Clustering for Processing Rate Optimization

Chuan Lin, Jia Wang, and Hai Zhou

Electrical Engineering and Computer Science
Northwestern University
Evanston, IL 60208

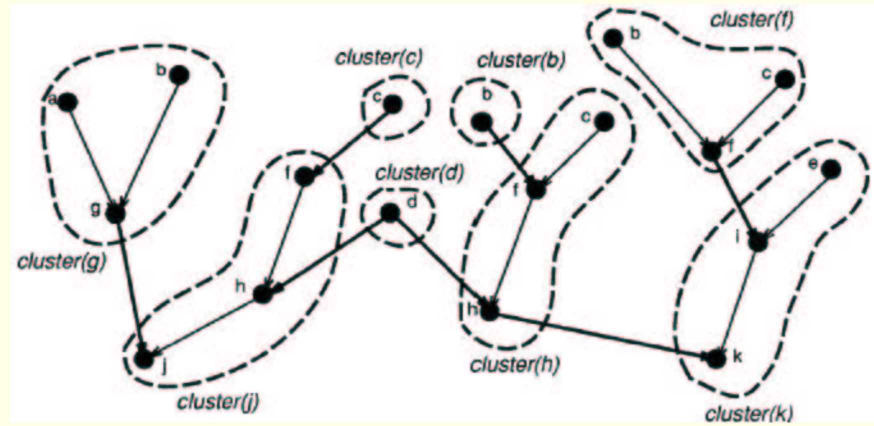
November 7, 2005

Outline

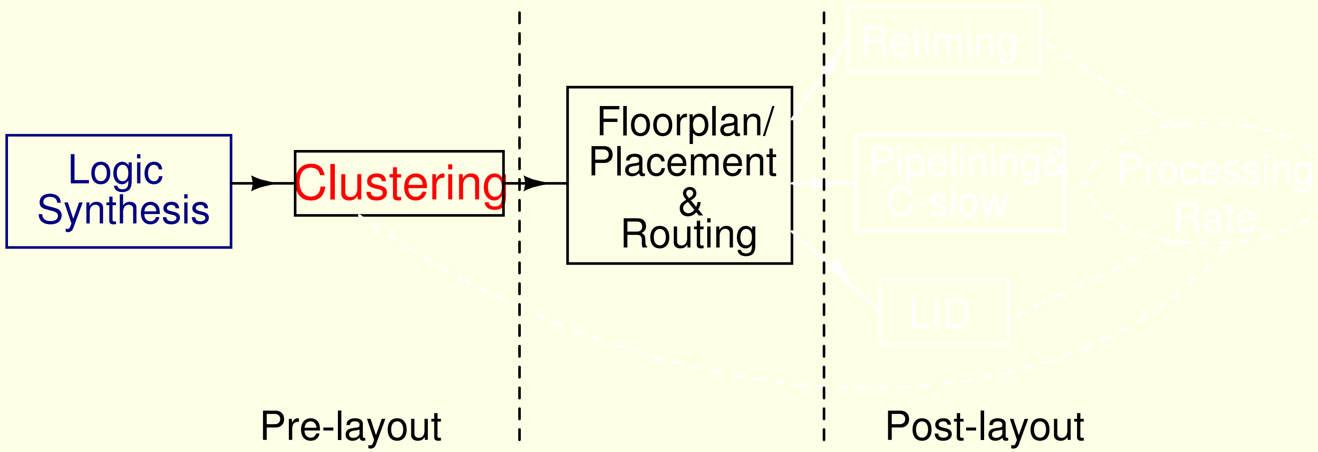
- Motivation
- Problem formulation
- Algorithm
- Experimental results
- Conclusions

A logical and physical design flow

- **Clustering** decomposes a circuit into clusters

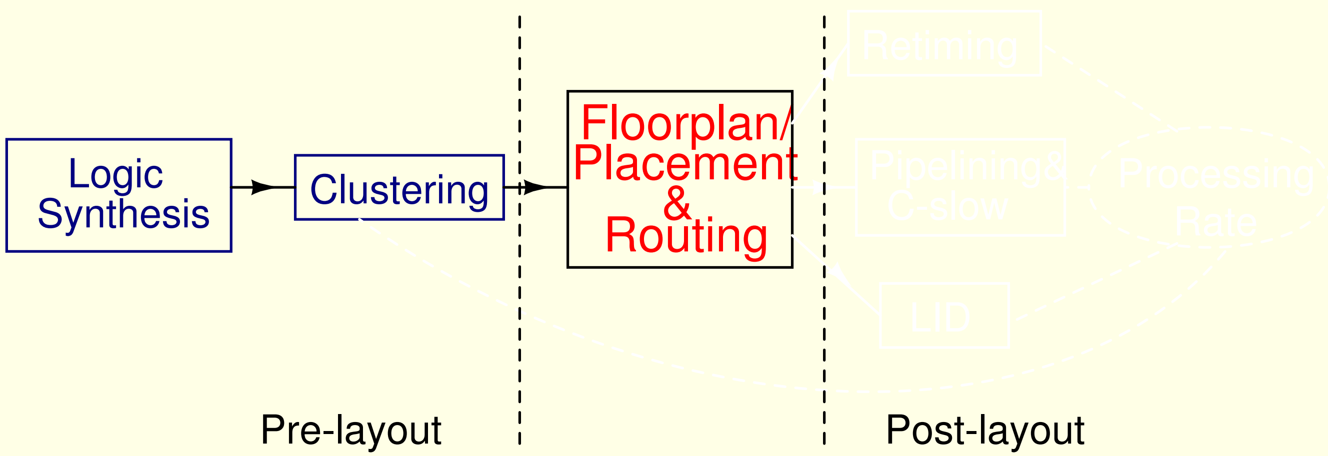


Rajaraman and Wong [DAC'93]



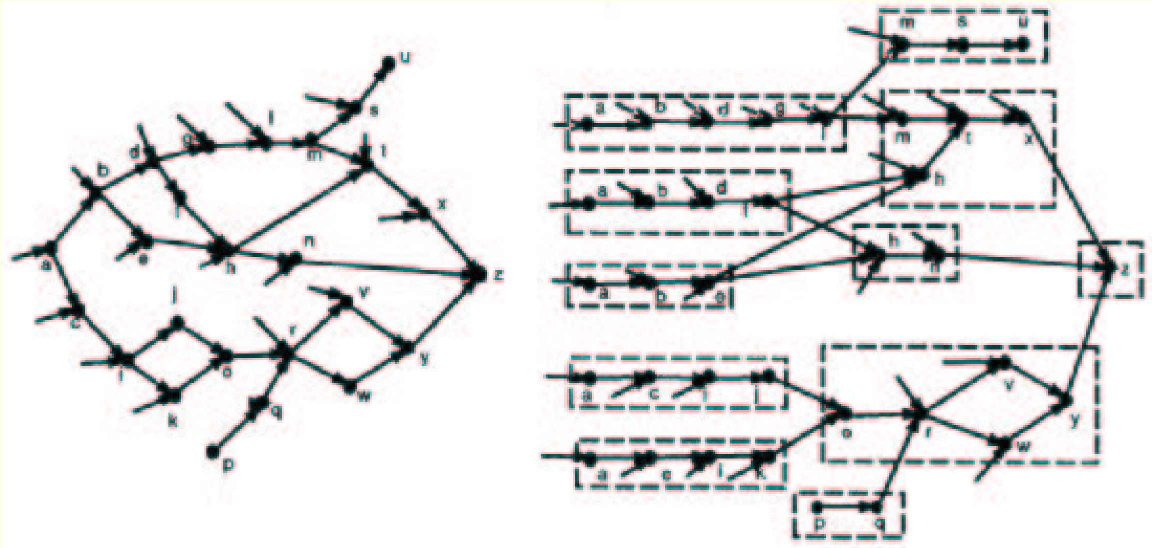
A logical and physical design flow

- **Floorplan/placement & routing** generate a layout



A logical and physical design flow

- **Floorplan/placement & routing** generate a layout
  - Intra-cluster interconnects are local (**short**)
  - Inter-cluster interconnects may become global (**long**)



Rajaraman and Wong [DAC'93]

A logical and physical design flow

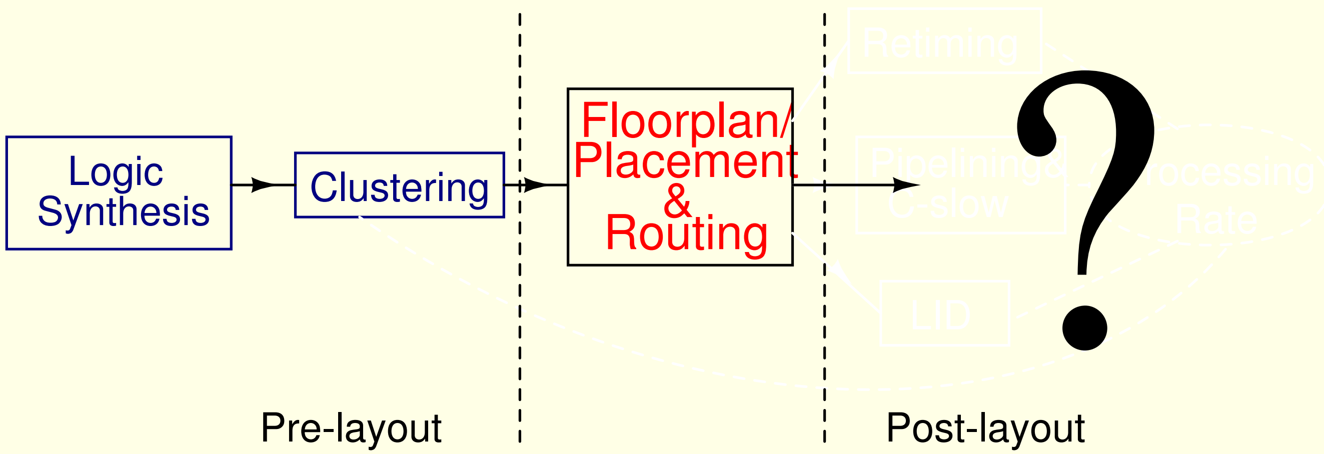
- **Floorplan/placement & routing** generate a layout
  - Intra-cluster interconnects are local (**short**)
  - Inter-cluster interconnects may become global (**long**)
    - Longer than one period even with buffer insertion

A logical and physical design flow

- **Floorplan/placement & routing** generate a layout
  - Intra-cluster interconnects are local (**short**)
  - Inter-cluster interconnects may become global (**long**)
    - ∗ Longer than one period even with buffer insertion
    - ∗ Delay NOT available at logic synthesis but after layout

