Statistical Timing Analysis With Coupling

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***Abstract*—As technology scales to smaller dimensions, increasing process variations and coupling induced delay variations make timing verification extremely challenging. In this paper, the authors establish a theoretical framework for statistical timing analysis with coupling. They prove the convergence of their proposed iterative approach and discuss implementation issues under the assumption of a Gaussian distribution for the parameters of variation. A statistical timer based on their proposed approach is developed and experimental results are presented for the International Symposium on Circuits and Systems benchmarks. They juxtapose their timer with a single pass, noniterative statistical timer that does not consider the mutual dependence of coupling with timing, and another statistical timer that handles coupling deterministically. Monte Carlo simulations reveal a distinct gain (up to 24%) in accuracy by their approach in comparison to the others mentioned.**

***Index Terms*—Coupling, fixpoint computation, statistical timing analysis, variability, very large scale integration (VLSI).**

I. Introduction

VARIABILITY in modern deep submicrometer very-large-scale-integration circuits has not scaled down in proportion to the scaling down of their feature sizes. Manufacturing process variations (e.g., V_T, L_e), environmental variations (e.g., V_{dd}, temperature), and device fatigue phenomenon contribute to uncertainties. These sources of variation are called parameters and the range in which they can collectively vary is called the parameter space. Uncertainty due to parametric variations deeply impacts the timing characteristics of a circuit and makes timing verification extremely difficult. This necessitates the consideration of the parametric variations in timing analysis for accurate timing estimation. Statistical timing analysis has thus emerged in an attempt to capture the statistical behavior of circuit delays under parametric variations.

With the progress of deep submicrometer technologies, shrinking geometries have led to a reduction in the self-capacitance of wires. Meanwhile, coupling-capacitances have increased as wires have a larger aspect ratio and are brought closer together. For 90-nm technologies, the ratio of an interconnect's parasitic coupling capacitance to its parasitic ground capacitance is nearly 5.5 (85% of the total parasitic capacitance) [2]. This signifies the increased dominance of coupling capacitances with technology scaling. Trends indicate that the role of coupling-capacitances will be even more dominant in the future as feature-sizes shrink [3]. Simultaneous switchings on coupled nets in a circuit affect the delay on each net and cause delay variations in the circuit. The delay variations are often significant (up to 40% stage delay error [4]) and should not be ignored. In addition, the mutual dependence of coupling effects and timing make timing analysis a chicken-and-egg problem. A coupling between two wires that lie in the fan-in and fan-out cone, respectively, of a logic gate causes timing dependencies that necessitates iterations during timing analysis. A situation of coupling is initially assumed (often the worst case situation) and the computed timing information is used to modify the coupling situation. This procedure is repeated until convergence.

The gate delay for multiple-input CMOS gates often depends on the number of inputs switching at the same time. For example, turning on two transistors in parallel is faster than using only one as the path resistance is lower. The assumption of only single input switchings for timing analysis and consequently ignoring the effect of multiple-input switchings (MISs) can result in significant errors (up to 20% stage delay error [5]). Although the probability of several MISs adding up along a path in a circuit is small, delay variations due to the temporal proximity of switching inputs should be considered in accurate timing analysis.

The increasing significance of the mentioned factors on timing accuracy motivates considering them in a unified timing analysis framework. Approaches to statistical timing analysis have gained wide acceptance recently [6]–[9]. Statistical timers treat delays as correlated random variables and propagate their distributions through the circuit. These approaches, however, do not discuss coupling induced delay variations or effects due to MIS. On the other hand, the majority of prevalent timing analysis approaches that consider coupling treat delays as deterministic values and do not discuss the impact of variability [10]–[15]. A statistical timing analysis algorithm that accounts for correlations and accommodates dominant interconnect coupling is proposed by Le *et al.* in [16]. While considering coupling in their proposed approach, they estimate switching window overlaps deterministically, based on worst case values. Our experiments conclude that such an approach is pessimistic and over estimates the switching windows at the outputs. A gate delay model that accounts for the effects of temporal proximity and input transitions is proposed in [17]. Another statistical gate delay model that considers MIS is proposed in [5]. Dartu *et al.* [18] present results on the significance of the mentioned effects on timing accuracy and recommend increased focus on consideration of these effects in timing analysis for modern circuits.

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In this paper, we establish a theoretical framework for statistical timing analysis with coupling. Our contributions are summarized as follows.

