That is, class n users at distances $d_0 \le r \le r^*_{n,i}$ are active, and users at distances $r^*_{n,i} < r \le 1$ are inactive. Furthermore, $u_k < u_l$ implies $r^*_{k,i} \le r^*_{l,i}$. Therefore, we have

$$A_{n,i} = \frac{p_n(r^*_{n,i} - d_0)}{1 - d_0}, \tag{9}$$

and

$$\overline{P}_{tot,i} = \frac{K}{M(1-d_0)} \sum_{n=1}^{N} \left(p_n \int_{d_0}^{r^*_{n,i}} P^*_{T,i}(r)\, dr \right), \tag{10}$$

where

$$P^*_{T,i}(r) = \frac{\xi_0[\sigma^2 + \overline{P}_{tot,j} h(2-r)]}{h(r)} \qquad i \ne j \tag{11}$$

is the transmitted power needed to achieve the threshold ξ_0 for users in cell i at distance r. It follows that the maximization in (4) reduces to a maximization over the set of radii of active users. When there is only one service class ($N = 1$), maximizing the total utility is equivalent to maximizing the total fraction of active users [5]. With multiple service classes, the total utility is maximized when the weighted sum of active users is maximized, where the weighting corresponds to the utility from each class.

III. Utility Maximization

Solutions to the utility maximization problem can be classified according to which constraints are binding. We consider these cases separately.

Demand-limited Solution: In this case, the supply of resources exceeds the demand, and all users are able to meet the SINR requirement, so that neither constraint is binding in either cell. Consequently, $r^*_{n,i} = 1$ and $\sum_{n=1}^{N} A_{n,i} = 1$ for all i and k, and $U_{tot} = \frac{2K}{M} \sum_{n=1}^{N} p_n u_n$.

Code-limited Solution: If codes are the limiting resource, then total utility is maximized by activating the users with the largest utility. Consequently, users in service class N are activated first, followed by users in service class $N-1$, and so forth until all codes are assigned. Let n^* be the class, which satisfies

$$\frac{K}{M(1-d_0)} \left[p_{n^*}(r - d_0) + \sum_{n=n^*+1}^{N} p_n(1-d_0) \right] = 1, \tag{12}$$

for some $r \in [d_0, 1]$. The solution is to activate all users in classes $n = n^* + 1, \dots, N$ (i.e., $r^*_{n,i} = 1$ for $n > n^*$), and partially activate users in class n^* (i.e., $r^*_{n^*,i} = r$) so that (12) is satisfied. All users in classes $n < n^*$ are inactive (i.e., $r^*_{l,i} = d_0$ for $l < n^*$).

Power-limited Solution: We first consider maximizing utility in cell 1 given fixed interference from cell 2. That is, for fixed $\overline{P}_{tot,2}$ we wish to determine the radii of active users in cell 1 which maximize $U_{tot,1}$ subject to code and power constraints.

Consider the Lagrangian

$$\begin{aligned} L(\{r^*_{n,1}\}; \alpha_{p,1}, \alpha_{c,1}) = &\quad U_{tot,1} + \alpha_{p,1}(\mathcal{P} - \overline{P}_{tot,1}) \\ + &\quad \alpha_{c,1}(1 - C_{tot,1}), \end{aligned} \tag{13}$$

where $\alpha_{p,1}$ and $\alpha_{c,1}$ are Lagrange multipliers. Substituting from (7), (8), and (10), and setting $\partial L / \partial r^*_{n,1} = 0$ gives the necessary conditions

$$u_n = \alpha_{p,1} P^*_{T,1}(r^*_{n,1}) + \alpha_{c,1} \quad \text{if } 1 > r^*_{n,1} > d_0, \tag{14}$$

where the Lagrange multipliers satisfy,

$$\begin{aligned} \alpha_{p,1} \quad &> 0 \quad \text{if } \mathcal{P} = \overline{P}_{tot,1} \\ &= 0 \quad \text{if } \mathcal{P} > \overline{P}_{tot,1} \end{aligned} \tag{15}$$

$$\begin{aligned} \alpha_{c,1} \quad &> 0 \quad \text{if } 1 = C_{tot,1} \\ &= 0 \quad \text{if } 1 > C_{tot,1}. \end{aligned} \tag{16}$$

When power is the only limiting resource, i.e., $\alpha_{p,1} > 0$ and $\alpha_{c,1} = 0$, we have

$$\frac{u_n}{P^*_{T,1}(r^*_{n,1})} = \alpha_{p,1}, \tag{17}$$

which implies $r^*_{k,1} < r^*_{l,1}$ if $u_k < u_l$.