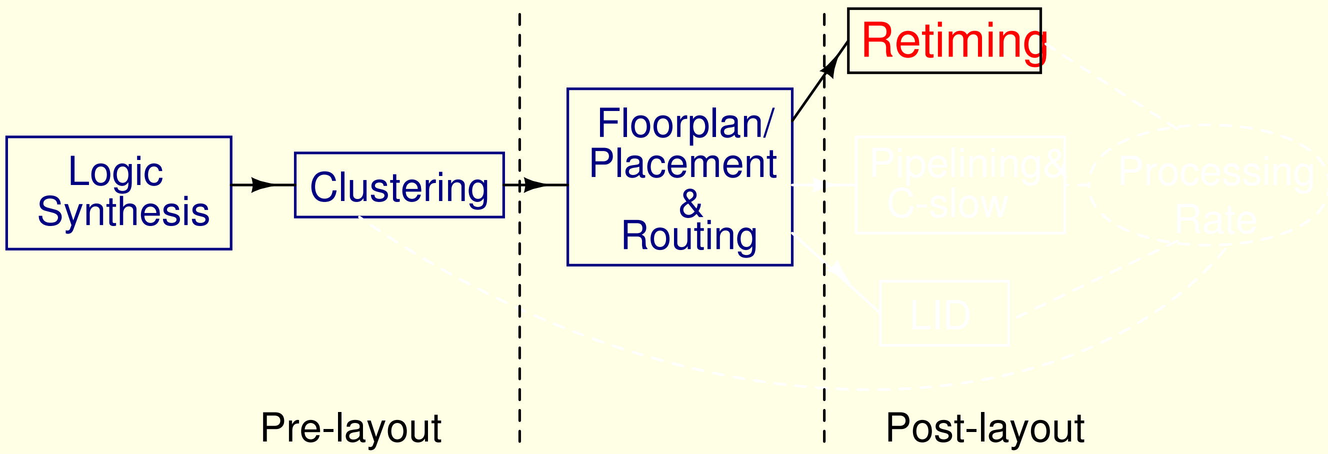
A logical and physical design flow

- **Floorplan/placement & routing** generate a layout
  - Intra-cluster interconnects are local (**short**)
  - Inter-cluster interconnects may become global (**long**)
- Correct design at logic synthesis may **FAIL** to meet timing



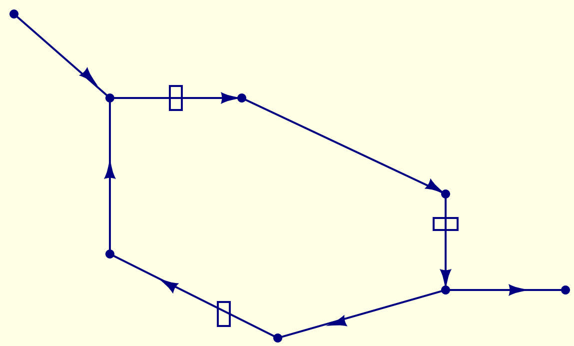
A logical and physical design flow

- **Retiming** moves flip-flops to pipeline global interconnects
  - Functionality is maintained



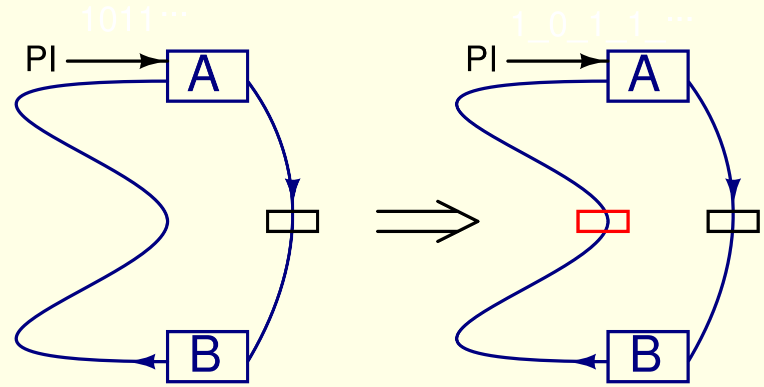
A logical and physical design flow

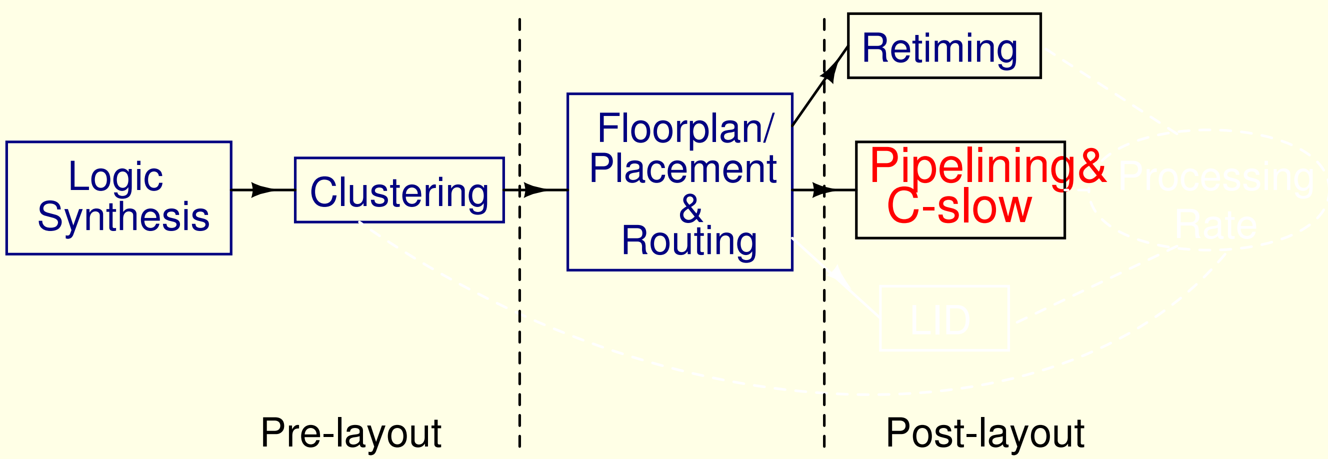
- **Retiming** moves flip-flops to pipeline global interconnects
  - Functionality is maintained
  - Period lower bounded



A logical and physical design flow

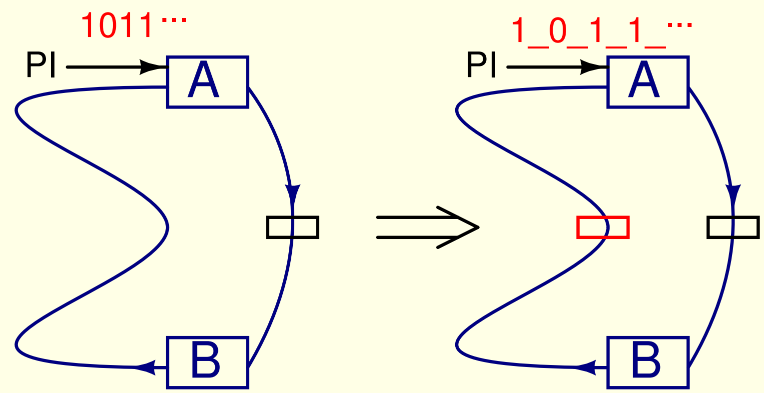
- **Pipelining** inserts FFs in a cycle
  - Function changed [Nookala and Sapatenkar DAC'04]

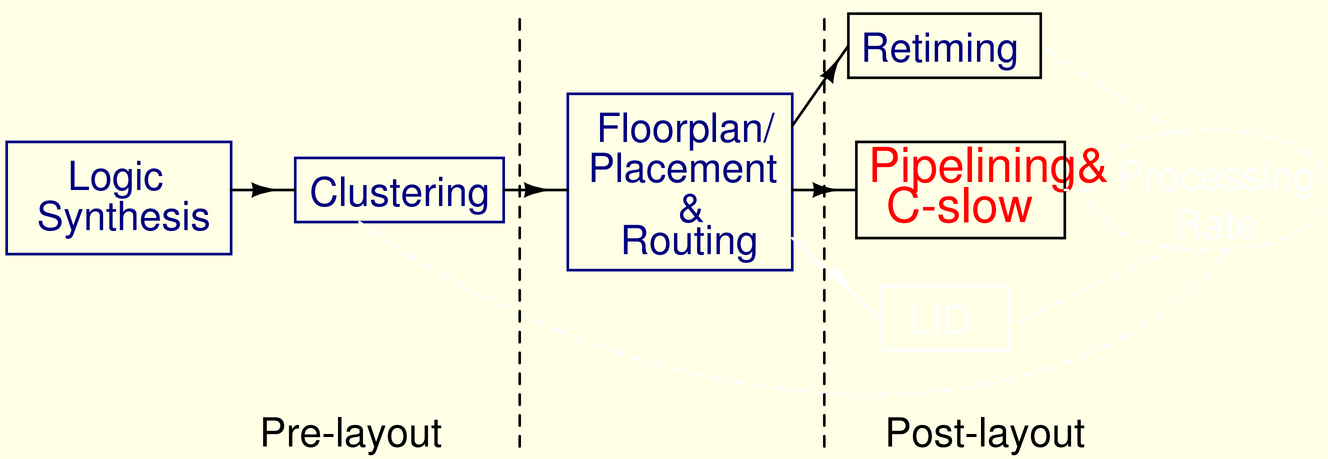




A logical and physical design flow

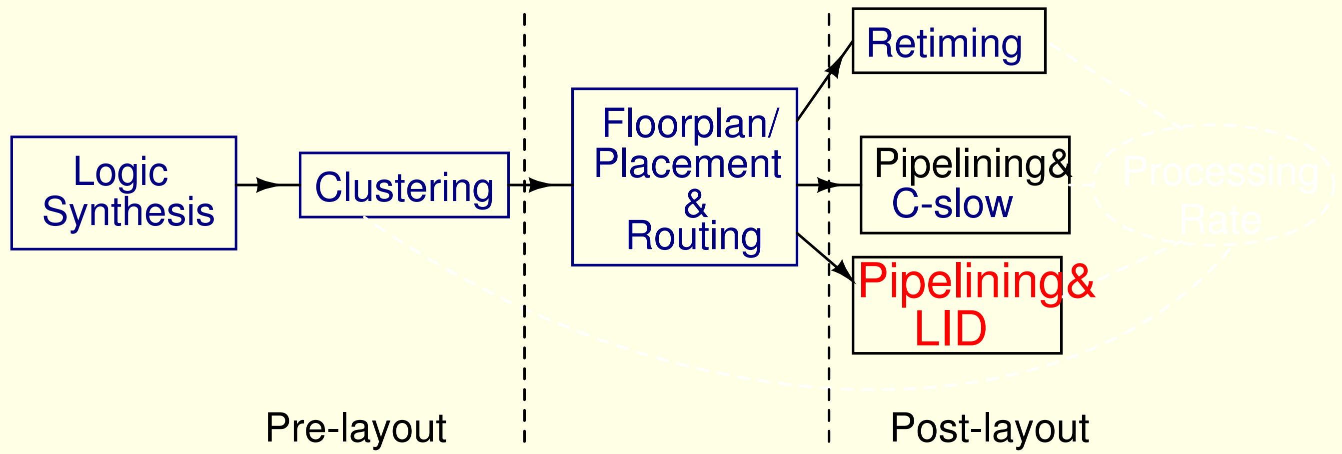
- **Pipelining** inserts FFs in a cycle
  - Function changed [Nookala and Sapatenkar DAC'04]
- C**-slow transformation**
  - Slows down input issue rate, i.e., throughput becomes $1/C$





