Global Routing with Crosstalk Constraints*

Hai Zhou and D.F. Wong
Department of Computer Sciences, University of Texas, Austin, TX 78712-1188

Abstract

Due to the scaling down of device geometry and increasing frequency in deep sub-micron designs, crosstalk between interconnection wires has become an important issue in VLSI layout design. In this paper, we consider crosstalk avoidance during global routing. We present a global routing algorithm based on a new Steiner tree formulation and the Lagrangian relaxation technique. We also give theoretical results on the complexity of the problem.

1 Introduction

With VLSI fabrication entering the deep sub-micron era, devices and interconnection wires are being placed at an ever increasing proximity. Reduction in the interconnection and transistor switching delays results in faster signal transition times. All these factors increase the coupling effect (inductive and capacitive) between interconnection wires. Increased coupling effect not only increases signal delays, but also decreases signal integrity due to transmission line behavior. This phenomenon is called *crosstalk* [2].

Most previous works on crosstalk avoidance are focused on detailed routing [3, 5, 6, 9, 11, 14], where the estimation of crosstalk is accurate but the flexibility to avoid it is restricted. Therefore, it is often not possible to find a crosstalk-feasible solution in detailed routing if the global routing is crosstalk-blinded.

In order to make a global routing solution crosstalk risk-free, Xue et al. [13] proposed a post global routing adjustment procedure to modify a given routing solution. It used a gridded model and assumed that the crosstalk only exists between adjacent wires. That is, if there is a spare track between two wires, then there is no crosstalk between them. The crosstalk risk estimation is divided into two coupled procedures: bound partitioning and net ordering. Bound partitioning partitions the crosstalk bound of each net into the regions it routes through. Net ordering orders the nets in each region to require as few spare tracks as possible. Their approach is totally region-based, that is, crosstalk risks are defined by the regions requiring extra tracks and the number of extra tracks needed. If a region still requires extra tracks after bound partitioning and net ordering, some nets in the region are ripped up and re-routed using other regions.

In this paper, we consider crosstalk avoidance during global routing. We use a crosstalk model which is more general than that of [13]. In order to have an accurate crosstalk estimation, we extend global routing to include layer and track assignments. Based on this extension, a novel *Crosstalk-Constrained Global Routing* problem is defined. Many subproblems of this problem are shown to be NP-hard. Therefore, a two-stage heuristic approach is used to control crosstalk. First, an initial solution is constructed based on a new Steiner tree formulation to minimize the total crosstalk. After that, crosstalk on each net is estimated. The nets having crosstalk violations are then ripped up. They are re-routed by a Lagrangian relaxation technique to satisfy crosstalk constraints.

*This work was partially supported by the Texas Advanced Research Program under Grant No. 003658288.

2 Crosstalk model

Consider two coupling nets as shown in Figure 1(a). We can model them by a circuit shown in Figure 1(b). In this figure, v_1, v_2 represents the voltages at the outputs of the drivers, R_1, R_2 represents the driver resistances, C represents the coupling capacitance between the nets, and C_1, C_2 represents all other capacitances besides C. The crosstalk effect on net 2 can be thought of as the difference of voltages on node O with or without the signal switching in net 1. Here, we can find that besides the coupling capacitance C, the crosstalk effect also depends on driver strengths (R_1 and R_2), other load capacitances (C_1 and C_2), and the voltages (v_1 and v_2).

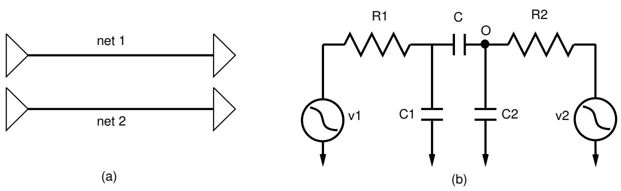

Figure 1: Modeling two coupling nets

Among all parameters affecting crosstalk effect, we assume that only the coupling capacitance is controllable in layout design. Other parameters are either not changeable or not preferred to be changed. However, they are useful information for crosstalk control in layout design. For example, if we know net 1 and net 2 will never switch at the same time, then we can safely route them together since the crosstalk effect will be zero even they are heavily coupled. We can model all parameters except coupling capacitance by a *crosstalk coefficient.* For each net i, the *crosstalk coefficient* from net j is a real number $e_{ij} \in [0, 1]$ which is the crosstalk on net i contributed by one unit of coupling capacitance from net j. For example, if $e_{ij} = 2e_{ik}$, then the crosstalk induced on net i by net j is twice of that by net k if they have the same coupling capacitance.