The Lagrange multipliers are shadow costs associated with the code or power consumption in each cell. This suggests a pricing scheme, in which the base station in cell i announces a price per code $\alpha_{c,i}$ and a price per unit transmitted power $\alpha_{p,i}$, and each user requests the transmitted power $P_{n,i}(r)$, which maximizes surplus, given by $u_n - (\alpha_{p,i} P_{n,i}(r) + \alpha_{c,i})$. The maximum surplus occurs at either $P^*_{T,i}(r)$, the power needed to achieve the SINR threshold, or zero. If $u_n \ge \alpha_{p,i} P^*_{T,i}(r) + \alpha_{c,i}$, then $P_{n,i}(r) = P^*_{T,i}(r)$, and the user is assigned a code. Otherwise, $P_{n,i}(r) = 0$ and the user is inactive.

If the optimal radius $r^*_{n,i} = 1$, then the prices are chosen so that $u_n \ge \alpha_{c,i} + \alpha_{p,i} P^*_{T,i}(1)$, so that the user at the cell boundary is activated. If $r^*_{n,i} < 1$, then the prices satisfy $u_n = \alpha_{c,i} + \alpha_{p,i} P^*_{T,i}(r^*_{n,i})$ and $\alpha_{p,i} > 0$. The prices which maximize utility are not unique in general.

We now turn to two-cell utility maximization. The necessary conditions (14)-(16) hold within each cell, so that the preceding pricing scheme can again achieve the optimal power allocation. If each cell uses up all available power, this corresponds to a *Nash Equilibrium (NE)*, in which each cell maximizes its own utility without regard to the effect on the other cell [7]. In general, the maximum utility is not achieved at the NE. This depends on the power constraint and interference between the cells.

Theorem 2: *There exists a $P_{L,U} > 0$ such that for any power constraint $\mathcal{P} \leq P_{L,U}$, the maximum utility is achieved when both cells are power-limited.*

Proof: It suffices to show that $\partial U_{tot}/\partial \overline{P}_{tot,i} > 0$ at ($\overline{P}_{tot,1} = 0, \overline{P}_{tot,2} = 0$), i.e., the marginal utility is positive at this point. Since $\partial U_{tot}/\partial \overline{P}_{tot,i}$ is a continuous function of $\overline{P}_{tot,i}$, $i = 1, 2$, it follows that there exists a $P_{L,U}$ such that $\partial U_{tot}/\partial \overline{P}_{tot,i} > 0$ for $\overline{P}_{tot,i} < P_{L,U}$, $i = 1, 2$. Hence, to maximize utility in this region, we can choose $\overline{P}_{tot,1} = \overline{P}_{tot,2} = \mathcal{P}$ for any $\mathcal{P} \leq P_{L,U}$.

Taking derivatives of (10) with respect to $\overline{P}_{tot,i}$ and $\overline{P}_{tot,j}$ for $i \neq j$ gives

$$1 = -\frac{K}{M(1-d_0)} \sum_{n=1}^{N} p_n P^*_{T,i}(r^*_{n,i}) \frac{\partial r^*_{n,i}}{\partial \overline{P}_{tot,i}} \quad (18)$$

$$0 = \sum_{n=1}^{N} p_n [P^*_{T,j}(r^*_{n,j}) \frac{\partial r^*_{n,j}}{\partial \overline{P}_{tot,i}} + \int_{d_0}^{r^*_{n,j}} \frac{\xi_0 h(2-r)}{h(r)}\, dr] \quad (19)$$

Taking the derivative of U_{tot} with respect to $\overline{P}_{tot,i}$ and substituting (17)-(19) gives

$$\begin{aligned} \frac{\partial U_{tot}}{\partial \overline{P}_{tot,i}} &= \frac{K}{M(1-d_0)} \sum_{n=1}^{N} p_n u_n [\frac{\partial r^*_{n,i}}{\partial \overline{P}_{tot,i}} + \frac{\partial r^*_{n,j}}{\partial \overline{P}_{tot,i}}] \\ &= \alpha_{p,i} - \frac{K \alpha_{p,j}}{M(1-d_0)} \sum_{n=1}^{N} p_n \int_{d_0}^{r^*_{n,j}} \frac{\xi_0 h(2-r)}{h(r)}\, dr \end{aligned} \quad (20)$$