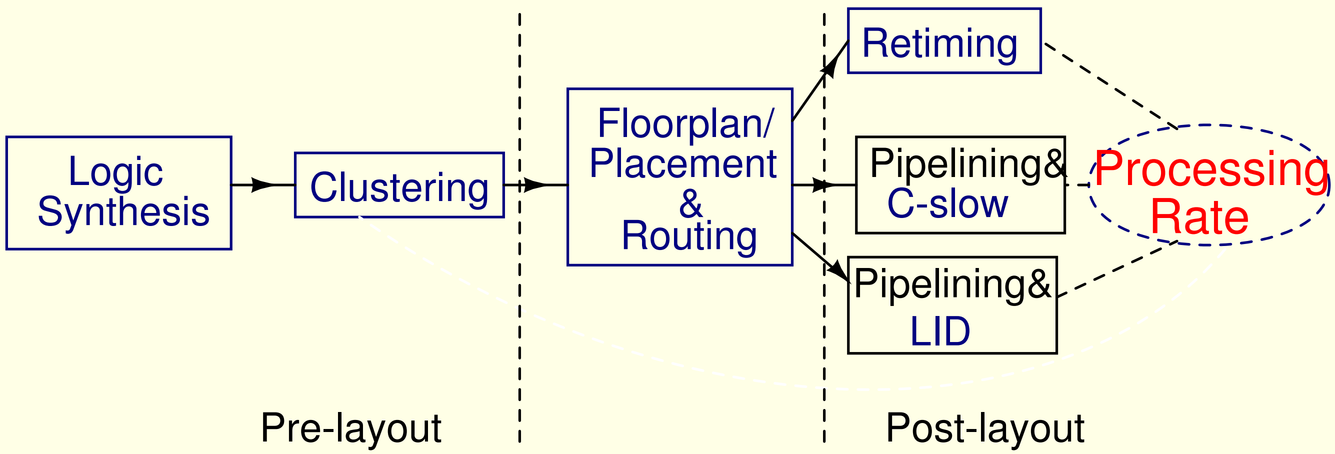
A logical and physical design flow

- **Latency insensitive design (LID)** [Carloni ICCAD'99]
  - Employ a protocol that slows down a part of the circuit only when necessary



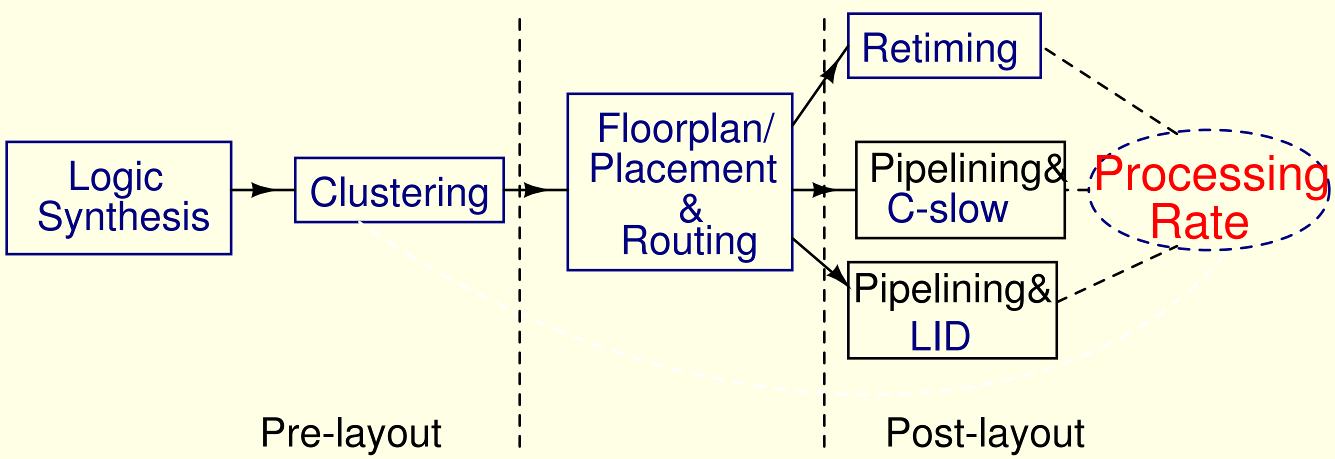
A logical and physical design flow

- **Processing rate** (throughput × frequency)
  - Common objective maximized by all three approaches



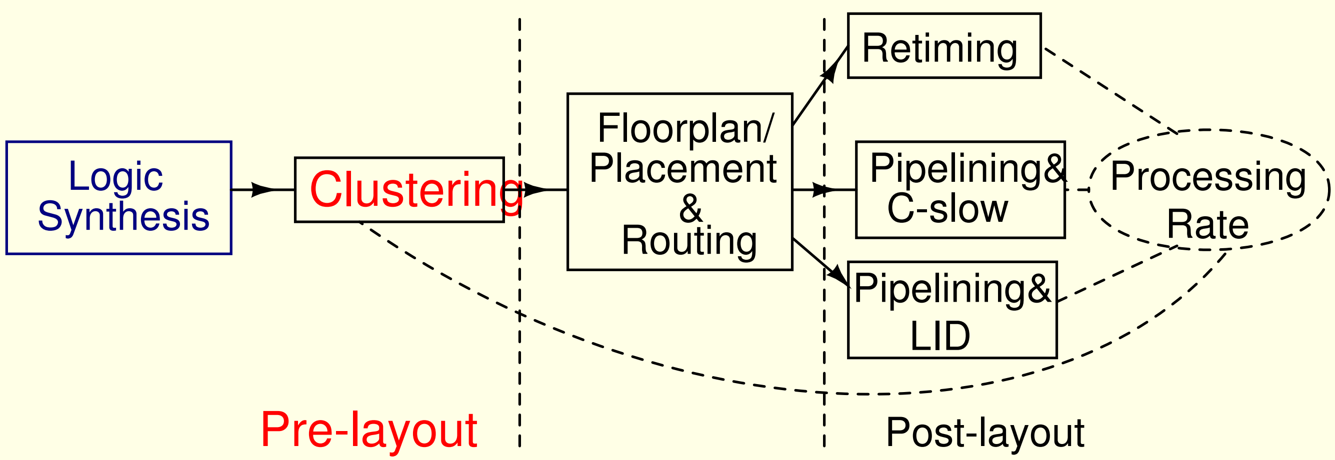
A logical and physical design flow

- **Processing rate** (throughput $\times$ frequency)
  - Common objective maximized by all three approaches
  - $\leq$ **1/max-cycle-ratio** [cycle ratio=cycle delay/cycle FFs] which is **only dependent on clustering**



A logical and physical design flow

- **Processing rate** (throughput $\times$ frequency)
  - Common objective maximized by all three approaches
  - $\leq$ **1/max-cycle-ratio** [cycle ratio=cycle delay/cycle FFs] which is affected by **clustering**



A logical and physical design flow

- **Processing rate** (throughput $\times$ frequency)
  - Common objective maximized by all three approaches
  - $\leq$ **1/max-cycle-ratio** [cycle ratio=cycle delay/cycle FFs] which is affected by **clustering**

- Our take
  - maximizing post-layout processing rate by **minimizing max-cycle-ratio** during pre-layout clustering

$$\text{max-cycle-ratio} \downarrow \;\Rightarrow \frac{1}{\text{max}-\text{cycle}-\text{ratio}} \uparrow \Rightarrow \text{max-processing-rate} \uparrow$$

Assumptions

- How to evaluate global interconnect delay BEFORE layout?

Assumptions

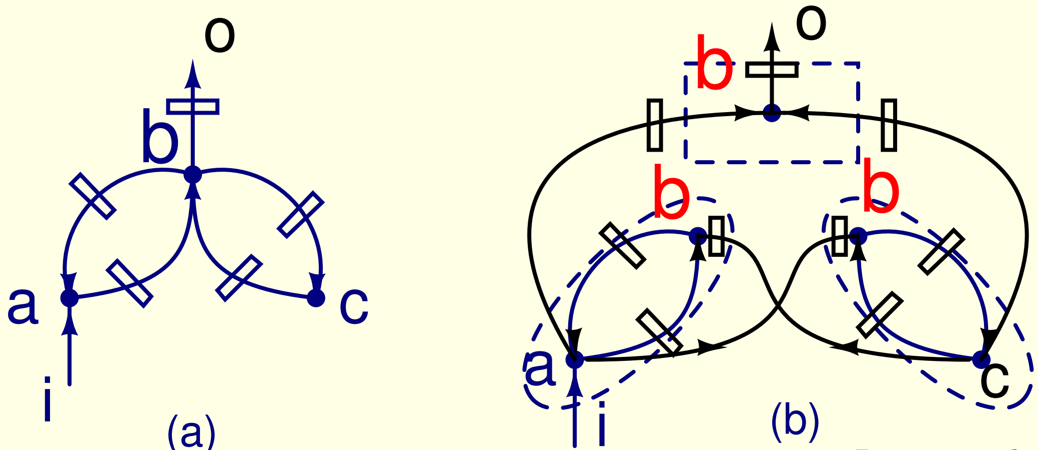
- How to evaluate global interconnect delay BEFORE layout?
  - Inter-cluster interconnect induces **extra constant delay** D

Assumptions

- How to evaluate global interconnect delay BEFORE layout?
  - Inter-cluster interconnect induces **extra constant delay** D
- **Gate replication** allows a gate (and its incident edges) to be assigned to more than one cluster

Assumptions

- How to evaluate global interconnect delay BEFORE layout?
  - Inter-cluster interconnect induces **extra constant delay** D
- **Gate replication** allows a gate (and its incident edges) to be assigned to more than one cluster
- Cluster size constraint



Pan *et al.* [TCAD'98]

Assumptions

- How to evaluate global interconnect delay BEFORE layout?
  - Inter-cluster interconnect induces **extra constant delay** D
- **Gate replication** allows a gate (and its incident edges) to be assigned to more than one cluster
- Cluster size constraint
- Ignore PI$\rightarrow$PO paths
  - FFs can be pipelined from PI to reduce period

Problem formulation

[Optimal Clustering]

- Given
  - $G = (V, E)$ strongly connected, V gates, E interconnects
  - Each $v \in V$ has a delay $d(v)$ and a size $s(v)$;
    Each $e \in E$ has a delay $d(e)$, a size $s(e)$ and a weight $w(e)$ (# of FFs).
  - Inter-cluster interconnect induces extra constant delay D
  - Size of a cluster $\leq$ constant A

Problem formulation

[Optimal Clustering]

