

### Homework 3

You may discuss the assignments with your classmates but need to write down your solutions independently. Be careful with your handwriting. Unclear solutions will be assumed wrong.

- (10 pts) The following binary numbers have a sign in the leftmost position and, if negative, are in 2s complement form. Perform the indicated arithmetic operations (show your work) and verify the answers.
  - $100111 + 111001$
  - $001011 + 100110$
  - $110001 - 010010$
  - $101110 - 110111$

- (25 pts) Design a LCM machine which is a sequential circuit that compute the Least Common Multiple of two 4-bit unsigned numbers  $a$  and  $b$ , using the following algorithm:

```
 $x, y, u, v := a, b, a, b$   
do  
   $x > y \rightarrow x, u := x - y, u + v$   
   $y > x \rightarrow y, v := y - x, v + u$   
od  
output  $((u + v)/2)$ ;
```

The available basic elements include full adders, D flip-flops, and multiplexers, in addition to Boolean gates. Assume that each basic element takes 1 unit of time, what is the minimum clock period you can use.

- (15 pts) Design a combinational circuit that compares two 4-bit unsigned numbers  $A$  and  $B$  to see whether  $B$  is greater than  $A$ . The circuit has one output  $X$ , such that  $X = 1$  if and only if  $A < B$ .
- (25 pts) Design a sequential multiplier that multiplies two 4-bit unsigned numbers  $A$  and  $B$ . The available basic elements include one 4-bit adder, D flip-flops, and multiplexers, in addition to Boolean gates. Assume that each basic element takes 1 unit of time, what is the minimum clock period you can use.
- (25 pts) Implement to the gate and full adder level an ALU bit slice with three operation selection inputs  $S_2, S_1, S_0$ , that implements the following eight functions of

the two data inputs  $A$  and  $B$  (and carry-in  $C_i$ ):

$S_2$	$S_1$	$S_0$	ALU operation
0	0	0	$F_i = 0$
0	0	1	$F_i = B - A$
0	1	0	$F_i = A - B$
0	1	1	$F_i = A + B$
1	0	0	$F_i = AXORB$
1	0	1	$F_i = AORB$
1	1	0	$F_i = AANDB$
1	1	1	$F_i = 1$