At ($\overline{P}_{tot,1} = 0, \overline{P}_{tot,2} = 0$) this reduces to $\partial U_{tot}/\partial \overline{P}_{tot,i} = \alpha_{p,i}$, where $\alpha_{p,i}$ denotes the optimal power price . For small enough $\mathcal{P}$, we must have $\alpha_{p,i} > 0$, and thus $\partial U_{tot}/\partial \overline{P}_{tot,i} > 0$ as desired. ■

To illustrate the solution for large $\mathcal{P}$, we consider two service classes. Let γ^*_i be the maximal utility in cell i,

$$\gamma^*_i = \max(p_1 u_1 r^*_{1,i} + p_2 u_2 r^*_{2,i}). \quad (21)$$

Maximizing the total utility is equivalent to maximizing $\gamma^*_1 + \gamma^*_2$. Figure 2 shows the feasible region for (γ^*_1, γ^*_2) for the parameters $u_1 = 10$, $u_2 = 30$, $p_1 = 0.5$, and $p_2 = 0.5$. The target SINR is $\xi_0 = 15$ dB and $h(r) = (d_0/r)^2$, $\mathcal{P} = 40$ dB.

The boundary of the feasible region is the solid curve in Figure 2. Consider the segment of the boundary curve

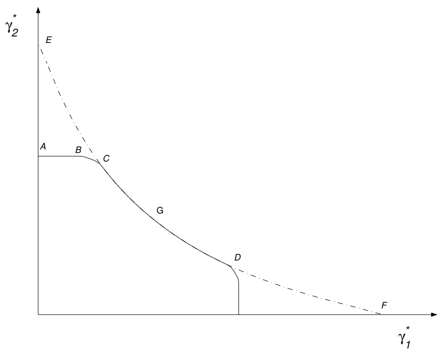

Fig. 2. Feasible region for (γ^*_1, γ^*_2).

between point A and B. In this region γ^*_2 is constant, corresponding to the situation in which cell 2 is demand-limited, so that $r^*_{i,2} = 1$, and cell 1 is not resource-limited. The segment between points B and C corresponds to the situation in which cell 2 is power-limited and all users in class 2 are active, i.e., $r^*_{2,2} = 1$. On the segment between C and D, one cell is power-limited and $r^*_{2,i} < 1$, i.e., not all class 2 users are active. Because the segment C-D is convex, it is easily verified that the maximum $(\gamma^*_1 + \gamma^*_2)$ occurs at points C and D, where cell i is power-limited and $r^*_{2,i} = 1$. The Nash equilibrium point is at G, i.e., both cells are power-limited; this does not achieve the maximum utility.

Suppose now that cell i can serve users in cell j where $i \neq j$. In other words, we allow $r^*_{n,i} > 1$. In that case, we get the dashed boundary curve shown in Figure 2. The dashed curve is identical to the original curve from C to D. The maximum $(\gamma^*_1 + \gamma^*_2)$ now occurs at points E and F in the figure, where cell i is power-limited and cell $j \neq i$ is turned off, i.e., $\overline{P}_{tot,j} = 0$.

We therefore conclude that for large $\mathcal{P}$ the Nash Equilibrium is not optimal in general. This is due to the loss in total utility caused by interference. Information exchange between the two cells is therefore needed so that one cell reduces the transmitted power, thereby reducing interference and achieving a higher total utility. Within the context of pricing, this is accomplished by charging a higher price per unit power in one cell, which reflects the externality cost due to interference.

IV. Revenue Maximization

Suppose that resources are allocated according to the pricing scheme previously presented; however now the prices are selected to maximize total *revenue*, rather than utility. In this case, the base station can search for the optimal prices in an adaptive mode without knowledge of the user utility functions. Since each active user at distance r contributes revenue $\alpha_{c,i} + \alpha_{p,i} P^*_{T,i}(r)$, the total revenue per code in cell i is

$$R_{tot,i} = \frac{K}{M(1-d_0)} \sum_{n=1}^{N} \left(p_n \int_{d_0}^{r^*_{n,i}} [\alpha_{c,i} + \alpha_{p,i} P^*_{T,i}(r)]\, dr \right) \tag{22}$$

The corresponding optimization problem is:

$$\begin{aligned} &\max_{\alpha_{c,i},\alpha_{p,i}} \quad R_{tot} = R_{tot,1} + R_{tot,2} \\ &\text{subject to:} \quad C_{tot,i} \le 1 \quad i = 1,2 \\ &\qquad\qquad\quad \overline{P}_{tot,i} \le \mathcal{P} \quad i = 1,2 \end{aligned} \tag{23}$$

where R_{tot} is the total revenue over two cells. Here we assume that the same price is charged to all users within a cell. In general, additional revenue can be obtained by *price discrimination*, i.e., charging different users *different* prices [8].