- Given
  - $G = (V, E)$ strongly connected, V gates, E interconnects
  - Each $v \in V$ has a delay $d(v)$ and a size $s(v)$;
    Each $e \in E$ has a delay $d(e)$, a size $s(e)$ and a weight $w(e)$ (# of FFs).
  - Inter-cluster interconnect induces extra constant delay D
  - Size of a cluster $\leq$ constant A
- Find
  - A clustering with possible gate replication
  - **Maximum-cycle-ratio is minimized**

- Pan *et al.* [TCAD'98]
  - Minimize period lower bound by binary search
  - Labeling for feasibility checking by binary search
    - ∗ Need to start from PI
  - Pre-computed all pair longest path matrix

- Cong *et al.* [DAC'99]
  - Accelerate label computation
  - No all-pair longest path matrix

- This work
  - Obj is max-cycle-ratio
  - Strongly connected graph without PI→PO paths
  - Incremental
  - No all-pair longest path matrix

- Pan *et al.* [TCAD'98]
  - Minimize period lower bound by binary search
  - Labeling for feasibility checking by binary search
    - Need to start from PI
  - Pre-computed all pair longest path matrix
- Cong *et al.* [DAC'99]
  - Accelerate label computation
  - No all-pair longest path matrix

- This work
  - Obj is max-cycle-ratio
  - Strongly connected graph without PI—PO paths
  - Incremental
  - No all-pair longest path matrix

- Pan *et al.* [TCAD'98]
  - Minimize period lower bound by binary search
  - Labeling for feasibility checking by binary search
    - Need to start from PI
  - Pre-computed all pair longest path matrix
- Cong *et al.* [DAC'99]
  - Accelerate label computation
  - No all-pair longest path matrix
- This work
  - Obj is max-cycle-ratio
  - Strongly connected graph without PI→PO paths
  - Incremental
  - No all-pair longest path matrix

Useful results from previous works

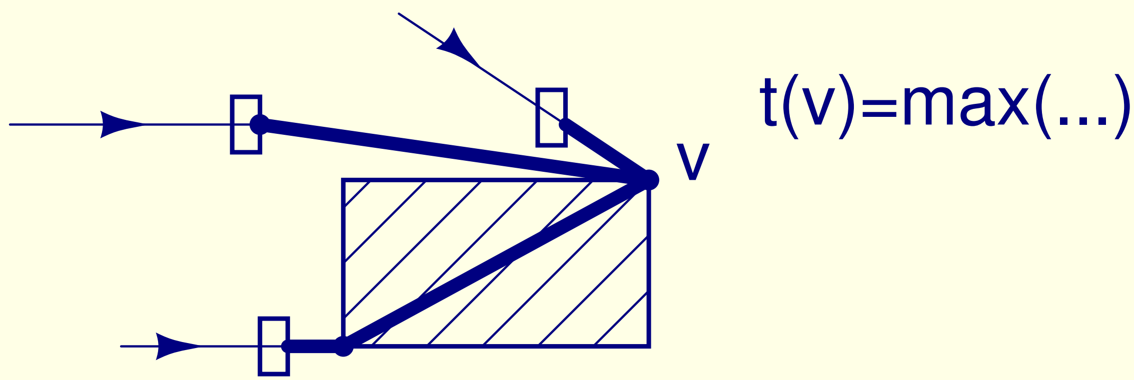
- Important results in Pan's work
  - A cluster has only one output, refer to as its root

Useful results from previous works

- Important results in Pan's work
  - A cluster has only one output, refer to as its root
  - A vertex $v \in V$ has at most one cluster c_v rooted at it

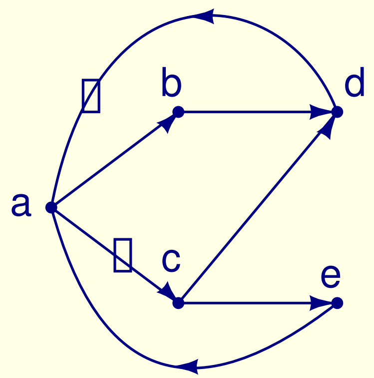
Useful results from previous works

- Important results in Pan's work
  - A cluster has only one output, refer to as its root
  - A vertex $v \in V$ has at most one cluster c_v rooted at it
  - The arrival time $t(v)$ of $v \in V$ is $\leq$ those of its replicas

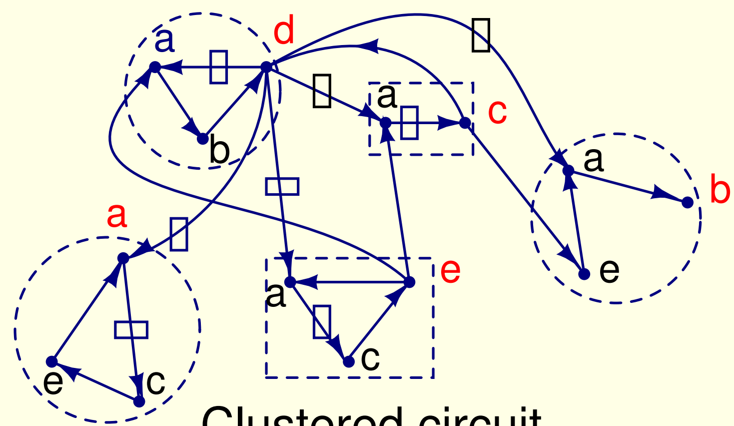


Useful results from previous works

- Important results in Pan's work
  - A cluster has only one output, refer to as its root
  - A vertex $v \in V$ has at most one cluster c_v rooted at it
  - The arrival time $t(v)$ of $v \in V$ is $\leq$ those of its replicas



Original circuit



Clustered circuit

Problem formulation

[Optimal Clustering]

(Objective)

$$\text{Minimize} \quad \phi$$

(Timing constraints)

$$t(v) \geq 0, \quad \forall v \in V \tag{1}$$

$$t(v) \leq t(v'), \quad \forall \text{ replicate } v' \text{ of } v \tag{2}$$

$$t(v) \geq t(u) + d(u,v) - w(u,v)\phi, \quad \forall \text{ local } (u,v) \in E_c \tag{3}$$

$$t(v) \geq t(u) + d(u,v) + D - w(u,v)\phi, \quad \forall \text{ global } (u,v) \in E_c \tag{4}$$

$$t(v) \geq t(u) + d(u,v) - w(u,v)\phi, \quad \forall (u,v) \in E \tag{5}$$

(Cluster size constraint)

$$|c_v| \leq A, \quad \forall v \in V \tag{6}$$

Problem formulation

[Optimal Clustering]

(Objective)

$$\text{Minimize} \quad \phi$$

(Timing constraints)

$$t(v) \geq 0, \quad \forall v \in V \tag{1}$$

$$t(v) \leq t(v'), \quad \forall \text{ replicate } v' \text{ of } v \tag{2}$$

$$t(v) \geq t(u) + d(u,v) - w(u,v)\phi, \quad \forall \text{ local } (u,v) \in E_c \tag{3}$$

$$t(v) \geq t(u) + d(u,v) + D - w(u,v)\phi, \quad \forall \text{ global } (u,v) \in E_c \tag{4}$$

$$t(v) \geq t(u) + d(u,v) - w(u,v)\phi, \quad \forall (u,v) \in E \tag{5}$$

(Cluster size constraint)

$$|c_v| \leq A, \quad \forall v \in V \tag{6}$$

Problem formulation

[Optimal Clustering]

(Objective)

$$\text{Minimize} \quad \phi$$

(Timing constraints)

$$t(v) \geq 0, \quad \forall v \in V \tag{1}$$

$$t(v) \leq t(v'), \quad \forall \text{ replicate } v' \text{ of } v \tag{2}$$

$$t(v) \geq t(u) + d(u,v) - w(u,v)\phi, \quad \forall \text{ local } (u,v) \in E_c \tag{3}$$

$$t(v) \geq t(u) + d(u,v) + D - w(u,v)\phi, \quad \forall \text{ global } (u,v) \in E_c \tag{4}$$

$$t(v) \geq t(u) + d(u,v) - w(u,v)\phi, \quad \forall (u,v) \in E \tag{5}$$

(Cluster size constraint)

$$|c_v| \leq A, \quad \forall v \in V \tag{6}$$

Problem formulation

[Optimal Clustering]

(Objective)

$$\text{Minimize} \quad \phi$$

(Timing constraints)

$$t(v) \geq 0, \quad \forall v \in V \tag{1}$$

$$t(v) \leq t(v'), \quad \forall \text{ replicate } v' \text{ of } v \tag{2}$$

$$t(v) \geq t(u) + d(u,v) - w(u,v)\phi, \quad \forall \text{ local } (u,v) \in E_c \tag{3}$$

$$t(v) \geq t(u) + d(u,v) + D - w(u,v)\phi, \quad \forall \text{ global } (u,v) \in E_c \tag{4}$$

$$t(v) \geq t(u) + d(u,v) - w(u,v)\phi, \quad \forall (u,v) \in E \tag{5}$$

(Cluster size constraint)

$$|c_v| \leq A, \quad \forall v \in V \tag{6}$$

Problem formulation

[Optimal Clustering]

(Objective)

$$\text{Minimize} \quad \phi$$

(Timing constraints)

$$t(v) \geq 0, \quad \forall v \in V \tag{1}$$

$$t(v) \leq t(v'), \quad \forall \text{ replicate } v' \text{ of } v \tag{2}$$

$$t(v) \geq t(u) + d(u,v) - w(u,v)\phi, \quad \forall \text{ local } (u,v) \in E_c \tag{3}$$

$$t(v) \geq t(u) + d(u,v) + \mathbf{D} - w(u,v)\phi, \quad \forall \text{ global } (u,v) \in E_c \tag{4}$$

$$t(v) \geq t(u) + d(u,v) - w(u,v)\phi, \quad \forall (u,v) \in E \tag{5}$$

(Cluster size constraint)

$$|c_v| \leq A, \quad \forall v \in V \tag{6}$$

Problem formulation

[Optimal Clustering]

(Objective)

$$\text{Minimize} \quad \phi$$

(Timing constraints)

$$t(v) \geq 0, \quad \forall v \in V \tag{1}$$

$$t(v) \leq t(v'), \quad \forall \text{ replicate } v' \text{ of } v \tag{2}$$

$$t(v) \geq t(u) + d(u,v) - w(u,v)\phi, \quad \forall \text{ local } (u,v) \in E_c \tag{3}$$

$$t(v) \geq t(u) + d(u,v) + D - w(u,v)\phi, \quad \forall \text{ global } (u,v) \in E_c \tag{4}$$

$$t(v) \geq t(u) + d(u,v) - w(u,v)\phi, \quad \forall (u,v) \in E \tag{5}$$

(Cluster size constraint)

$$|c_v| \leq A, \quad \forall v \in V \tag{6}$$

Problem formulation

[Optimal Clustering]

