Theory of Computation Final

Due Thursday, June 11th at 4 PM. One point deduction for each minute late.

You may only use your textbook, assignments and notes from the course. In particular you should not use the Internet in any way. You may not discuss these questions with anyone except the instructor.

Each main question is 20 points for a total of 100 points.

Prove all of your answers.

1. What classes are defined by each of the following, where L is the class DSPACE($O(\log n)$):

   (a) The set of languages A such that for some B in L.

   $$x \in A \Leftrightarrow \exists y \in \Sigma^* \ (x, y) \in B$$

   for all $x \in \Sigma^*$.

   (b) The set of languages A such that for some B in L and some polynomial p,

   $$x \in A \Leftrightarrow \exists y \in \Sigma^* \ |y| = p(|x|) \text{ and } (x, y) \in B$$

   for all $x \in \Sigma^*$.

2. Show Set Splitting, as defined below, is NP-complete.

   Input: A collection $\mathcal{C}$ of subsets of $\{1, \ldots, n\}$.

   Question: Is there a partition of $\{1, \ldots, n\}$ into S_1 and S_2 such that no $C \in \mathcal{C}$ is a subset of S_1 or S_2.

3. A linear program is a set of variables $x_1, \ldots, x_n$ and a list of linear inequalities among these variables such as

   $$5x_1 - 4x_2 + 9x_3 + 6x_7 - 4x_2 \leq -5$$

   A linear program is feasible if there exists a setting of the x_i's to real values that fulfill all of the equations. Show that there is a log-space function f that reduces the monotone-CVAL problem to feasible linear programs.

   With the fact that one can check feasibility of linear programs in P (hard proof), this result shows that linear program feasibility is P-complete.

   Recall monotone-CVAL is the set of circuits of AND and OR gates (with no negations) and TRUE/FALSE inputs that evaluate to TRUE.

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4. Show that L is in NEXP if and only if there is a polynomial-time oracle Turing machine M and a polynomial q such that for all $x \in \Sigma^*$,

$$x \in L \Leftrightarrow \exists A \subseteq \Sigma^* \forall y \in \Sigma^* \text{ such that } |y| = q(|x|), M^A(x, y) \text{ accepts.}$$

Hint: Think tableau.

5. We say a function $f : \Sigma^* \rightarrow \mathbb{N}$ is in the class #P (pronounced "Sharp-P") if there is a nondeterministic polynomial time Turing machine M such that $f(x)$ is the number of possible accepting tableaus for $M(x)$.

   (a) Show that if every function f in #P is also computable in polynomial time then P = NP.

   (b) Show that if f and g is in #P then the function h is also in #P where $h(x) = f(x) + g(x)$.

   (c) Show that if f and g is in #P then the function r is also in #P where $r(x) = f(x)g(x)$ (ordinary integer multiplication).