Let

$$\overline{u}_{max} = \max_l (u_l \sum_{n=l}^{N} p_n) \tag{24}$$

$$m = \arg\max_l (u_l \sum_{n=l}^{N} p_n) \tag{25}$$

We will refer to a set of prices $(\alpha_{p,i}, \alpha_{c,i})$, $i = 1,2$, as *feasible* if code and power constraints are not violated when power and codes are allocated to maximize surplus given these prices.

Theorem 3: *$R_{tot} \le 2K\overline{u}_{max}/M$ with equality if $\alpha_{p,i} = 0$ and $\alpha_{c,i} = u_m$, $i = 1,2$, are feasible.*

Proof: Since $r^*_{k,i} \le r^*_{l,i}$ when $k < l$, and $\alpha_{c,i} + \alpha_{p,i} P^*_{T,i} \le u_n$ for each active user in class n, we have from (22),

$$\begin{aligned} R_{tot,i} &= \frac{K}{M(1-d_0)} [\int_{d_0}^{r^*_{1,i}} [\alpha_{c,i} + \alpha_{p,i} P^*_{T,i}(r)]\, dr \\ &\quad + \sum_{n=2}^{N} (\sum_{k=n}^{N} p_k) \int_{r_{n-1,i}}^{r^*_{n,i}} [\alpha_{c,i} + \alpha_{p,i} P^*_{T,i}(r)]\, dr \\ &\le \frac{K}{M(1-d_0)} [u_1 (r^*_{1,i} - d_0) \\ &\quad + \sum_{n=2}^{N} u_n (r^*_{n,i} - r^*_{n-1,i}) (\sum_{k=n}^{N} p_k)] \\ &\le \frac{K}{M(1-d_0)} \overline{u}_{max} (r^*_{N,i} - d_0) \\ &\le \frac{K \overline{u}_{max}}{M} \end{aligned}$$

■

Equality holds when $\alpha_{p,i} = 0$ and $\alpha_{c,i} = u_m$. In that case, all class l users are active for $l \ge m$, i.e., $r^*_{l,i} = 1$, and all class k users are inactive for $k < m$, i.e., $r^*_{k,i} = d_0$. In contrast with utility maximization, revenue maximization does not always activate all users, even if additional users can be activated without violating any constraints. This is because when the prices decrease, more users are active; however, each active user contributes less revenue. Therefore the optimal solution depends on the utility u_n and fraction of users p_n in each service class.

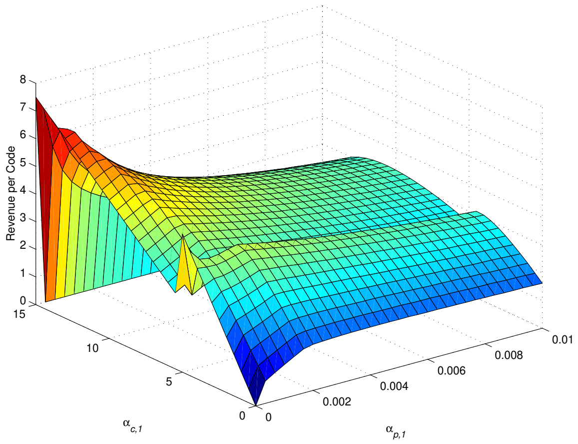

Fig. 3. Revenue versus $(\alpha_{c,1}, \alpha_{p,1})$ with two service classes.

Figure 3 shows the total revenue per code vs. prices $(\alpha_{c,1}, \alpha_{p,1})$ for cell 1 with fixed interference. Two service classes are assumed with $u_1 = 5$, $u_2 = 15$, $p_1 = 0.5$, and $p_2 = 0.5$. The target SINR is $\xi_0 = 5dB$ and $h(r) = (d_0/r)^4$. The revenue per code has two local maxima, one at $(\alpha_{c,1} = u_1, \alpha_{p,1} = 0)$, and the other at $(\alpha_{c,1} = u_2, \alpha_{p,1} = 0)$. Since $\overline{u}_{max} = \max(u_1, u_2 p_2) = 7.5$ and $m = 2$, the maximum occurs at $(\alpha_{c,1} = u_2, \alpha_{p,1} = 0)$, and $R_{tot,1} = K u_2 / 2M = 7.5$. The maximum revenue is achieved by activating only class 2 users. All class 1 users are inactive.