(Objective)

$$\text{Minimize} \quad \phi$$

(Timing constraints)

$$t(v) \geq 0, \quad \forall v \in V \tag{1}$$

$$t(v) \leq t(v'), \quad \forall \text{ replicate } v' \text{ of } v \tag{2}$$

$$t(v) \geq t(u) + d(u,v) - w(u,v)\phi, \quad \forall \text{ local } (u,v) \in E_c \tag{3}$$

$$t(v) \geq t(u) + d(u,v) + D - w(u,v)\phi, \quad \forall \text{ global } (u,v) \in E_c \tag{4}$$

$$t(v) \geq t(u) + d(u,v) - w(u,v)\phi, \quad \forall (u,v) \in E \tag{5}$$

(Cluster size constraint)

$$|c_v| \leq A, \quad \forall v \in V \tag{6}$$

Problem formulation

[Optimal Clustering]

(Objective)

$$\text{Minimize} \quad \phi$$

(Timing constraints)

$$t(v) \geq 0, \quad \forall v \in V \tag{1}$$

$$t(v) \leq t(v'), \quad \forall \text{ replicate } v' \text{ of } v \tag{2}$$

$$t(v) \geq t(u) + d(u,v) - w(u,v)\phi, \quad \forall \text{ local } (u,v) \in E_c \tag{3}$$

$$t(v) \geq t(u) + d(u,v) + D - w(u,v)\phi, \quad \forall \text{ global } (u,v) \in E_c \tag{4}$$

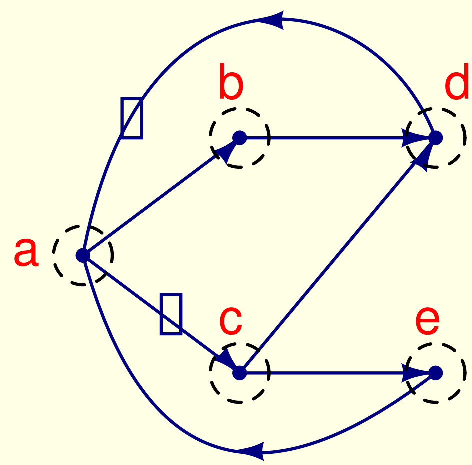
$$t(v) \geq t(u) + d(u,v) - w(u,v)\phi, \quad \forall (u,v) \in E \tag{5}$$

(Cluster size constraint)

$$|c_v| \leq A, \quad \forall v \in V \tag{6}$$

Algorithm outline

- Start with a feasible ϕ
  - Treat each gate as a cluster
  - Compute the maximum cycle ratio (Howard's algorithm)



Algorithm outline

- Start with a feasible ϕ
  - Treat each gate as a cluster
  - Compute the maximum cycle ratio (Howard's algorithm)
- Iteratively push ϕ down while keeping timing and cluster size constraints satisfied

Algorithm outline

- Start with a feasible ϕ
  - Treat each gate as a cluster
  - Compute the maximum cycle ratio (Howard's algorithm)
- Iteratively push ϕ down while keeping timing and cluster size constraints satisfied
  - Checking the existence of a smaller ϕ by **constructive clustering**

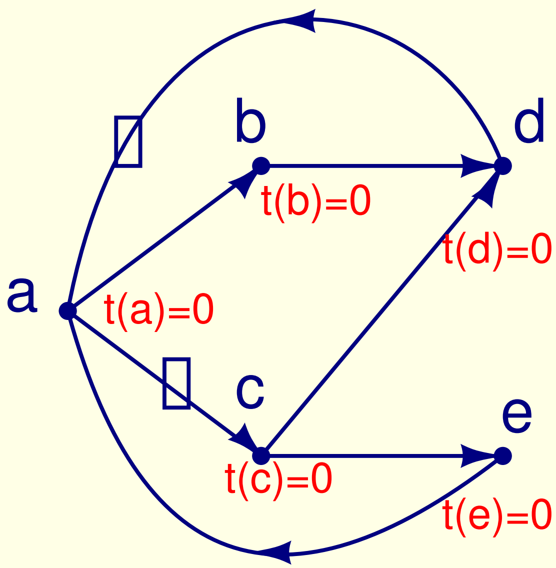
Algorithm outline

- Start with a feasible ϕ
  - Treat each gate as a cluster
  - Compute the maximum cycle ratio (Howard's algorithm)
- Iteratively push ϕ down while keeping timing and cluster size constraints satisfied
  - Checking the existence of a smaller ϕ by **constructive clustering**
    - If yes, new clustering has a smaller ϕ
    - If no, current ϕ is optimal

Constructive clustering under ϕ

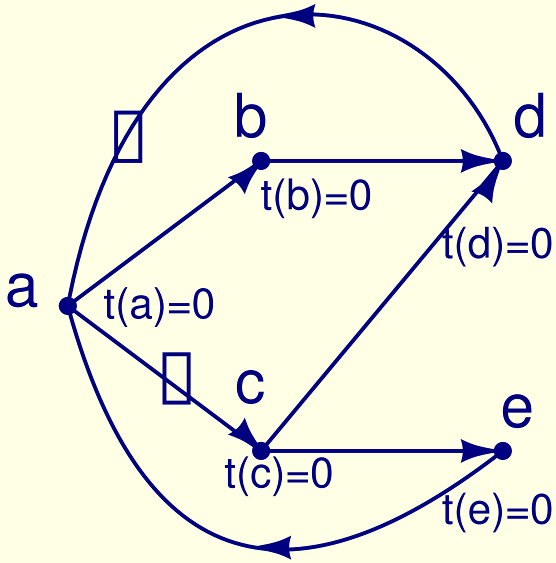
- Notations
  - Arrival time vector T $= \Big(t(1), t(2), ..., t(|V|)\Big)$
  - Bottom element ($\perp$): $t(v) = 0$, $\forall v \in V$

Constructive clustering under ϕ



1. **Start with** $\top = \bot$

Constructive clustering under ϕ

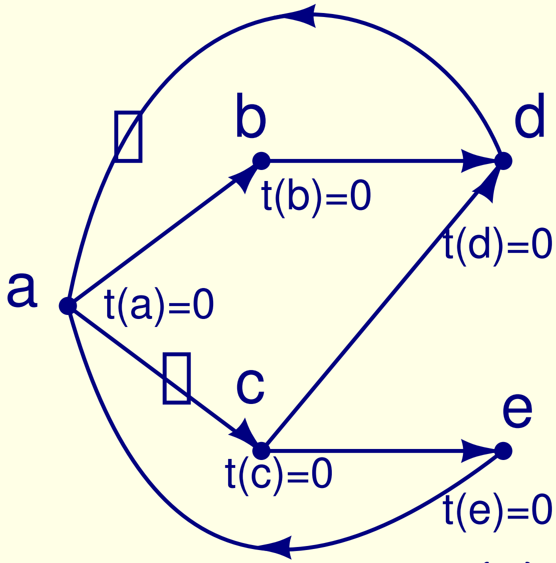


1. Start with $\top = \bot$
2. **Satisfy (1) and (5) by Bellman-Ford's algorithm**
   - **Denoted as** $BF(\top, \phi)$

$$t(v) \geq 0, \quad \forall v \in V \tag{1}$$

$$t(v) \geq t(u) + d(u,v) - w(u,v)\phi, \quad \forall (u,v) \in E \tag{5}$$

Constructive clustering under ϕ



1. Start with $\mathsf{T} = \perp$
2. Satisfy (1) and (5) by Bellman-Ford's algorithm
   - Denoted as $BF(\mathsf{T}, \phi)$
3. **Satisfy (2), (3), (4) and (6)**

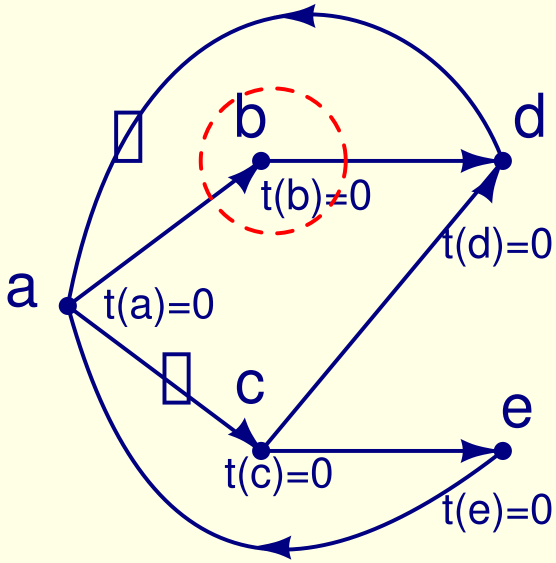
$$t(v) \leq t(v'), \quad \forall \text{ replicate } v' \text{ of } v \tag{2}$$

$$t(v) \geq t(u) + d(u,v) - w(u,v)\phi, \quad \forall \text{ local } (u,v) \in E_c \tag{3}$$

$$t(v) \geq t(u) + d(u,v) + D - w(u,v)\phi, \quad \forall \text{ global } (u,v) \in E_c \tag{4}$$

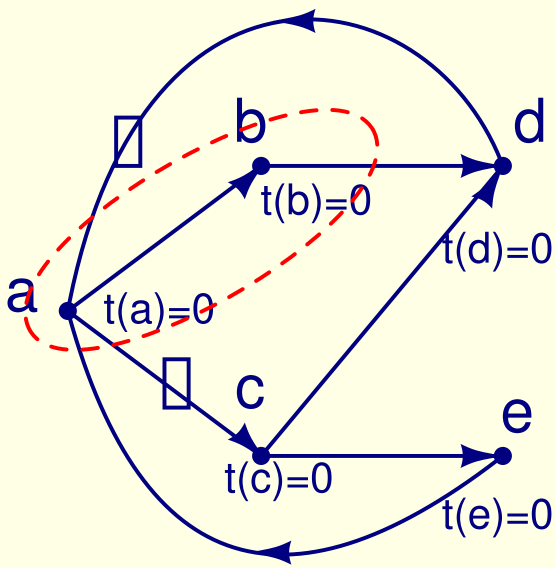
$$|c_v| \leq A, \quad \forall v \in V \tag{6}$$

Constructive clustering under ϕ



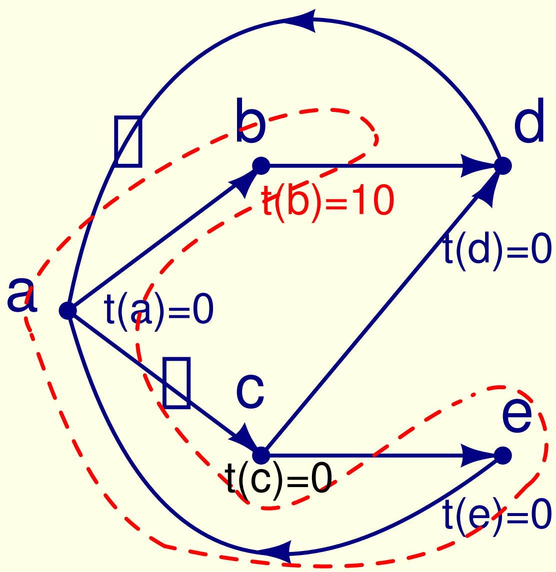
1. Start with $\mathsf{T} = \perp$
2. Satisfy (1) and (5) by Bellman-Ford's algorithm
   - Denoted as $BF(\mathsf{T}, \phi)$
3. **Construct c_v rooted at v, $\forall v \in V$**
   - **Include one critical input at a time**
     - path $p : u \rightsquigarrow v$ is in c_v
     - $t(v) = t(u) + D + d(p) - w(p)\phi$

