

Theory of Computing - Assignment 5

1. Suppose L is accepted by a multi-tape nondeterministic Turing machine M using constructible time $t(n)$. Show that there is a 2-tape nondeterministic Turing machine N using time $O(t(n))$ also accepting L .

Hint: Use one tape of N to guess the state and the symbol under each tape head at every time step of $M(x)$. Use the other tape to simulate each of M 's tapes, one at a time.

2. Let $\kappa(G)$ be the size of the largest clique in graph G .
 - (a) Suppose $P=NP$. Show that $\kappa(G)$ is computable in polynomial time.
 - (b) Suppose $P=NP$. Show that there is a polynomial-time computable function f such that $f(G)$ outputs a clique of G of size $\kappa(G)$.
3. A Boolean formula is in Disjunctive Normal Form (DNF) if it is the OR of clauses, each of which is an AND of literals. Show that given a formula ϕ in DNF, we can determine if ϕ is satisfiable in polynomial-time.
4. Let C be a collection of subsets of a set S . A hitting set is a subset S' of S such that S' contains at least one element from each set in C . For example if $S = \{a, b, c, d, e\}$ and $C = \{\{a, b, c\}, \{c, d, e\}, \{d\}\}$ then $\{c, d\}$ is a hitting set of C .
Let $L = \{(C, k) \mid C \text{ has a hitting set of size } k\}$. Show that L is NP-complete.