1) We describe the generic problem of timing analysis under uncertainty from the parameters of variation and delay variations due to coupling. We compare and contrast the uncertainty introduced by each of these effects (due to variability and ignorance of functional information, respectively) and propose considering them in a unified framework.
2) We introduce the concept of statistical switching windows, which is a novel extension to the idea of switching windows in deterministic static timing analysis.
3) We propose an iterative approach for statistical timing analysis with coupling as a fixpoint computation on a lattice and establish its convergence.
4) We present a probabilistic view of the overlap between two statistical switching windows. Using a generic coupling model, we discuss the computation of a coupling induced delay pushout as a random variable.
5) We consider the implications of the assumption of Gaussian distributions for the parameters of variation in statistical timing analysis with coupling. The concept of a Gaussian switching window is described and we present the computation of a delay pushout under the given assumption.
6) We compare and contrast the effect of MIS with coupling. Using a simple MIS model as an example, we illustrate that our framework is amenable to incorporating that effect in a unified way.

We develop a statistical timer that considers coupling based on our proposed approach and present results obtained for the International Symposium on Circuits and Systems (ISCAS'85) benchmarks [19]. We additionally develop the following two timers for comparison:

1) single pass statistical timer based on [9], which does not consider the mutual dependence of coupling and timing;
2) iterative statistical timer that considers coupling deterministically, based on [16].

Monte Carlo simulations reveal a distinct gain in accuracy (up to 24%) of our approach in comparison to the others.

The rest of this paper is organized as follows. Section II describes statistical timing analysis with coupling as a fixpoint computation problem on a lattice. Section III presents the generalized computation of coupling induced delay pushouts as random variables. We discuss the implications of the assumption of Gaussian distributions for the parameters of variation and simplify the computation of the delay pushout under the given assumption in Section IV. We show that the effects of MIS are amenable to our framework in Section V, and present experimental results and comparisons to other approaches in Section VI. Conclusions are drawn in Section VII.

II. Statistical Timing as Fixpoints

We consider a combinational circuit and select a set of interconnect wires where timing information needs to be computed. This set includes the primary inputs, the primary outputs, and the inputs and outputs of all gates in the circuit. In statistical timing, we use random variables with known distributions to denote timing information. Depending on the purpose of timing analysis, the timing information could be a delay, a slew, a switching window, or a combination of them. Symbolically, we use a variable x_i to denote the timing information on a wire i. Furthermore, we use X to represent the vector $(x_1, x_2, \ldots, x_n)$, that is the timing information of the whole circuit.

Depending on the actual delay model, the timing information on a wire i depends only on a subset of other wires $i_1, i_2, \ldots, i_k$. These wires may include all inputs of the gate fanning out to i, and the wires coupled to i. This implies that x_i can be computed as

$$x_i \;=\; t_i(x_{i_1}, x_{i_2}, \ldots, x_{i_k})$$

where the function t_i is decided by the physical configuration of the circuit and the delay model used. Such a function for computing the timing information on a wire is called a local timing transformation. Putting all local transformations together, we get a global timing transformation T for the entire circuit, which can be written as

$$X = T(X). \tag{1}$$

A solution of timing analysis must be an X satisfying (1). Such a solution is also called a fixpoint of T. This formulation is amenable to timing analysis considering variability, coupling, and MIS. Coupling causes mutual dependence of the timing information on coupled wires and may create cyclic dependencies during timing analysis. For a complex transformation T, an iterative method is perhaps the only possible way to find its fixpoint. First, an initial solution X_0 is guessed, then new solutions are iteratively computed from previous solutions $X_1 = T(X_0)$, $X_2 = T(X_1) = T^2(X_0), \ldots$ until we find a fixpoint, that is, $X_m = T^m(X_0)$ such that $T(X_m) = X_m$.

A. Statistical Timing With Coupling

Considering functional information in timing analysis involves enumerating all input vectors which is exponential in computational complexity. As a result, static timing analysis does not use functional information of a circuit. Such a treatment makes timing information on each wire to be a set of possible signal switchings, instead of a single signal switching. The set is represented by a switching window and a set of slew rates such that any signal switching falling in the window and having a slew in the range is in the set. The switching window x_i on a wire i denotes an interval $[x_i^l, x_i^u]$, such that any actual signal switching x_i^* lies in the interval. Thus

$$\{x_i^* : x_i^l \;\le\; x_i^* \;\le\; x_i^u\}$$

denotes the set of all possible signal switchings. The switching window representation is a worst case representation as it only gives bounds (best and worst case) on the elements of the set. It does not provide any information on the distribution of signal switchings within the window.

Statistical timing analysis considers the uncertainty in circuit component delays due to parametric variations. Assumptions on the distribution of the parameters of variation and on the delay model as a function of these parameters provide a known distribution of signal switchings on every wire in the circuit.

Statistical timing with coupling involves uncertainties from both variability and ignorance of functional information. Although both uncertainties prohibit the solution to timing analysis on a wire from being a single signal switching, we do not treat them in the same way. This is because we have information about the delay distribution due to uncertainty from variability, but not so for uncertainty due to ignorance of functional information. Consequently, we cannot obtain the solution to statistical timing analysis with coupling as a known distribution of signal switchings on each wire of the circuit.