Constructive clustering under ϕ



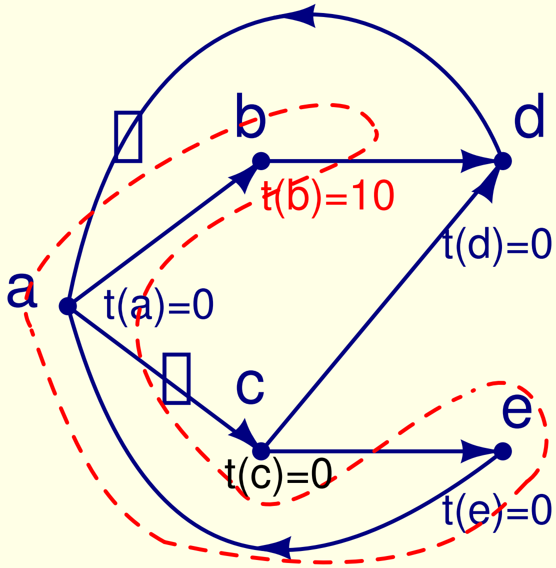
1. Start with $\mathsf{T} = \perp$
2. Satisfy (1) and (5) by Bellman-Ford's algorithm
   - Denoted as $BF(\mathsf{T}, \phi)$
3. **Construct c_v rooted at v, $\forall v \in V$**
   - **Include one critical input at a time**
     - path $p : u \rightsquigarrow v$ is in c_v
     - $t(v) = t(u) + D + d(p) - w(p)\phi$

Constructive clustering under ϕ



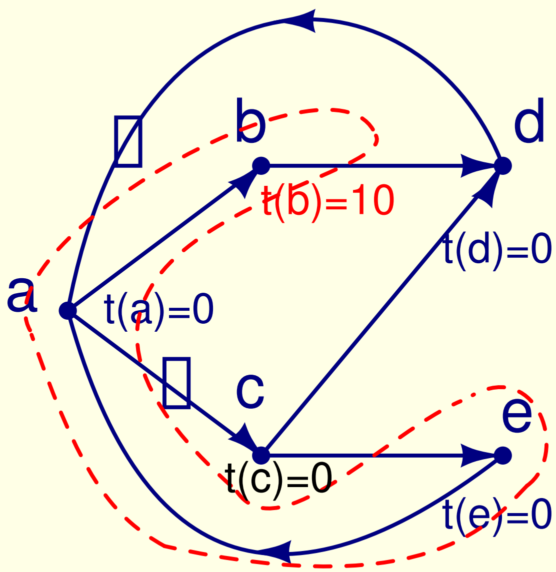
1. Start with $\mathsf{T} = \perp$
2. Satisfy (1) and (5) by Bellman-Ford's algorithm
   - Denoted as $BF(\mathsf{T}, \phi)$
3. **Construct c_v rooted at v, $\forall v \in V$**
   - **Include one critical input at a time**
     - path $p : u \rightsquigarrow v$ is in c_v
     - $t(v) = t(u) + D + d(p) - w(p)\phi$

Constructive clustering under ϕ



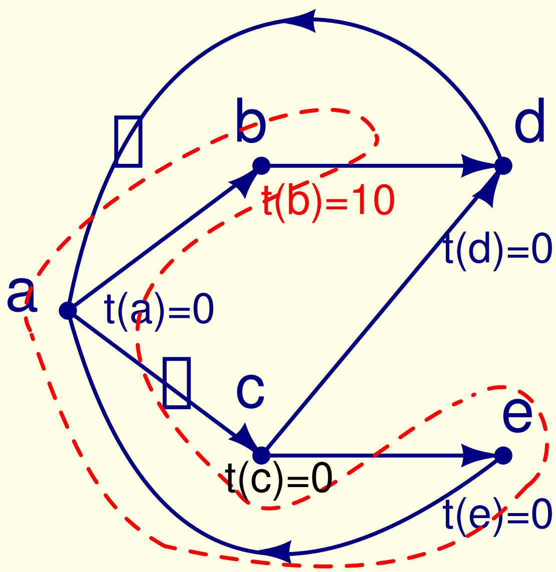
1. Start with $\mathsf{T} = \perp$
2. Satisfy (1) and (5) by Bellman-Ford's algorithm
   - Denoted as $BF(\mathsf{T}, \phi)$
3. **Construct c_v rooted at v, $\forall v \in V$**
   - **Include one critical input at a time**
   - **Size $|c_v| = A$, or no critical input**

Constructive clustering under ϕ



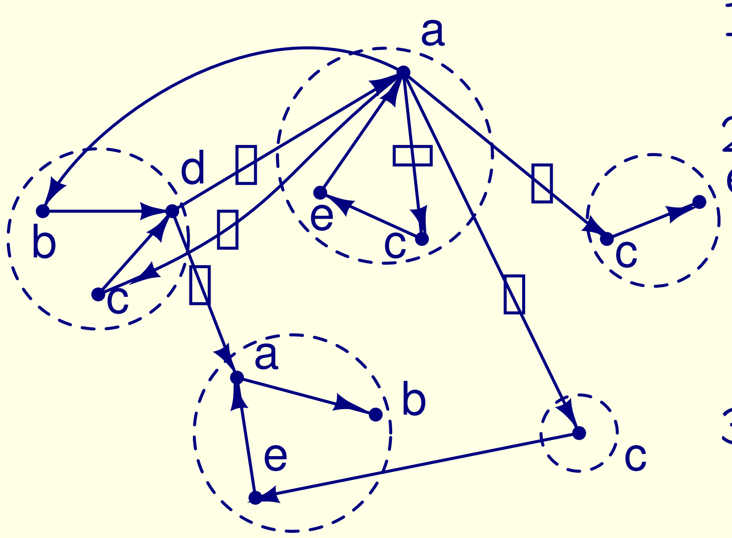
1. Start with $\mathsf{T} = \perp$
2. Satisfy (1) and (5) by Bellman-Ford's algorithm
   - Denoted as $BF(\mathsf{T}, \phi)$
3. **Construct c_v rooted at v, $\forall v \in V$**
   - **Denoted as $\mathcal{L}_v(\mathsf{T}, \phi)$**

Constructive clustering under ϕ



1. Start with $\mathsf{T} = \perp$
2. Satisfy (1) and (5) by Bellman-Ford's algorithm
   - Denoted as $BF(\mathsf{T}, \phi)$
3. **Construct c_v rooted at v, $\forall v \in V$**
   - **Denoted as $\mathcal{L}_v(\mathsf{T}, \phi)$**
   - **(2), (3), (4) and (6) are satisfied at c_v**
   - **BUT (1) and (5) may be violated**

Constructive clustering under ϕ



1. Start with $\mathsf{T} = \perp$
2. Satisfy (1) and (5) by Bellman-Ford's algorithm
   - Denoted as $BF(\mathsf{T}, \phi)$
3. Construct c_v rooted at v, $\forall v \in V$
   - Denoted as $\mathcal{L}_v(\mathsf{T}, \phi)$
4. **Iterate 2 and 3 until all satisfied**
   - **Denote one iteration as**

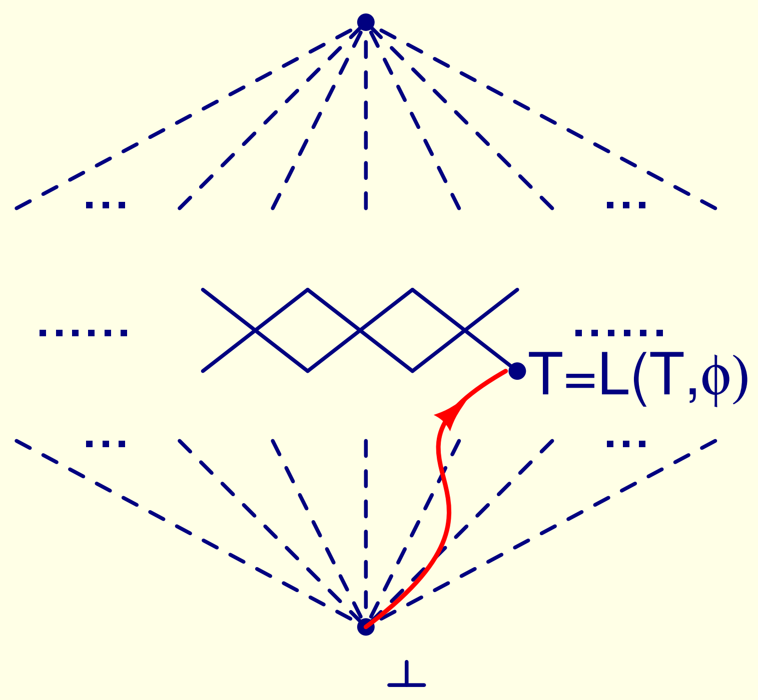
$$\mathcal{L}(\mathsf{T}, \phi) \triangleq BF\Big(\big(\mathcal{L}_1(\mathsf{T}, \phi), ..., \mathcal{L}_{|V|}(\mathsf{T}, \phi)\big), \phi\Big)$$

Constructive clustering under ϕ

- All constrains are satisfied $\iff \mathsf{T} = \mathcal{L}(\mathsf{T}, \phi)$
  - T is called a **fixpoint** of $\mathcal{L}$ under ϕ

Constructive clustering under ϕ

- ϕ is feasible $\iff$ $\mathcal{L}$ has a fixpoint under ϕ $\iff$ Applying $\mathcal{L}$ from $\perp$ is finitely convergent to a fixpoint



Constructive clustering under ϕ

- ϕ is feasible $\iff$ $\mathcal{L}$ has a fixpoint under ϕ $\iff$ Applying $\mathcal{L}$ from $\perp$ is finitely convergent to a fixpoint
- ϕ is infeasible $\Leftarrow$ $\mathsf{T} > BF(\perp, \phi)$

Optimality checking

- ϕ is optimal $\Leftarrow$ $\phi - 1/(|V|N_{\mathsf{ff}})^2$ is infeasible
  - ϕ is the maximum-cycle-ratio of a clustering satisfying size constraint
  - N_{ff} is the max # of FF on any acyclic path in G

Optimality checking

- ϕ is optimal $\Leftarrow$ $\phi - 1/(|V|N_{\mathsf{ff}})^2$ is infeasible
  - ϕ is the maximum-cycle-ratio of a clustering satisfying size constraint
  - N_{ff} is the max # of FF on any acyclic path in G
- Optimal $\phi \geq$ maximum-cycle-ratio of G

Optimal Clustering Algorithm

$c_v^{\mathsf{opt}} \leftarrow c_v \leftarrow \{v\}$, $\forall v \in V$;

$\phi^{\mathsf{opt}} \leftarrow \phi \leftarrow$ `maximum cycle ratio of` G_c;

