

Kolmogorov Complexity – Assignment 1

1. Let p_x be the shortest program that generates the string x . Show that for some constant c independent of x , p_x is c -random, i.e. $C(p_x) \geq |p_x| - c$.
2. Show that for every c there are strings x and y such that $C(xy) \geq C(x) + C(y) + c$.
3. Show that for large enough n , if $|x| = n$, $|z| > 3 \log n$ and $C(x|z) \geq n$ then z is not a substring of x .
4. Let $R = \{x \mid C(x) \geq |x|\}$. In class we showed that R has at least one string of every length.
 - (a) Let m be the number of strings in $R \cap \Sigma^n$. Show that $C(m) \geq n - c \log n$ for some constant c .
Hint: Let y be the lexicographically least random string of length n . Think about $C(y|m)$.
 - (b) Use the above to show that for some constant k , for every n there are at least $2^n/n^k$ strings of length n in R .

For extra credit show there are at least $2^n/c$ strings of length n in R for some constant c