For a net, the crosstalk effects from other nets may not happen at the same time. But characterizing all cases requires exhaustive timing analysis, which in turn depends on crosstalk. Therefore, in the worse case, we can use the summation of all the effects from other nets as the total crosstalk on one net. That is,

$$X_i = \sum_{j \neq i} e_{ij} C_{ij}, \tag{1}$$

where C_{ij} is the coupling capacitance between net i and net j.

Generally speaking, each element in a chip is coupled with every other element. But coupling capacitance decreases rapidly when an element is out of the neighborhood of the other element. For interconnection, coupling capacitance between perpendicular wires is also very small. Therefore, we assume coupling capacitance only exists between neighboring parallel wires and is given by the following formula:

$$C = \alpha \frac{length}{distance^{\beta}}, \tag{2}$$

where β is a constant which was estimated to be about 2 in [10].

3 Problem formulation

In traditional design flow, routing is divided into global and detailed routing. In global routing, the set of regions a net goes through is decided. In detailed routing, the actual route in each region is then computed. From the previous section, we know coupling is critically dependent on wire adjacency. However, in traditional global routing, for each net, only routing regions it goes through are decided. The adjacency information in each region is not available. Therefore, it is not possible to get accurate crosstalk estimation in that situation.

In order to estimate crosstalk during global routing, some "detailed" information must be known. Of course, doing the whole task of global and detailed routing together would be very complicated. Therefore, the idea is to do a "simplified" detailed routing "on the fly" in the process of global routing. This detailed routing must be simple enough to be merged into the global routing process, and at the same time, must be detailed enough for crosstalk estimation.

In fact, each region in the global routing stage can be viewed as a multi-layer rectangular area. We also assume that each layer is composed of tracks with equal spacing and each track can be used only by one net. During global routing, besides deciding which regions a net goes through, layer and track assignments in each region are also decided. Formally, we are given an undirected graph $G = (V, E)$, where each edge e represents a region $r(e)$ which has a length $l(e)$ and may have k layers each with a capacity $c_i(e)$, for $1 \le i \le k$. Also specified are n nets each is a subset of V. An *extended global routing solution* is an edge labeling S of the form $S(e) = (s_1, s_2, \ldots, s_k)$, where each s_i for $1 \le i \le k$ is a sequence of nets. For $1 \le i \le n$, the edges including i must form a Steiner tree for net i. If $s_i = (\ldots, 2, 1, 3, \ldots)$, this means that nets 1, 2, and 3 are routed through region $r(e)$ and placed in layer i in the order 2, 1, 3. For example, an extended global routing solution is shown in Figure 2, where each region has only one layer.

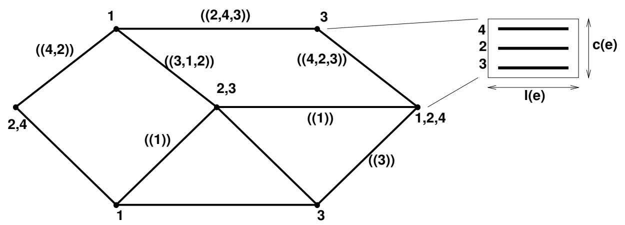

Figure 2: An extended global routing solution

Given an extended global routing solution, the adjacency information is known in each edge. Suppose nets i and j are adjacent on layer l of edge e, the coupling capacitance between them can be computed as $C = \alpha \frac{l(e)}{(c_l(e)/|N_l(e)|)^{\beta}}$, where $N_l(e)$ represents the set of nets on layer l of edge e. Therefore, crosstalk on each net can be computed. Using $X_i(S)$ to represent crosstalk on net i under an extended global routing solution S, the problem we need to solve can be defined as follows.