$\phi_{\mathsf{lb}} \leftarrow$ `maximum cycle ratio of` G;

`While` $\left(\phi \geq \phi_{\mathsf{lb}} + 1/(|V|N_{\mathsf{ff}})^2\right)$ `do`

    $\phi \leftarrow \phi - 1/(|V|N_{\mathsf{ff}})^2$;

    $\top \leftarrow BF(\perp, \phi)$;

    `While` $\left(\left(\top \neq \mathcal{L}(\top, \phi)\right) \wedge \left(\neg(\top > BF(\perp, \phi))\right)\right)$ `do`

        $\top \leftarrow \mathcal{L}(\top, \phi)$;

    `If` $\left(\top > BF(\perp, \phi)\right)$ `then`

        `break;`

    `Update` ϕ, c^{opt} `and` ϕ^{opt};

`Return` c^{opt} `and` ϕ^{opt};

Optimal Clustering Algorithm

$c_v^{\mathsf{opt}} \leftarrow c_v \leftarrow \{v\}$, $\forall v \in V$;

$\phi^{\mathsf{opt}} \leftarrow \phi \leftarrow$ `maximum cycle ratio of` G_c;

$\phi_{\mathsf{lb}} \leftarrow$ `maximum cycle ratio of` G;

`While` $\left(\phi \geq \phi_{\mathsf{lb}} + 1/(|V|N_{\mathsf{ff}})^2\right)$ `do`

  $\phi \leftarrow \phi - 1/(|V|N_{\mathsf{ff}})^2$;

  $\top \leftarrow BF(\bot, \phi)$;

  `While` $\left(\left(\top \neq \mathcal{L}(\top, \phi)\right) \wedge \left(\neg(\top > BF(\bot, \phi))\right)\right)$ `do`

    $\top \leftarrow \mathcal{L}(\top, \phi)$;

  `If` $\left(\top > BF(\bot, \phi)\right)$ `then`

    `break;`

  `Update` ϕ, c^{opt} `and` ϕ^{opt};

`Return` c^{opt} `and` ϕ^{opt};

Optimal Clustering Algorithm

$c_v^{\text{opt}} \leftarrow c_v \leftarrow \{v\}$, $\forall v \in V$;

$\phi^{\text{opt}} \leftarrow \phi \leftarrow$ maximum cycle ratio of G_c;

$\phi_{\text{lb}} \leftarrow$ maximum cycle ratio of G;

While $\left(\phi \geq \phi_{\text{lb}} + 1/(|V|N_{\text{ff}})^2\right)$ do

$\quad \phi \leftarrow \phi - 1/(|V|N_{\text{ff}})^2$;

$\quad \top \leftarrow BF(\bot, \phi)$;

$\quad$ While $\left(\left(\top \neq \mathcal{L}(\top, \phi)\right) \wedge \left(\neg(\top > BF(\bot, \phi))\right)\right)$ do

$\quad\quad \top \leftarrow \mathcal{L}(\top, \phi)$;

$\quad$ If $\left(\top > BF(\bot, \phi)\right)$ then

$\quad\quad$ break;

$\quad$ Update ϕ, c^{opt} and ϕ^{opt};

Return c^{opt} and ϕ^{opt};

Optimal Clustering Algorithm

```
c_v^opt ← c_v ← {v}, ∀v ∈ V;
φ^opt ← φ ← maximum cycle ratio of G_c;
φ_lb ← maximum cycle ratio of G;
While (φ ≥ φ_lb + 1/(|V|N_ff)^2) do
    φ ← φ − 1/(|V|N_ff)^2;
    ⊤ ← BF(⊥, φ);
    While ((⊤ ≠ L(⊤, φ)) ∧ (¬(⊤ > BF(⊥, φ)))) do
        ⊤ ← L(⊤, φ);
    If (⊤ > BF(⊥, φ)) then
        break;
    Update φ, c^opt and φ^opt;
Return c^opt and φ^opt;
```

$c_v^{\text{opt}} \leftarrow c_v \leftarrow \{v\},\ \forall v \in V$;

$\phi^{\text{opt}} \leftarrow \phi \leftarrow$ maximum cycle ratio of G_c;

$\phi_{\text{lb}} \leftarrow$ maximum cycle ratio of G;

While $\left(\phi \geq \phi_{\text{lb}} + 1/(|V|N_{\text{ff}})^2\right)$ do

  $\phi \leftarrow \phi - 1/(|V|N_{\text{ff}})^2$;

  $\top \leftarrow BF(\bot, \phi)$;

  While $\left(\left(\top \neq \mathcal{L}(\top, \phi)\right) \wedge \left(\neg(\top > BF(\bot, \phi))\right)\right)$ do

    $\top \leftarrow \mathcal{L}(\top, \phi)$;

  If $\left(\top > BF(\bot, \phi)\right)$ then

    break;

  Update ϕ, c^{opt} and ϕ^{opt};

Return c^{opt} and ϕ^{opt};

Optimal Clustering Algorithm

$c_v^{\text{opt}} \leftarrow c_v \leftarrow \{v\}$, $\forall v \in V$;

$\phi^{\text{opt}} \leftarrow \phi \leftarrow$ `maximum cycle ratio of` G_c;

$\phi_{\text{lb}} \leftarrow$ `maximum cycle ratio of` G;

`While` $\left(\phi \geq \phi_{\text{lb}} + 1/(|V|N_{\text{ff}})^2\right)$ `do`

$\quad\phi \leftarrow \phi - 1/(|V|N_{\text{ff}})^2$;

$\quad\top \leftarrow BF(\perp, \phi)$;

$\quad$`While` $\left(\left(\top \neq \mathcal{L}(\top, \phi)\right) \wedge \left(\neg(\top > BF(\perp, \phi))\right)\right)$ `do`

$\quad\quad\top \leftarrow \mathcal{L}(\top, \phi)$;

$\quad$`If` $\left(\top > BF(\perp, \phi)\right)$ `then`

$\quad\quad$`break;`

$\quad$`Update` ϕ, c^{opt} `and` ϕ^{opt};

`Return` c^{opt} `and` ϕ^{opt};

Optimal Clustering Algorithm

$c_v^{\mathsf{opt}} \leftarrow c_v \leftarrow \{v\}$, $\forall v \in V$;

$\phi^{\mathsf{opt}} \leftarrow \phi \leftarrow$ `maximum cycle ratio of` G_c;

$\phi_{\mathsf{lb}} \leftarrow$ `maximum cycle ratio of` G;

`While` $\left(\phi \geq \phi_{\mathsf{lb}} + 1/(|V|N_{\mathsf{ff}})^2\right)$ `do`

  $\phi \leftarrow \phi - 1/(|V|N_{\mathsf{ff}})^2$;

  $\top \leftarrow BF(\bot, \phi)$;

  `While` $\left(\left(\top \neq \mathcal{L}(\top, \phi)\right) \wedge \left(\neg(\top > BF(\bot, \phi))\right)\right)$ `do`

    $\top \leftarrow \mathcal{L}(\top, \phi)$;

  `If` $\left(\top > BF(\bot, \phi)\right)$ `then`

    `break;`

  `Update` ϕ, c^{opt} `and` ϕ^{opt};

`Return` c^{opt} `and` ϕ^{opt};

Optimal Clustering Algorithm

$c_v^{\mathsf{opt}} \leftarrow c_v \leftarrow \{v\}$, $\forall v \in V$;

$\phi^{\mathsf{opt}} \leftarrow \phi \leftarrow$ maximum cycle ratio of G_c;

$\phi_{\mathsf{lb}} \leftarrow$ maximum cycle ratio of G;

While $\left(\phi \geq \phi_{\mathsf{lb}} + 1/(|V|N_{\mathsf{ff}})^2\right)$ do

    $\phi \leftarrow \phi - 1/(|V|N_{\mathsf{ff}})^2$;

    $\top \leftarrow BF(\bot, \phi)$;

    While $\left(\left(\top \neq \mathcal{L}(\top, \phi)\right) \wedge \left(\neg(\top > BF(\bot, \phi))\right)\right)$ do

        $\top \leftarrow \mathcal{L}(\top, \phi)$;

    If $\left(\top > BF(\bot, \phi)\right)$ then

        break;

    Update ϕ, c^{opt} and ϕ^{opt};

Return c^{opt} and ϕ^{opt};

Optimal Clustering Algorithm

$c_v^{\mathsf{opt}} \leftarrow c_v \leftarrow \{v\}$, $\forall v \in V$;

$\phi^{\mathsf{opt}} \leftarrow \phi \leftarrow$ maximum cycle ratio of G_c;

$\phi_{\mathsf{lb}} \leftarrow$ maximum cycle ratio of G;

While $\left(\phi \geq \phi_{\mathsf{lb}} + 1/(|V|N_{\mathsf{ff}})^2\right)$ do

    $\phi \leftarrow \phi - 1/(|V|N_{\mathsf{ff}})^2$; $\triangleleft$ *Resultant improvement is not small*

    $\top \leftarrow BF(\bot, \phi)$;

    While $\left(\left(\top \neq \mathcal{L}(\top, \phi)\right) \wedge \left(\neg(\top > BF(\bot, \phi))\right)\right)$ do

        $\top \leftarrow \mathcal{L}(\top, \phi)$;