The solution to statistical timing analysis on any wire i is a distribution obtained from a statistical sampling of the timing information on i at various corners in the parameter space. For any corner in the parameter space, that is, a given assignment of parametric values, the timing information on i is in the form of a switching window. Consequently, the solution to statistical timing analysis with coupling is a distribution of switching windows on each wire of the circuit. This view of the solution is obtained by first considering the uncertainty from ignorance of functional information, and then the uncertainty from variability.

Each window in the above view of the solution is denoted by a best and a worst case value. Consequently, the distribution of the windows contains the distributions of these best and worst case values. The two distributions thus obtained are represented as the distributions of two correlated random variables. The window formed using these random variables as the best and worst case value, respectively, contains all possible signal switching distributions in the solution. This transformation of the original view of the solution gives an alternate view of the solution to statistical timing analysis with coupling as a window of signal switching distributions.

The latter view of the solution is also obtained by considering the uncertainty from variability first. Temporarily assuming that functional information and input configuration of each gate is known, statistical timing analysis on a wire i computes a distribution of a single signal switching. However, functional information and input configuration is not considered in reality. The timing information on i is therefore a set of signal switching distributions, which is represented by a window of signal switching distributions.

B. Statistical Switching Windows

Each view of the solution to statistical timing analysis contains the exact same information. We consider the solution as a window of signal switching distributions, since it allows us to leverage the theoretical foundations of timing analysis with coupling as a fixpoint computation developed in [15]. We cannot use traditional switching windows in our formulation as they represent a set of deterministic quantities, while our set

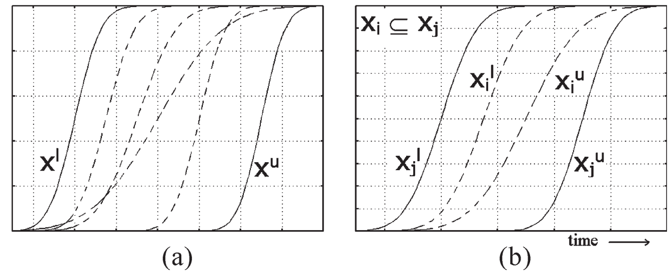

Fig. 1. (a) Statistical switching window for a set of distributions. (b) Inclusion relation between two switching windows.

consists of signal switchings as random variables with known distributions.

We introduce a statistical switching window as a representation for a set of random variables. For any random variable x_i^* in the set, we consider providing a lower and an upper bound using two correlated random variables x_i^l and x_i^u, respectively. We call these two random variables the lower and upper bounding random variable (or bounding RV) of the statistical switching window x_i. The bounding RVs are correlated since they are functions of common parameters of variation. Mathematically, the statistical switching window x_i is defined as

$$x_i = \left[x_i^l, x_i^u\right] \stackrel{\triangle}{=} \left\{x_i^* : Pr\left(x_i^l \leq x_i^* \leq x_i^u\right) = 1\right\} \quad (2)$$

where, $Pr(k)$ denotes the probability of an event k. A statistical switching window is illustrated graphically in Fig. 1(a). In this figure, the distributions of various random variables are shown to be bounded within the distributions of x^l and x^u. In the rest of this paper, we consider the solution to statistical timing analysis with coupling on a wire i as a statistical switching window x_i of signal switching distributions.

C. Structure of Fixpoints

An inclusion relation $(\subseteq)$ between two statistical switching windows x_i and x_j is formally defined as

$$x_i \subseteq x_j \text{ def } \left(Pr\left(x_j^l \leq x_i^l \leq x_i^u \leq x_j^u\right) = 1\right). \quad (3)$$

Fig. 1(b) illustrates the definition graphically. In the figure, the statistical switching window x_i is shown to be contained within the statistical switching window x_j, that is, the distribution of x_i^l is upper bounded by that of x_j^l, while the distribution of x_i^u is lower bounded by that of x_j^u.

We consider the family of all sets of signal switching distributions on a wire. The inclusion relation $(\subseteq)$ forms a partial order on the family as it satisfies the properties of reflexivity, transitivity, and antisymmetry [15].

A set of signal switching distributions on a wire is actually a subset of the whole set that consists of all possible signal-switching distributions. The inclusion relationship between statistical switching windows is the inclusion relationship between these subsets and thus forms a partial order. According to lattice theory [20], a partially ordered set forms a complete lattice if any subset has a least upper bound and a greatest lower bound of its members. The family of all subsets of a given set with an inclusion relation forms a complete lattice. The proof is provided in [15]. Timing information of a circuit is a vector of timing information on all wires. The partial order on each wire can be extended pointwise to get a partial order on vectors. We say that two timing information vectors $X = (x_1, x_2, \ldots, x_n)$ and $Y = (y_1, y_2, \ldots, y_n)$ satisfy $X \subseteq Y$ if and only if $x_i \subseteq y_i$ for all $1 \leq i \leq n$. It can be shown that the vectors with such a partial order also form a complete lattice. We consider the complete lattice structure since the existence of a fixpoint to (1) under certain conditions in the lattice is guaranteed.