Problem 1 (Crosstalk-Constrained Global Routing (CCGR)) *Given a routing graph $G = (V, E)$, n nets, and the crosstalk constraints $C_1, \ldots, C_n$, find an extended global routing solution S such that $X_i(S) \le C_i$ for all $1 \le i \le n$.*

4 A global router with crosstalk control

As will be shown in the next section, many sub-problems of the CCGR problem are NP-hard. We use a heuristic to deal with this problem. It is a sequential approach, that is, routing is done net by net. The approach consists of two stages. The first stage constructs a routing solution to minimizes the summation of crosstalk on all nets. Each time, a Steiner tree is computed for a net which minimizes the total crosstalk and its topology is then fixed for the remaining nets. In the second stage, crosstalk on each net in the initial solution is estimated. The nets having constraint violations are then ripped up. During re-routing, a method based on Lagrangian relaxation is used to balance crosstalk according to each net's constraint. Because there is no forbidden region in the process, a new route can have the same topology as the old one but with different layer/track assignment. This means layer/track re-assignment is also included in this procedure.

4.1 Minimum crosstalk Steiner tree

Since we use a sequential approach in our routing, the kernel procedure is how to construct a Steiner tree and decide the layer/track assignment in each region such that the total crosstalk is minimize. Formally, this can be defined as the following problem.

Problem 2 (Minimum Crosstalk Steiner Tree) *Given a routing graph $G = (V, E)$ and an extended global routing solution S for nets $1, \ldots, m-1$ which minimized the total crosstalk, find an extended global routing solution S' for nets $1, \ldots, m$ such that routing topologies for nets $1, \ldots, m-1$ are kept and $X(S') = \sum_{i=1}^{m} X_i(S')$ is minimized.*

Let $X^e(S)$ denote the summation of crosstalk on all nets in edge e under solution S. The objective function in the above problem can be written as follows.

$$\begin{aligned} X(S') = \sum_{e \in E} X^e(S') &= \sum_{e \in T_m} X^e(S') + \sum_{e \notin T_m} X^e(S') \\ &= \sum_{e \in T_m} (X^e(S') - X^e(S)) + \sum_{e \in E} X^e(S) \end{aligned} \tag{3}$$

where T_m represents the Steiner tree for net m. For each edge e, $X^e(S') - X^e(S)$ is the difference of the minimum total crosstalks in edge e before or after inserting net m in the edge. Given a set of nets going through an edge, the minimum crosstalk in the edge can be computed by an optimal layer/track assignment. Therefore, we can compute $X^e(S') - X^e(S)$ and use it as the cost of edge e. Since $\sum_{e \in E} X^e(S)$ in (3) is a constant, the minimum crosstalk Steiner tree problem becomes a minimum Steiner tree problem.

Unfortunately, according to Theorem 1, layer/track assignment for crosstalk minimization in one region is NP-hard. Therefore, a heuristic is designed to do the job. It works as follows. First, we decide an ordering on all nets. Then, each net is added sequentially to the region according to its order. When adding one net, the existing nets keep their relative ordering, and the new net is inserted to a position which has the minimum increase of crosstalk.

If we use the same net ordering for sequential routing in the above heuristic, the layer/track assignments can be merged into the routing process. In this case, when computing edge costs, there is no need to do layer/track re-assignments for the existing nets. Suppose S is the given solution for nets $1,\ldots,m-1$, the cost of routing net m on edge e is the minimum increase of crosstalk by inserting net m in one sequence of $S(e)$. If e is chosen by a minimum Steiner tree algorithm, such insertion will give $S'(e)$. An example of the procedure is given in Figure 3.