If price discrimination is allowed, then given sufficient codes and power, the prices which maximize revenue for service class n in cell i are $\alpha_{c,i,n} = u_n$ and $\alpha_{p,i,n} = 0$. The prices then depend on the service class. In that case, all users in the system are active, and each user receives zero surplus. We also observe that the power allocation which maximizes revenue with price discrimination also maximizes utility.

Theorem 4: *When each cell has sufficient power and more than M class N users, that is, the load $K/M > 1/p_N$, revenue is maximized by setting $\alpha_{c,i} = u_N - \epsilon$, $\alpha_{p,i} = \epsilon / P^*_{T,i}(r^*_{N,i})$, and letting $\epsilon \to 0$ with $r^*_{N,i} = \frac{p_N K(1-d_0)}{M} + d_0$. Furthermore, the system is code-limited.*

Proof: To maximize the revenue in this situation, the cell activates exactly M class N users closest to the desired base station, and charges each active user u_N. We therefore have $r^*_{k,i} = d_0$ for any $k < N$, and $r^*_{N,i} = p_N K(1-d_0)/M + d_0$. If $r^*_{N,i} < 1$ at the optimum, then $\alpha_{p,i}$ must be positive. Since our objective is to charge each active user u_N, we let $\alpha_{c,i}$ approach u_N from below, and let $\alpha_{p,i}$ tend to zero in such a way as to maintain the optimal $r^*_{N,i}$. This is accomplished by the prices in the Theorem. ■

Notice the load satisfies $K/M < 1/\sum_{n=m}^{N} p_n$ in Theorem 3. In Theorem 4, we have $K/M > 1/p_N$. When $1/p_N > K/M > 1/\sum_{n=m}^{N} p_n$, the solution to maximize revenue varies. In this case, we have observed examples where the cells either allocate all the codes, or reserve some codes, depending on the parameters. When all the codes are allocated, the cells activate the users with largest utility, and charge each active user u_{n^*}, where n^* is the class satisfying (12). Similar to Theorem 4, we let $\alpha_{c,i}$ approach u_{n^*} from below, and let $\alpha_{p,i}$ tend to zero in such a way as to maintain the optimal $r^*_{n^*,i}$. When some codes are reserved, the cells activate all the users in class $n > n^*$, and charge each active user u_{n^*+1}. Thus, the optimal prices are $\alpha_{c,i} = u_{n^*+1}$ and $\alpha_{p,i} = 0$.

A complete characterization of the solution when power is the limiting resource appears to be difficult. The following theorem characterizes the solution for a single service class.

Theorem 5: *For a single service class, the power distribution which achieves the maximum revenue also achieves the maximum total utility.*

Proof: From (22), we have

$$R_{tot} \leq \frac{K u_1 (r^*_{1,1} + r^*_{1,2} - 2d_0)}{M(1-d_0)} \tag{26}$$

Maximizing revenue is therefore equivalent to maximizing $r^*_{1,1} + r^*_{1,2}$, and charging each active user u_1. Maximizing $r^*_{1,1} + r^*_{1,2}$ is equivalent to maximizing the number of active users, which also maximizes utility [5]. ■

The power allocation which maximizes $r^*_{1,1} + r^*_{1,2}$ is presented in [5]. If the optimal $r^*_{1,i} = 1$, then the optimal prices are $\alpha_{c,i} = u_1$ and $\alpha_{p,i} = 0$, corresponding to a demand-limited cell. That is, all K users are activated. If the optimal $r^*_{1,i} < 1$, then the revenue is maximized by setting $\alpha_{c,i} = u_1 - \epsilon$, and $\alpha_{p,i} = \epsilon / P^*_{T,i}(r^*_{1,i})$, and letting ϵ go to zero.

V. Conclusion

We have studied forward-link resource allocation for a two-cell voice CDMA network with multiple service classes. The utility is maximized when both cells are either demand- or code-limited. When the available transmitted power is sufficiently small, the utility is maximized when both cells are power-limited. This is not true in general, which means that the two cells must exchange information to maximize total utility. In all cases, the optimal power allocation can be achieved with a pricing scheme. For a system with one service class, the power allocation that maximizes utility can also achieve maximum revenue, albeit with a specific set of prices. With multiple service classes, revenue maximization generally activates fewer users than utility maximization.

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