We now consider the transformation T (as defined in Section II), which transforms a vector of sets of signal switching distributions to another vector of sets of signal switching distributions. We say that T is a monotonic transformation when, for any two vectors X and Y, if $X \subseteq Y$, then $T(X) \subseteq T(Y)$. The monotonicity of T is proved from the monotonicity of its member (or local) timing transformations $t_1, t_2, \ldots, t_n$ (as defined in Section II). We do not restrict our approach to any particular timing model. Theoretically, any timing model must satisfy the monotonic property for each local timing transformation. If a transformation t_i is not monotonic, it implies that some switching distributions on the wire i disappear under the condition of the same or more possible switching distributions on wires in its fan-in cone and those coupled to i. Such a model is not theoretically sound as it contradicts with the causality of physical effects. Under such a timing model, iterations for timing analysis with coupling may not converge. For such models, iterations are often started with an initial worst case assumption of maximum possible coupling induced delay variations. A finite number of iterations are performed to reduce pessimism in the solution, but not until convergence. However, for a theoretically correct timing model, the existence of a fixpoint of the monotonic transformation T is guaranteed by Knaster–Tarski theorem [20].

D. Searching for a Fixpoint

We use an iterative approach to compute the fixpoint of our transformation. The assumed initial solution in the iterative approach is critical to reaching the desired fixpoint. A random initial solution may keep the iterations going on forever. Based on the monotonicity of the timing transformation T, the top and bottom elements of our lattice are good candidates for the initial solution. In traditional lattice theory, $\perp$ and $\top$ denote the bottom and the top element of a lattice, respectively. In our lattice, we have $\perp = (\emptyset, \emptyset, \ldots, \emptyset)$ and $\top = (P_1, P_2, \ldots, P_n)$, where $\emptyset$ denotes an empty set while P_i is the set of all possible signal switching distributions on wire i and can be computed by assuming the effect of coupling everywhere. Since $\perp \subseteq T(\perp)$, based on the monotonicity of T, we have

$$\perp \subseteq T(\perp) \subseteq T^2(\perp) \subseteq T^3(\perp) \cdots$$

which gives us an ascending chain of timing information vectors. Similarly, a descending chain is obtained if we start with $\top$. We next consider the convergence of these chains to a fixpoint.

E. Convergence to a Fixpoint

A discrete delay model has a finite number of statistical switching windows on each wire ideally. For example, we consider a simple coupling model that gives the worst case additional coupling induced delay on a wire as either of two random variables depending on whether the switching window on that wire overlaps with that of a coupled wire or not. In this case, the maximum possible number of delay random variables for any wire is 2^k, where k is the total number of couplings in the circuit. Under similar discrete models, each wire has ideally a finite number of statistical switching windows. Any chain of timing information for a wire in the solution space has thus a finite number of elements. This implies that the iterative process will always reach a fixpoint in finite steps. The monotonic property of T when we start from $\perp$ or $\top$ is sufficient to prove convergence. In fact, any solution X_0 such that $X_0 \subseteq T(X_0)$ or $X_0 \supseteq T(X_0)$ can be used as an initial solution to reach a fixpoint. The number of iterations to reach a fixpoint is upper bounded by the sum of the lengths of the longest chains of all timing information variables in the lattice.

Under a continuous delay model, it is possible for the chains generated in the iterative process to have infinite elements. This implies that the generation of an exact fixpoint in finite steps is not guaranteed. However, if the iteration does theoretically converge to a fixpoint, an approximation to the fixpoint can be found. For such an approximated converged fixpoint to be a real fixpoint, stronger properties than the monotonicity are needed for the transformation T.

Theoretically, a statistical gate delay model defines a response waveform distribution as a function of switching waveform distributions on a gate's fan-ins and wires coupled to its fan-out net. We do not restrict the distributions to independent ones, and assume that the model considers correlations as well. In statistical static timing analysis, this model needs to be expanded to compute a set of response distributions as a function of sets of waveform distributions on the fan-ins and coupling wires. A general way to expand a function from a set to its power set is by natural lifting.

