ECE510: Homework 4
Temporal Logic

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Due 5/29 Thursday

1. Suppose the netlist of Figure 1 was combinational. Describe how would you use BDDs to verify that for each output $\mathbf{x}$ that has even parity, there exists some input $\mathbf{u}$ whose parity is odd, such that N outputs $\mathbf{x}$ on input $\mathbf{u}$.

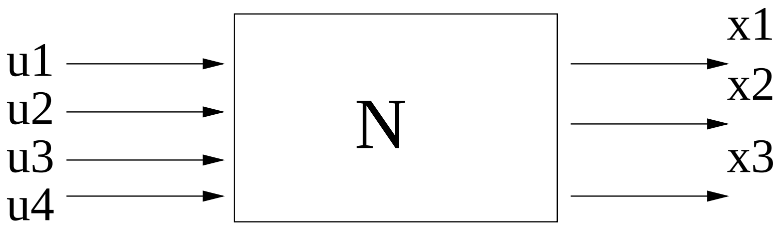

Figure 1: A netlist.

2. Two CTL formulas f, g are said to be *logically equivalent* if for every netlist η and every state s in η, it is the case that $\eta, s \models f$ if and only if $\eta, s \models g$.

   Rewrite the following CTL formulas as logically equivalent formulas not involving the operators EF, AX, AG, AF, AU; justify your transformations.

   (a) EFf
   (b) AXf
   (c) AGf
   (d) $AfUg$
   (e) AFf

3. Write CTL formula that express the following properties for the netlist in Figure 1.

   (a) the outputs x_1 and x_2 are always complementary

   (b) whenever x_1 high, sometime later x_2 becomes high

   (c) it is possible for x_1 to go high and remain high forever

   (d) there is no path on which x_1 eventually becomes low and stays low

   (e) whenever x_1 is high within two or three time steps it becomes low

4. Let ϕ be an arbitrary CTL formula. Prove that, in general, there does not exist a formula ψ such that, in any netlist, the set of states satisfying ψ is exactly the set of states which can be reached in one step from the states satisfying ϕ. (In essence, this shows the image operation can not be expressed using CTL.)

5. Let S be the set of states in a given a Moore netlist (i.e., one in which every output is driven only by latches). A relation $\mathcal{E} \subset S \times S$ is said to be a *bisimulation* if the following conditions are true for all states s and t in S:

   (a) the output at state s is equal to the output at state t, and

   (b) for every state s' that s can make a transition to, there is a state t' that t can make a transition to such that $(s', t') \in \mathcal{E}$, and

   (c) for every state t' that t can make a transition to, there is a state s' that s can make a transition to such that $(t', s') \in \mathcal{E}$.

   Two states s and t are defined to be *bisimilar* if there is a bisimulation relation $\mathcal{D}$ such that $(s, t) \in \mathcal{D}$.

   Prove that if states s and t are bisimilar, then for any CTL formula ϕ, state $s \models \phi$ if and only if $t \models \phi$.[1]

   Hints: Use induction — take as your induction hypothesis that s and t agree on all CTL formula of length less than or equal to n. Also, the fact that EF, AX, AU, AF, AG operations can be expressed in terms of EX, EG, EU (cf. Problem 2 in this homework) can be used to simplify the proof.

[1] The converse is also true, but the proof is much more involved.