Theory of Computation – Assignment 1

Read Chapters 1.1-1.3, 2 (except for 2.3.2)

1. Exhibit a Turing machine that decides the language

$$\{0^n 1^n \mid n \geq 0\}.$$

   0^n is the string of n zeros. For example $0^3 1^4$ is the string 0001111.

   This will be the only time I want you to describe an actual Turing machine. In the future you only need give an informal algorithm, as in the next problem.

2. Show the language

$$\text{PRIMES} = \{x \mid x \text{ is the binary representation of a prime number}\}$$

   is decidable.

3. We can consider a Turing machine with a infinite 2-dimensional tape where the head can move up and down as well as left and right. The tape is infinite in both dimensions.

   (a) Give a formal definition of 2-dimensional Turing machines.

   (b) Show that any language accepted by a 2-dimensional Turing machine is also accepted by a standard Turing machine.

4. Describe a bijection (1-1 and onto function) mapping $\mathcal{N}$ to Σ^*. $\mathcal{N}$ is the set of non-negative integers.