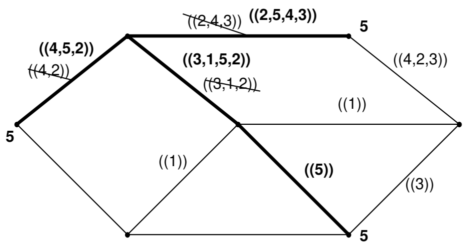

Figure 3: Routing net 5

In practice, crosstalk is not the only objective we want to minimize. We also need to consider wire length and congestion. In such case, we can use the following edge cost function.

$$cost(e) = \alpha \cdot length(e) + \beta \cdot overflow(e)^2 + \gamma \cdot xtalk(e)$$

where $overflow(e)$ is the extra space needed when compared with the capacity of the edge, and $xtalk(e)$ is the increase of crosstalk on the edge. Based on this cost function, any Steiner tree heuristic on general weighted graph can be used to construct a Steiner tree which simultaneously optimizes the wire length, congestion, and crosstalk.

The Steiner tree heuristic we use in our global router is based on the shortest path and the minimum spanning tree. According to [8], these heuristics are guaranteed to have solutions within $2(1-1/l)$ of the optimum, where l is the number of leaves in the optimal solution. We use Dijkstra's algorithm [4] for the shortest path and Prim's algorithm [4] for the minimum spanning tree. The reason we use Prim's algorithm instead of Kruskal's is that Dijkstra's algorithm and Prim's algorithm are both working in a tree expansion fashion, hence can be easily combined.

4.2 Rip-up and re-route based on Lagrangian relaxation

Given an initial solution constructed by the minimum crosstalk Steiner tree algorithm, nets with crosstalk violations are ripped up. We re-route them one net at a time in the order of decreasing violations. When routing a net, besides crosstalk on the current net, extra crosstalk can be induced on previously routed nets. Therefore, we need to solve the following problem.

Problem 3 (Re-Routing for Crosstalk Control) *Given a routing graph $G=(V,E)$ and an extended global routing solution S for nets $1,\ldots,m-1$ such that $X_i(S) < C_i$ for $1 \le i \le m-1$, find an extended global routing solution S' for nets $1,\ldots,m$ such that routing topologies of nets $1,\ldots,m-1$ are kept, the wire length is minimized, and $X_i(S') < C_i$ for all $1 \le i \le m$.*

The above problem requires to balance crosstalk on each net according to its constraint. Similar as in [15], we can use Lagrangian relexation to satisfy all constraints simultaneously. By relaxing constraints into objective function, we have the following problem.

Problem 4 (Lagrangian Sub-Problem (LSP)) *Given a routing graph $G=(V,E)$, an extended global routing solution S for nets $1,\ldots,m-1$, and constants $\mu_1,\ldots,\mu_m$, find an extended global routing solution S' for nets $1,\ldots,m$ such that the routing topology for nets $1,\ldots,m-1$ is kept, and $length(S')+\sum_{i=1}^{m}\mu_i(X_i(S')-C_i)$ is minimized. Where $length(S')$ is the wire length in solution S'.*

Let $length_i(S')$ be the wire length of net i under solution S'. The objective function of above problem can be written as follows.

$$\begin{aligned} & length(S') + \sum_{i=1}^{m}\mu_i(X_i(S')-C_i) \\ &= \sum_{i=1}^{m}(length_i(S') + \mu_i X_i(S')) - \sum_{i=1}^{m}\mu_i C_i \end{aligned}$$

Since $\sum_{i=1}^{m}\mu_i C_i$ is a constant, by treating $length_i(S') + \mu_i X_i(S')$ as crosstalk on net i, the Lagrangian sub-problem becomes the minimum crosstalk Steiner tree problem, hence can be solved by the same approach.

