Northwestern University
Electrical Engineering and Computer Science
EECS310: Foundations of Computer Science
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Handout #6

Midterm Exercises

1. Decide whether each of the following statements is TRUE or FALSE.
   (a) $p \vee \neg p$ and $p \Rightarrow (q \Rightarrow r) \equiv (p \Rightarrow q) \Rightarrow (p \Rightarrow r)$ are theorems.
   (b) The divisibility relation | is a partial order on positive integers.
   (c) Let $i \rho j$ denote "i is a divisor of j and $i < j$", then $< N, \rho >$ is well founded.
   (d) An algorithm is partially correct if it is correct on some but not all of the inputs.
   (e) There are relations that are irreflexive, symmetric, and transitive.
2. Prove $(p \wedge q) \vee (p \wedge \neg q) \equiv p$.
3. Prove $(\exists\, x|\ :\ R) \Rightarrow ((\forall\, x|R\ :\ P) \Rightarrow Q \equiv (\exists\, x|R\ :\ P \Rightarrow Q))$ (provided x does not occur free in P).
4. Prove that
$$(\forall i|0 \leq i < n\ :\ A_i \subseteq B_i) \Rightarrow (\cup i|0 \leq i < n\ :\ A_i) \subseteq (\cup i|0 \leq i < n\ :\ B_i)$$
5. Prove that for any two consecutive Fibonacci numbers $F(n)$ and $F(n{+}1)$, we have $GCD(F(n), F(n{+}1)) = 1$.