Theory of Computing - Assignment 7

For a 3-CNF formula ϕ, let MAX-SAT(ϕ) be the maximum number of clauses satisfiable by any assignment of the variables.

Theorem 1 (PCP Theorem) *For any L in NP there is a polynomial-time computable function f such that for all x, $f(x)$ is a 3-CNF formula with m clauses and*

- *If x is in L then MAX-SAT(ϕ)= m.*
- *If x is not in L then MAX-SAT(ϕ)$\leq \frac{7m}{8}$.*

Recall in class we gave a poly-time computable function f such that if ϕ a 3-CNF formula with m clauses, $G = f(\phi)$ has $7m$ vertices and the largest Clique of G is MAX-SAT(ϕ).

1. Show there is some constant $\alpha > 1$ so that we cannot find a Vertex Cover by a ratio better than α unless P = NP.

2. Show that for *all* constants $\beta > 1$, we cannot find a clique with a ratio better than β unless P = NP.

   Hint: Consider the product graph G^k where the vertices are tuples $(v_1, \ldots, v_k)$ of vertices of G and there is an edge between two tuples $(u_1, \ldots, u_k)$ and $(v_1, \ldots, v_k)$ if for all i either $u_i = v_i$ or (u_i, v_i) is an edge of G.

3. A regularized QBF ψ has an even number n variables is of the form (note the alternating quantifiers)
   $$\psi = \exists x_1 \forall x_2 \exists x_3 \forall x_4 \ldots \exists x_{n-1} \forall x_n \phi(x_1, \ldots, x_n)$$
   where ϕ is a 3-CNF. Show the set of true regularized QBFs is PSPACE-complete.

4. Generalized Geography is the following two player game on a directed graph G and a start node s. The players alternate by following an edge to a new node. Once an edge is used it cannot be used again by either player. A player loses when they have no legal move.

   Show that the set of pairs (G, s) where the player that moves first has a winning strategy is PSPACE-complete.