Let $L(\mu_1,\ldots,\mu_m)$ denote the optimal solution of the Lagrangian sub-problem. For any $\mu_i \ge 0, 1 \le i \le m$, we have

$$\begin{aligned} L(\mu_1,\ldots,\mu_m) &= \min_{S'}\{length(S') + \sum_{i=1}^{m}\mu_i(X_i(S')-C_i)\} \\ &\le \min_{S'}\{length(S') + \sum_{i=1}^{m}\mu_i(X_i(S')-C_i) : \\ &\qquad\qquad X_i(S') \le C_i, 1 \le i \le m\} \\ &\le \min_{S'}\{length(S') : X_i(S') \le C_i, 1 \le i \le m\}. \end{aligned}$$

Therefore, if we have $\mu_1^*,\ldots,\mu_m^*$ such that

$$L(\mu_1^*,\ldots,\mu_m^*) = \min_{S'}\{length(S') : X_i(S') \le C_i, 1 \le i \le m\},$$

we know that S' is an optimal solution. Hence we need to solve the following problem, which is actually a convex program.

Problem 5 (Lagrangian Multiplier Problem (LMP))

$$\begin{aligned} &\textit{Maximize} && L(\mu_1,\ldots,\mu_m) \\ &\textit{subject to} && \mu_i \ge 0, \forall 1 \le i \le m \end{aligned}$$

Since $L(\mu_1,\ldots,\mu_m)$ as a function is not everywhere differentiable, we will use sub-gradient method described as follows to solve it.

Algorithm Sub-gradient Method for LMP
1. $k=0; \mu_i = 0, \forall 1 \le i \le m$;
2. Solve the LSP by the Steiner tree heuristic;
3. If stop criteria is satisfied, halt;
4. Else $\mu_i = \max\{0, \mu_i + \theta_k(X_i(S') - C_i)\}$,
   $\forall 1 \le i \le m; k = k+1$; goto 2.

The general theory for sub-gradient method states that if $\theta_k \to 0$ and $\sum_{j=1}^{k}\theta_j \to \infty$ as $k \to \infty$, it will converge to an optimal solution [1]. But we must notice that an optimal solution to the LMP does not always give an optimal solution to the re-routing for crosstalk control problem.

Table 1: Experimental results

test problems	Traditional GR #vio.	Traditional GR max. vio	post-GR layer/track assignment #vio.	post-GR layer/track assignment Max. vio	our approach first stage #vio.	our approach first stage Max. vio	our approach second stage #vio.	our approach second stage Max. vio
ami49.1.a	329	285.29	301	191.91	121	81.46	1	0.84
ami49.1.b	252	86.27	143	35.46	46	15.23	1	0.14
ami49.1.c	303	114.50	265	46.78	146	25.93	3	0.90
ami49.2	325	100.46	283	46.16	156	16.92	2	0.19
ami33.1	70	82.15	65	31.90	35	8.97	5	0.39
ami33.2	64	83.07	55	31.87	24	8.89	1	0.19
apte.a	19	21.93	11	7.56	3	4.54	0	0.00
apte.b	17	5.15	11	2.10	5	1.74	0	0.00
xerox.1	49	5.16	9	4.34	3	3.83	2	0.08
xerox.2	57	17.17	18	5.87	2	0.32	0	0.00
xerox.3	70	49.70	34	31.23	22	15.98	0	0.00

5 Complexity results

In the CCGR problem, both routing topology and adjacency relations in regions can be changed. Therefore, the following methods can be use to reduce crosstalk: *shielding* (inserting a spare wire between coupling nets), *track assignment* (changing neighboring relations of nets), *layer assignment* (distributing coupling wires to different layers), But all of them are difficult problems. We summarize them into the following theorem.

Theorem 1 *All of the following problems are NP-hard.*

Problem 6 (Shielding for Crosstalk Control) *Suppose the routing topology, layer/track assignments in all regions are fixed, and only one shielding wire can be inserted in each region. Find a shielding position in each region such that each net satisfies its crosstalk constraint.*

Problem 7 (Track Assignment for Crosstalk Minimization) *Suppose the routing topology is fixed, and so are track assignments in all regions except one. Find a track assignment in that region such that the total crosstalk on the nets is minimized.*

Problem 8 (Track Assignment for Crosstalk Control) *Suppose the routing topology is fixed, and so are track assignments in all regions except one. Find a track assignment in that region such that each net satisfies its constraint.*

