Theory of Computing – Assignment 4

1. Show that for all $\epsilon > 0$,
$$n \log n = O(n^{1+\epsilon}).$$

2. Consider a Turing machine on a 2-way infinite tape, with blanks on the left of the input and the head is initially on the first character of the input.

   Let L be a language accepted by a Turing machine with a 2-way infinite tape in time $t(n) \geq n$. Show that there is a Turing machine with a 1-way infinite tape that accepts L in time $O(t(n))$.

3. Show the following are equivalent for every language L.

   (a) L is in NTIME(n^k) for some k.

   (b) There are constants c and d and a language A in DTIME(n^c) such that for all x,
   $$x \in L \text{ if and only if for some } y \text{ with } |y| = |x|^d,\ (x, y) \in A$$

   (c) There is a constant d and a language A in DSPACE$(\log n)$ such that for all x,
   $$x \in L \text{ if and only if for some } y \text{ with } |y| = |x|^d,\ (x, y) \in A$$

   Hint: Let M be the nondeterministic machine accepting L and let y be the tableau for $M(x)$.

4. Do you think 3(c) above is also equal to NSPACE$(\log n)$? Why or why not?