Lecture #1:

EECS 395: Introduction to Computational Complexity

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1 Building Blocks

1.1 Alphabets

An *alphabet* is a finite, nonempty set of symbols. Conventionally, an alphabet is represented by Σ. Common alphabets are:

1. $\Sigma = \{0, 1\}$, the binary alphabet.
2. $\Sigma = \{a, b, ..., z\}$, the set of all lower-case letters.
3. The set of all ASCII characters.

1.2 Strings

A *string* is finite sequence of symbols chosen from some alphabet Σ. For example, 001101 is string from the binary alphavet $\Sigma = \{0,1\}$. ϵ is used to denote an empty string. The length of a string is number of symbols in the string, and the standard notation for length of a string x is $\mid x \mid$. For example, $\mid 0110 \mid = 4$. By definition $\mid \epsilon \mid = 0$.

The set of all strings over an alphabet Σ is conventionally denoted by Σ^*. For instance, $\{0,1\}^* = \{\epsilon$, 0, 1, 00, 10, 01, 11, 000, 001, ...}.

1.3 Languages

A language is a finite or infinite set of strings over some alphabet Σ. The set of all strings is Σ^*. ϕ is the empty set, not to be confused with $\{\epsilon\}$, the set consisting of the single empty string.

Languages can be defined implicitly. For example,

L = { x | x is the binary representation of a prime}

If L is a language over Σ, then $\overline{L} = \Sigma^* - L$.

2 What is a Turing Machine?

- Simple machine with n states.
- Start in state 0.
- Input on an arbitrarily large TAPE that can be read from *and* written to.
- Read an alphabet from the tape.
- Depending on current state and input alphabet
  - write a alphabet to the tape
  - move the tape right or left
  - move to a new state
- Stop if enter yes or no state.
- Accept if yes, reject if no or does not terminate.

2.1 Notation for a Turing Machine

A Turing Machine is represented using a 8-tuple.

M = (Σ, Q, Γ, δ, q_0, q_{acc}, r_{rej}, B)

whose components has the following meanings:

Σ : The finite set of input symbols.

Γ : The complete set of tape symbols; $\Sigma \subseteq \Gamma$.

B : The blank symbol. $B \in \Gamma - \Sigma$.

Q : The finite set of state of finite control.

q_0 : The start state, a member of Q.

q_{acc} : Accept states in Q.

q_{rej} : Reject states in Q.

δ : The transition function. The arguments of $\delta(q_a, \alpha)$ are a state q and a tape symbol α. The value of $\delta(q_a, \alpha)$, if it is defined, is a triple (q_b, β, D), where

1. q_b is a state in Q.
2. β is symbol in Γ, written in the cell being scanned, replacing whatever was there.
3. D is the direction, either L or R, standing for "left" or "right", telling us the direction in which the head moves.

2.2 The Language of a Turing Machine

The language of a Turing Machine is the set of input strings when placed on the tape, and the tape head initially at the leftmost input symbol; the machine eventually enters an accepting state. More formally, L(M), language of a machine, is defined as follows:

L(M) = { x | M starting on input x eventually ends up in a state q_{acc} }.