Definition 1: Given f: $S \rightarrow S$, a natural lifting F: $2^S \rightarrow 2^S$ is

$$F(A) = \bigcup_{a \in A} f(a), \qquad \forall A \in 2^S.$$

Given a subset S of elements in a complete lattice L, we use $\bigvee S$ to represent the least upper bound of elements in S. A function $T : L \rightarrow L$ is said to be or-continuous if for any chain C, $T(\bigvee C) = \bigvee T(C)$. A timing transformation defined by a natural lifting is shown to be or-continuous [15]. Given that T is or-continuous, it is known that $\bigvee_{n \geq 0} T^N(\perp)$ is a fixpoint. This establishes that our iterative approach always converges to a fixpoint, given that we start our iteration from the bottom element of the lattice. A fixpoint may however, not be reached if we start our iterations from the top element [15]. This provides a strong reason why timing analysis iterations should start from the bottom element.

F. Optimal Fixpoint

It is known that if fixpoints of a monotonic transformation are found by the iterative method starting from the bottom element, it must be the least fixpoint. It is also known that the fixpoints of a monotonic transformation form a complete lattice. The least fixpoint is therefore the intersection of all the fixpoints. In terms of statistical switching windows, a fixpoint with the smallest window will result from an initial assumption that the probability of any two windows overlapping with each other is zero. Therefore, in order to find the optimal fixpoint, that is the vector of tightest switching windows on the real solution (the one with minimal pessimism), we should start with an initial solution where the probability of an overlap between the statistical switching windows on any two coupled wires is zero. Subsequent application of T until convergence guarantees the optimal solution.

III. Coupling Induced Delay Pushouts as Random Variables

A. Overlap Between Statistical Switching Windows

The overlap between two statistical switching windows is not a deterministic quantity. For two statistical switching windows x_i and x_j, we denote the overlap between the windows by a random variable O_{ij}, which is defined as

$$O_{ij} \stackrel{\triangle}{=} \min\left(x_i^u, x_j^u\right) - \max\left(x_i^l, x_j^l\right). \tag{4}$$

We consider a probabilistic view of the overlap O_{ij}. If we denote the cumulative distribution function (cdf) of O_{ij} by $\Psi_{ij}(t)$, the probability of an overlap between x_i and x_j is given by

$$Pr(O_{ij} > 0) \; = \; 1 - \Psi_{ij}(0). \tag{5}$$

B. Coupling Induced Delay Pushout Computation

Simultaneous switchings on a pair of coupled nets cause delay variations on the nets. The amount of additional delay (positive or negative) induced is called the coupling induced delay pushout and is a function of the overlap between the switching windows on the coupled nets. Since the overlap between two statistical switching windows is a random variable, the coupling induced delay pushout is consequently a random variable even if the delay pushouts for a deterministic overlap are constants. Considering the timing information on any wire as a set of signal switching distributions, coupling possibly induces additional signal switching distributions in the set. The new set must have an inclusion relationship with the original since any conservative coupling model will not remove any original switching distribution from the set. The delay pushout thus widens the switching window on a wire, that is, if x_i and x_i' denote the switching windows on a wire i when effects due to coupling are not considered and are considered, respectively, we have

$$x_i \;\subseteq\; x_i'. \tag{6}$$

Without any loss of generality, we consider the computation of the worst case delay on a wire i when coupling effects are considered. The worst case delay on i is given by the sum of the delay on the wire when coupling effects are not considered and the coupling induced delay pushout due to each wire it couples with.

As an example, we illustrate the computation of a coupling induced delay pushout D as a random variable based on a simple coupling model, which is given by

$$D \stackrel{\triangle}{=} \begin{cases} D^O, & \text{overlap} \\ D^N, & \text{no overlap} \end{cases} \tag{7}$$

where, D^O and D^N are the values assigned to D, depending on whether the statistical switching windows between the coupled wires overlap or not, respectively. We denote the overlap as a random variable O.

Based on prior work by [8] and [9], we assume that all random variables are expressed as a weighted sum of independent and orthonormal random variables $\xi_i, i = 1, 2, \ldots, n$, such that $E[\xi_i \xi_j] = 0$ if $i \neq j$, and $= 1$, otherwise. The delay pushouts D^O and D^N are random variables, and are expressed as

$$\begin{aligned} D^O &= d_0^O + \sum_{i=1}^{n} d_i^O \xi_i \\ D^N &= d_0^N + \sum_{i=1}^{n} d_i^N \xi_i \\ \Rightarrow D &= d_0 + \sum_{i=1}^{n} d_i \xi_i \stackrel{\triangle}{=} \begin{cases} d_0^O + \sum_{i=1}^{n} d_i^O \xi_i, & \text{overlap} \\ d_0^N + \sum_{i=1}^{n} d_i^N \xi_i, & \text{no overlap.} \end{cases} \end{aligned} \tag{8}$$

We define a new set of random variables $\mathbf{d_i}$s, $i = 0, 1, \ldots, n$ as

$$\mathbf{d}_i \stackrel{\triangle}{=} \begin{cases} d_i^O, & \text{overlap} \\ d_i^N, & \text{no overlap.} \end{cases} \tag{9}$$

