EECS 395-23 – Assignment 4

1. Generalized Geography is a game played on a directed graph with a specific start vertex s. The first player picks an edge (s, a). The second player picks an edge (a, b). The first player now picks an edge (b, c) and so on. In other words each player must pick an edge starting from where the previous edge left off. No edge can ever be picked twice (though the same vertex can be used multiple times). The player that cannot pick a legal edge loses.

   Show the language $GG = \{ (G, s) \mid \text{The first player has a win} \}$ is PSPACE-complete. Hint: Reduce from TQBF, the set of true quantified Boolean formula.

2. Consider the cat and mouse game on an undirected graph. The cat and mouse are on some specified initial vertex. The cat and mouse alternate each moving along a single edge. They may repeat edges during the game. The cat wins if it ever can occupy the same vertex as the mouse.

   Show that deciding whether the cat has a winning strategy is computable in polynomial time in the size of the graph.

3. Show that PH = PSPACE implies PH = Σ_k^p for some k.

4. Consider the knapsack problem: There is a capacity c, and n objects where object i has size s_i and value v_i. The c, s_i and v_i are integers. The goal is to find a set $S \subseteq \{1, \ldots, n\}$ that maximizes $\sum_{i \in S} v_i$ with the constraint $\sum_{i \in S} s_i \leq c$. Give an algorithm that finds the best S in time polynomial in n and $\max s_i$.

   Note: This is called a pseudopolynomial-time algorithm because we can run in time polynomial in the values as opposed to polynomial in the length of the binary representation of the values.