Problem 9 (Layer Assignment for Crosstalk Control) *Suppose there are only two layers in each region, and the routing topology and net orderings in all regions are fixed. Find a layer assignment in each region such that each net satisfies its crosstalk constraint.*

6 Experimental Results

We implement the global router described in Section 4 on a Solaris/PC platform. Five circuits from the CBL/NCSU building block benchmarks: ami33, ami49, apte, hp, and xerox are used. Since these benchmarks do not come with placement information, we use a floorplaner based on simulated annealing [12] to give placements. Using different parameters, we obtain two different placements for both ami33 and ami49: ami33.1, ami33.2, ami49.1, ami49.2, and three different placements for xerox: xerox.1, xerox.2, xerox.3. Using different minimum track spacing and crosstalk constraints, different test problems are produced for the same placement, which are indicated by the suffixes: a, b, etc.

In order to measure the effectiveness of our global router, we compare it with other routing approaches. "Traditional GR" only considers wire length and overflow, and nets are ordered sequentially in each region. "Post-GR layer/track assignment" uses the same topologies as in traditional GR but use the greedy heuristic given in Section 4.1 to do layer/track assignment in each region. For each approach, the numbers of nets having crosstalk violations and the maximum violations are reported in Table 1. For our global router, both results after the first stage (i.e., minimizing total crosstalk) and the second stage (i.e., rip-up & re-route by Lagrangian relaxation) are reported. As we can see, after rip-up and re-route, almost all violations are solved. For those few nets which still have violations, the violations are so small that it could be easily solved in detailed routing.

Acknowledgements

We would like to thank Dr. Y.-Z. Liao and Dr. Yang Cai for their stimulating discussions.

References

[1] R.K. Ahuja, T.L. Magnanti and J. B. Orlin, *Network Flows: Theory, Algorithms, and Applications.* Prentice Hall, 1993.

[2] H.B. Bakoglu, *Circuits, interconnections and packaging for VLSI.* Addison-Wesley, 1990.

[3] K. Chaudhary, A. Onozawa and E.S. Kuh, A Spacing Algorithm for Performance Enhancement and Cross-talk Reduction. *ICCAD,* 1993.

[4] T.H. Cormen, C.E. Leiserson, and R.L. Rivest, *Introduction to Algorithms.* The MIT Press, 1990.

[5] T. Gao and C.L. Liu, Minimum Crosstalk Channel Routing. *ICCAD,* 1993.

[6] T. Gao and C.L. Liu, Minimum Crosstalk Switchbox Routing. *ICCAD,* 1994.

[7] M.S. Garey and D.S. Johnson, *Computers and Intractability: A Guide to the Theory of NP-Completeness.* W. H. Freeman, New York, 1979.

[8] F.K. Hwang, D.S. Richards, and P. Winter, *The Steiner Tree Problem.* North-Holland, 1992.

[9] D.A. Kirkpatrick and A.L. Sangiovanni-Vincentelli, Techniques for Crosstalk Avoidance in the Physical Design of High-Performance Digital Systems. *ICCAD,* 1994.

[10] T. Sakurai and K. Tamaru, Simple formulas for two and three dimensional capacitance. *IEEE Trans. Electron Devices,* 1993.

[11] S. Thakur, K.-Y. Chao and D.F. Wong, An Optimal Layer Assignment Algorithm for Minimizing Crosstalk for Three Layer VHV Channel Routing. *ISCAS,* 1995.

[12] D.F. Wong and C.L. Liu, A New Algorithm for Floorplan Design. *DAC,* 1986.

[13] T. Xue, E.S. Kuh, and D. Wang, Post Global Routing Crosstalk Risk Estimation and Reduction. *ICCAD,* 1996.

[14] H. Zhou and D.F. Wong, An Optimal Algorithm for River Routing with Crosstalk Constraints. *ICCAD,* 1996.

[15] H. Zhou and D.F. Wong, Crosstalk Constrained Maze Routing Based on Lagrangian Relaxation. *ICCD,* 1997.