We therefore have

$$d_0 + \sum_{i=1}^{n} d_i \xi_i \; = \; \mathbf{d}_0 + \sum_{i=1}^{n} \mathbf{d}_i \xi_i. \tag{10}$$

We multiply (10) by a new independent and orthogonal random variable ξ_{n+1} and take the expected value ($E[\cdot]$) on both sides. We consider the orthogonality property of ξ_i, and that the overlap is a function of $\xi_i, i = 1, \ldots, n$ but independent of ξ_{n+1}. Using conditional expectation, we obtain

$$\begin{aligned} d_0 &= \frac{1}{E[\xi_{n+1}]}\left(E[\mathbf{d}_0\xi_{n+1}] + \sum_{i=1}^{n} E[\mathbf{d}_i\xi_i\xi_{n+1}]\right) \\ &= \frac{1}{E[\xi_{n+1}]}\left(E[\mathbf{d}_0] + \sum_{i=1}^{n} E[\mathbf{d}_i\xi_i]\right) E[\xi_{n+1}] \\ &= E[\mathbf{d}_0] + \sum_{i=1}^{n} E[\mathbf{d}_i\xi_i] \\ &= d_0^O Pr(\text{overlap}) + d_0^N Pr(\text{no overlap}) \\ &\quad + \sum_{i=1}^{n} E[\mathbf{d}_i\xi_i] \\ &= d_0^O Pr(O > 0) + d_0^N Pr(O \le 0) \\ &\quad + \sum_{i=1}^{n} E[\mathbf{d}_i\xi_i] \quad \text{where} \end{aligned} \tag{11}$$

$$\begin{aligned} E[\mathbf{d}_i\xi_i] = d_i^O E[\xi_i | O > 0] Pr(O > 0) +& \\ d_i^N E[\xi_i | O \le 0] Pr(O \le 0).& \end{aligned} \tag{12}$$

We multiply (10) again by ξ_k for $k = 1, 2, \ldots, n$ and match the expectations on both sides to obtain

$$d_k = \frac{1}{E\left[\xi_k^2\right]}\left(E[\mathbf{d}_0\xi_k] + \sum_{i=1}^{n} E[\mathbf{d}_i\xi_i\xi_k] - d_0 E[\xi_k]\right) \tag{13}$$

where

$$\begin{aligned} E[\mathbf{d}_i\xi_i\xi_k] &= d_i^O E[\xi_i\xi_k |\, O > 0] Pr(O > 0) \\ &\quad + d_i^N E[\xi_i\xi_k |\, O \le 0] Pr(O \le 0) \end{aligned} \tag{14}$$

$$\begin{aligned} E[\mathbf{d}_0\xi_k] &= d_0^O E[\xi_k |\, O > 0] Pr(O > 0) \\ &\quad + d_0^N E[\xi_k |\, O \le 0] Pr(O \le 0). \end{aligned} \tag{15}$$

The previous illustration of the computation of the delay pushout is extensible to an arbitrary coupling model and does not imply that the proposed approach is limited to a given coupling model. The model can be trivially extended to having M delay pushouts for given ranges of an overlap value and expressed as

$$D \triangleq \begin{cases} D^N & \text{no overlap} \\ D_i^O & \text{overlap} \in (O_i, O_{i+1}],\ \forall i = 1, 2, \ldots, M-1\ . \\ D_M^O & \text{overlap} > O_M \end{cases}$$

Under such a model, the equations are modified to obtain the desired coefficients d_is. The conditional expectations are now evaluated for each of the possible overlap intervals. Even for a continuous coupling model (for example, the delay pushout as a exponential function of overlap), a similar approach is used to compute the desired coefficients. The conditional expectation computations in this case involves the probability density function of the overlap instead of probability that the overlap lies in some interval. We do not discuss details of such models as this paper is model independent and guarantees convergence under the monotonicity properties of the timing transformation.

IV. Practical Considerations Under a Gaussian Assumption

A. Gaussian Statistical Switching Windows

The parameters of variation in statistical timing are often assumed to be random variables having a Gaussian distribution. Gate delays are expressed as a linear weighted sum of these parameters and thereby have a Gaussian distribution too. We consider the following definition:

$$\Phi(t) \triangleq \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{t} \exp\left(-\frac{x^2}{2}\right) dx. \tag{16}$$

$\Phi(t)$ denotes the cdf of a unit normal Gaussian. It is known that the cdf of an arbitrary Gaussian $X \sim N(\mu, \sigma^2)$ with mean μ and variance σ^2 is equivalent to $\Phi((t-\mu)/\sigma)$, where $\Phi(t)$ is defined as in (16).

The assumption of a Gaussian distribution for the bounding RVs when used in describing statistical switching windows prohibits the generic use of the $\subseteq$ relation between the windows.