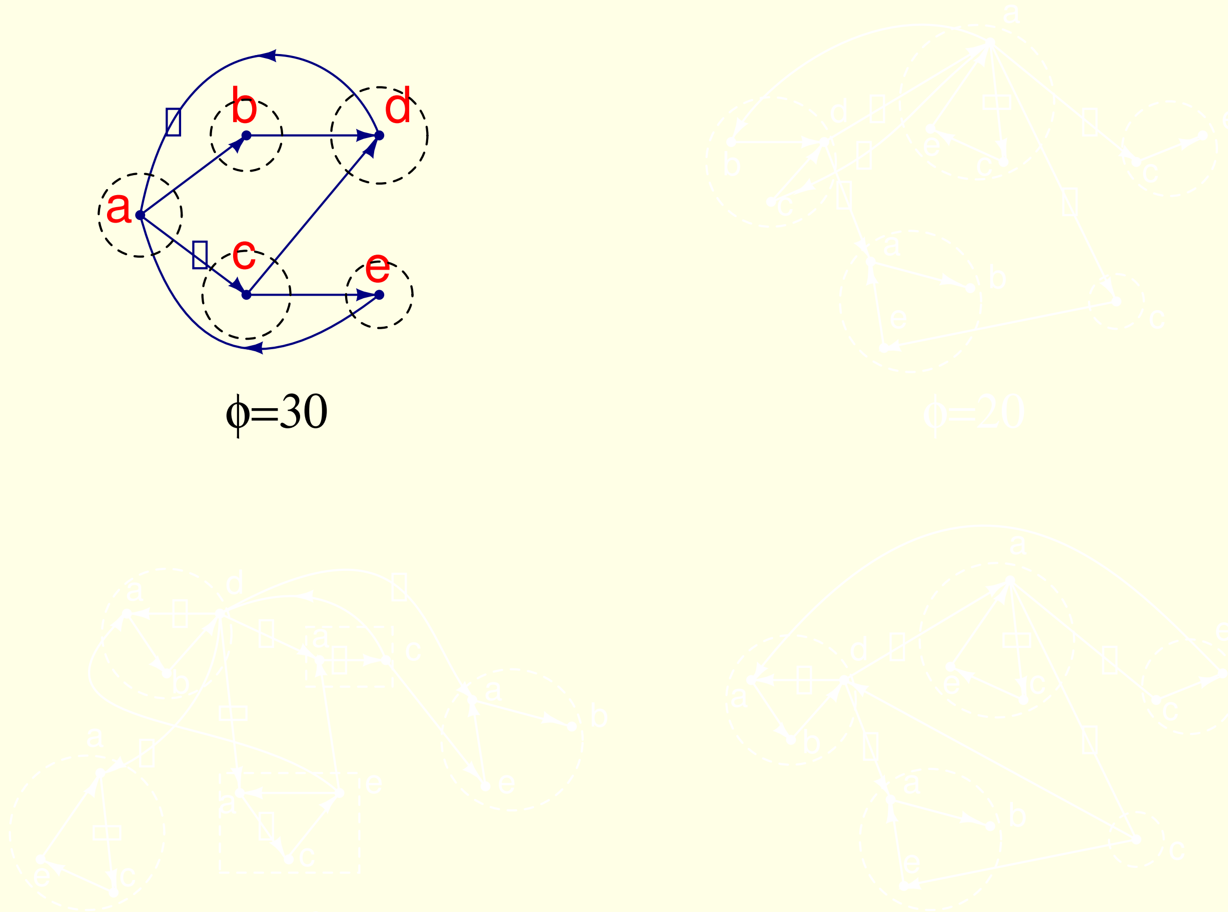
    If $\left(\top > BF(\bot, \phi)\right)$ then

        break;

    Update ϕ, c^{opt} and ϕ^{opt};

Return c^{opt} and ϕ^{opt};

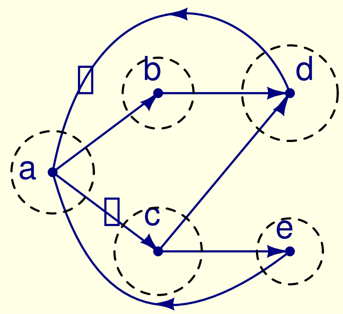
An example (A=3, D=10)



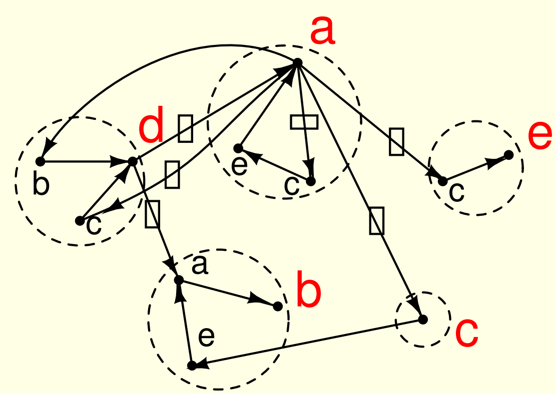
ϕ=30 ϕ=20

ϕ=10 ϕ=15

An example (A=3, D=10)



ϕ=30

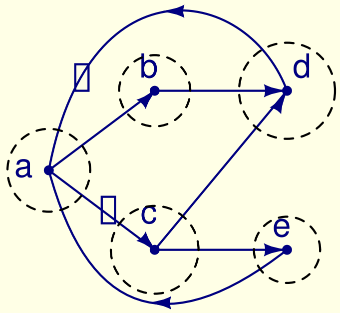


ϕ=20

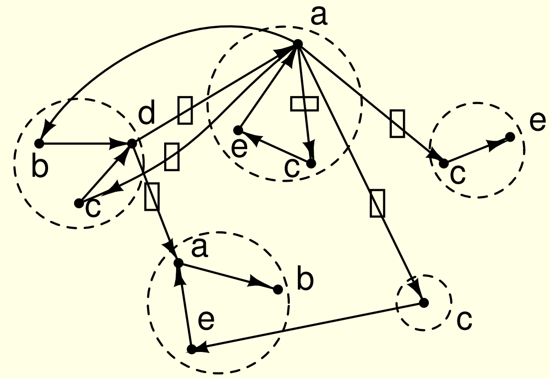
ϕ=10

ϕ=15

An example (A=3, D=10)

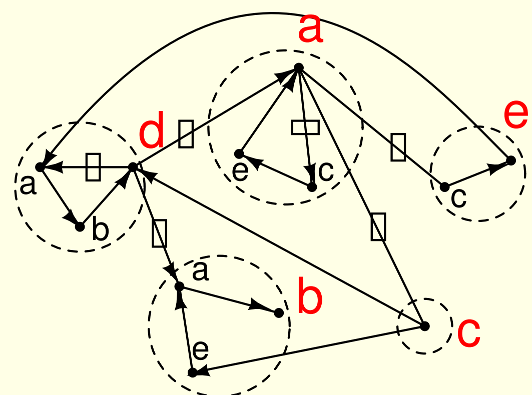


ϕ=30



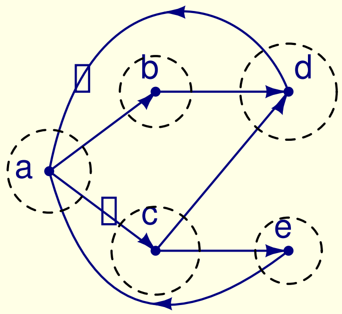
ϕ=20

ϕ=10

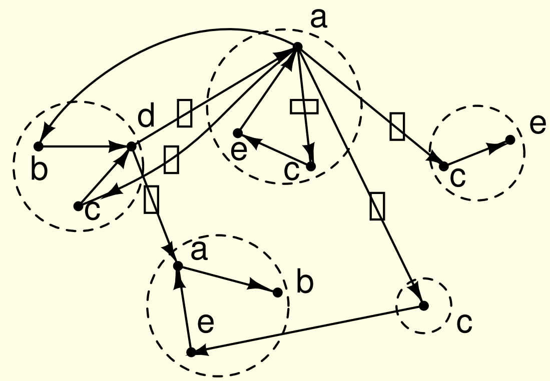


ϕ=15

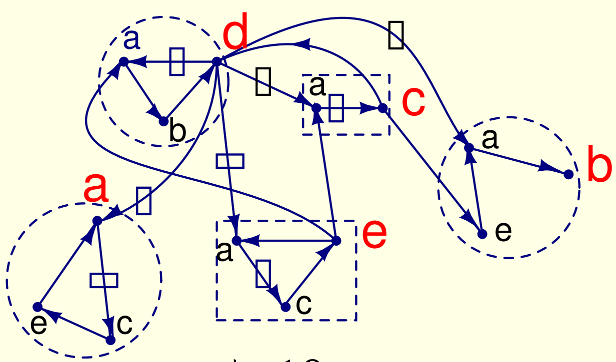
An example (A=3, D=10)



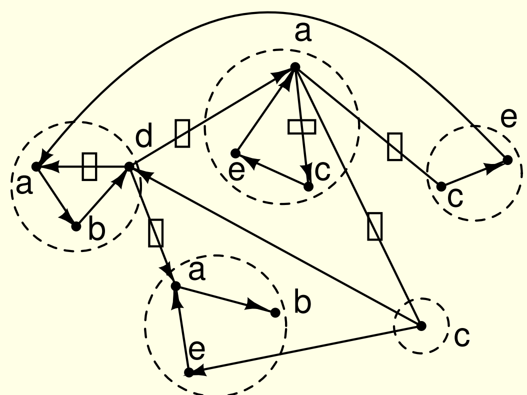
ϕ=30



ϕ=20



ϕ=10



ϕ=15

Experiment setup

- ISCAS89 benchmark
- Connect PO$\rightarrow$FF$\rightarrow$PI
- Gate size 1, gate delay 1
- Interconnect size 0, intra-cluster delay 0, inter-cluster $D = 2$
- $A = 5\%|V|, 10\%|V|, 20\%|V|$

Optimal Maximum-cycle-ratio

Circuit	$A = 5\%\|V\|$			$A = 10\%\|V\|$			$A = 20\%\|V\|$		
	ϕ^{opt}	**#iter**	time(s)	ϕ^{opt}	#iter	time(s)	ϕ^{opt}	#iter	time(s)
s208	13.00	**4**	0.20	11.00	4	0.09	10.00	3	0.04
s349	18.00	**14**	1.73	16.00	21	0.47	14.67	14	1.39
s420	14.00	**3**	0.33	13.00	3	0.34	12.00	2	0.00
s635	75.00	**4**	5.71	70.00	4	8.02	68.00	4	6.43
s838	17.00	**3**	0.69	16.00	2	0.01	16.00	2	0.01
s1196	26.00	**3**	2.86	25.00	3	3.40	24.00	2	0.02
s1423	55.00	**3**	2.77	53.00	2	0.07	53.00	2	0.06
s1512	23.78	**15**	166.36	22.50	8	0.79	22.50	8	0.68
s3330	14.33	**11**	11.34	14.00	10	1.82	14.00	10	1.82
s4863	30.25	**8**	133.54	30.00	5	3.85	30.00	5	3.70
s5378	21.00	**3**	0.94	21.00	3	0.73	21.00	3	0.73
s9234	38.00	**3**	3.78	38.00	3	2.98	38.00	3	2.98
s35932	27.00	**4**	48.13	27.00	4	47.79	27.00	4	46.59
s38584	48.00	**2**	21.92	48.00	2	21.90	48.00	2	21.46

Run-time comparison with Pan *et al.*'s work

Circuit	time/step (s)	
	Pan's	presented
s208	0.01	0.00
s349	0.19	0.02
s420	0.03	0.00
s635	0.10	0.03
s838	0.13	0.01
s1196	0.08	0.02
s1423	0.65	0.04
s1512	1.22	0.06
s3330	1.07	0.17
s4863	82.30	1.04
s5378	7.58	0.31
s9234	n/a	1.20
s35932	n/a	11.46
s38584	n/a	11.07
arith	19.39X	1
geo	12.78X	1

Techniques for replication reduction

- We did not do experiments on area overhead
- Previous works [Sangiovanni-Vincentelli *et al.* ICCAD'91, Pan *et al.* TCAD'98, Cong *et al.* DAC'99]
  - Allow a cluster to have multiple outputs
  - Relax timing on non-critical paths
  - Merge small clusters

Summary

- Processing rate is identified as an important metric for sequential circuits
- max processing rate $\leq 1/$ maximum-cycle-ratio (affected by clustering)
- A new algorithm is presented
  - Finds a clustering with minimal maximum-cycle-ratio
  - Efficient in practice
  - Incremental

Erratum

- The algorithm presented in this talk is a little different from the one in the paper
  - There is a slip in the original proof of the termination criterion
  - The correct version can also be found in the technical report

Thank you !