Lemma 1: No Gaussian can be a bounding RV for a set of Gaussians.

Proof: We use contradiction to prove our claim and assume that a Gaussian $X \sim N(\mu_x, \sigma_x^2)$ is an upper bounding RV for another Gaussian $Y \sim N(\mu_y, \sigma_y^2)$. From (3), we have

$$\begin{aligned} & Pr(X \ge Y) = 1 \\ \Rightarrow\ & Pr(X - Y \ge 0) = 1 \\ \Rightarrow\ & Pr(Z \ge 0) = 1, \text{ where } Z \sim N\left(\mu_z, \sigma_z^2\right) \triangleq X - Y \\ \Rightarrow\ & 1 - \Phi_z(0) = 1, \text{ where } \Phi_z(t) = \Phi\left(\frac{t - \mu_z}{\sigma_z}\right) \\ \Rightarrow\ & \Phi_z(0) = 0 \end{aligned}$$

which is a contradiction, since $\Phi(t) \in (0, 1)$ for real t. ■

As a heuristic, we now consider a weaker condition on the bounding RVs x_i^l and x_i^u for any statistical switching window x_i as

$$Pr(x_i^l \le c) \ge Pr(x_i^* \le c) \ge Pr(x_i^u \le c), \qquad \forall c \in \Re. \tag{17}$$

It can be trivially proved (using contradiction) that the condition is a necessary condition to (2). Agarwal *et al.* denote such bounding RVs as statistical upper and lower bounds in [7], [21]. We observe that the cumulative distribution functions (cdfs) of x_i^l and x_i^u provide a statistical interval for the cdf of the uncertain actual timing variable x_i^*. Similarly, we present a weaker condition on the bounding RVs of two statistical switching windows x_i and x_j to have an inclusion relationship $(x_i \subseteq x_j)$ as

$$\begin{aligned} & Pr\left(x_i^l \le c\right) \le Pr\left(x_j^l \le c\right) \\ & \quad \wedge Pr\left(x_i^u \le c\right) \ge Pr\left(x_j^u \le c\right), \qquad \forall c \in \Re. \end{aligned} \tag{18}$$

It can be trivially proved (using contradiction) that the condition is a necessary condition to (3). We next show that the assumption of a Gaussian distribution for the bounding RVs even under the above weaker conditions prohibits the generic use of the $\subseteq$ relation between the windows. We discuss the cause of this problem and propose a remedy in the remainder of this section.

Definition 1: We say that a random variable X strictly dominates another random variable Y if and only if the cdf of X is upper bounded by the cdf of Y, that is

$$Pr(X \leq c) \leq Pr(Y \leq c), \qquad \forall c \in \Re.$$

Lemma 2: The cdfs of two arbitrary Gaussians with nonequal variances always intersect at exactly one point.

Proof: We consider two arbitrary Gaussians having means μ_1, μ_2 and variances σ_1^2, σ_2^2, respectively. The point(s) of intersection of the cdfs is(are) found by equating the cdfs and solving for c. We thus solve for c in

$$\Phi\left(\frac{c-\mu_1}{\sigma_1}\right) = \Phi\left(\frac{c-\mu_2}{\sigma_2}\right). \tag{19}$$

It is known that $\Phi(y)$ is strictly increasing with y, that is $\Phi(x) > \Phi(y)$ if and only if $x > y$. The previous equation is therefore equivalent to solving

$$\frac{c-\mu_1}{\sigma_1} = \frac{c-\mu_2}{\sigma_2}. \tag{20}$$

Equation (20) is a linear equation in c and has a single root at $c^* = (\sigma_2\mu_1 - \sigma_1\mu_2)/(\sigma_2 - \sigma_1)$, given $\sigma_1 \neq \sigma_2$. Since both the cdfs are monotonically increasing, c^* denotes the single point of intersection of the cdfs. ■

Corollary 1: For two arbitrary Gaussians with equal variance, the cdf of the Gaussian with a larger mean strictly dominates the cdf of the other.

Proof: A real solution to (20) does not exist if $\sigma_1 = \sigma_2$. Consequently, $\Phi((c-\mu_1)/\sigma_1) \geq \Phi((c-\mu_2)/\sigma_2)$, $\forall c \in (-\infty, \infty)$ if $\mu_1 \leq \mu_2$, and vice versa. ■

Lemma 3: An arbitrary Gaussian can never strictly dominate another Gaussian, given that their variances are nonidentical.

Proof: For two arbitrary Gaussians having means μ_1, μ_2 and variances σ_1^2, σ_2^2, respectively, we assume without any loss of generality that $\sigma_2 > \sigma_1$. Based on Lemma 2 and given the monotonicity of the cdfs, the following results are immediate:

$$\Phi\left(\frac{c-\mu_1}{\sigma_1}\right) \geq \Phi\left(\frac{c-\mu_2}{\sigma_2}\right) \qquad \forall c \in \left[\frac{\sigma_2\mu_1-\sigma_1\mu_2}{\sigma_2-\sigma_1}, \infty\right)$$
$$\Phi\left(\frac{c-\mu_1}{\sigma_1}\right) \leq \Phi\left(\frac{c-\mu_2}{\sigma_2}\right) \qquad \forall c \in \left(-\infty, \frac{\sigma_2\mu_1-\sigma_1\mu_2}{\sigma_2-\sigma_1}\right].$$

Thus, neither of the Gaussians strictly dominate the other. ■

Corollary 2: If the ratio of the means of two arbitrary Gaussians is equal to the ratio of their variances, the cdf of the Gaussian with the smaller mean (or variance) strictly dominates the other in the interval $c \in [0, \infty)$.

Proof: Given $(\mu_1/\mu_2) = (\sigma_1/\sigma_2)$ implies that $\sigma_2\mu_1 = \sigma_1\mu_2$. The proof is immediate when this result is substituted in the above inequalities. ■

Corollary 3: For two statistical switching windows x_i and y_i, $x_i \subseteq y_i$ implies that x_i^l strictly dominates y_i^l and y_i^u strictly dominates x_i^u.

Proof: Immediate from (3) and Definition 2. ■

Definition 3: A statistical switching window x_i is said to be a Gaussian switching window if and only if both x_i^l and x_i^u have a Gaussian distribution.

Corollary 4: An arbitrary Gaussian switching window x_i cannot have an inclusion relation to another Gaussian switching window x_j, unless the variances of x_i^l, x_j^l, x_i^u, and x_j^u are identical.

Proof: Immediate from Lemma 3 and Corollary 3. ■

Consequently, Gaussian switching windows for statistical timing analysis with coupling may not guarantee convergence. The Gaussian distribution spans the entire range of real numbers and the point of intersection of the cdfs of two Gaussian may lie far away from their mean values (or the region of interest). In addition, realistic parametric variations have a distribution constrained in a finite region. We therefore consider a truncated Gaussian distribution for the bounding RVs in our approach. The cdf of a truncated Gaussian with mean μ and variance σ^2 as a function of c is assumed to be 0 for all $c \leq \mu - k\sigma$ and is assumed to be 1 for all $c \geq \mu + k\sigma$, for a given k (typically $k \in [3, 7]$). Truncated Gaussian distributions have earlier been assumed for parametric distributions in [5], [22].

Lemma 4: An arbitrary truncated Gaussian with mean μ_1 and variance σ_1^2 can strictly dominate another truncated Gaussian with mean μ_2 and variance σ_2^2 under the following condition:

$$\left(\max(\mu_1 + k\sigma_1, \mu_2 + k\sigma_2) \leq \frac{\sigma_2\mu_1 - \sigma_1\mu_2}{\sigma_2 - \sigma_1}\right) \bigvee$$
$$\left(\min(\mu_1 - k\sigma_1, \mu_2 - k\sigma_2) \geq \frac{\sigma_2\mu_1 - \sigma_1\mu_2}{\sigma_2 - \sigma_1}\right).$$

This condition implies that a truncated Gaussian can dominate another if the point of intersection of their cdfs lies outside the range $[\mu - k\sigma, \mu + k\sigma]$ for both the truncated Gaussians. Although we consider delay random variables to be Gaussians, we assume truncated Gaussian distributions for the bounding RVs x_i^l and x_i^u of a statistical switching window x_i to maintain the monotonic property of T for convergence. We denote such a statistical switching window as a truncated Gaussian switching window. A truncated Gaussian switching window x_i can have an inclusion relation with another truncated Gaussian switching window x_j if both pairs of truncated Gaussians x_i^l, x_j^l and x_i^u, x_j^u satisfy the condition mentioned in Lemma 4 individually. The timing transformation T must ensure this condition for monotonicity and convergence of our approach. It is easy to ensure the condition by adjusting the means (or variances) of the statistical switching window bounds pessimistically after each application of T during iterative timing analysis. This is done by rewriting the condition in Lemma 4 as inequality conditions on μ_2 (or σ_2). We do not limit our approach to a given delay model. Given a gate and the statistical switching windows on all its fan-ins and wires coupled to its fan-out net, we assume that the delay model provides the switching window at the fan-out of the gate. We merely adjust the bounding RVs pessimistically so that the above conditions are satisfied, that is, the upper bounding RV strictly dominates the lower in the interval $[\mu - k\sigma, \mu + k\sigma]$.

B. Coupling Induced Delay Pushout as a Gaussian

We now consider the implications of the assumption of a unit normal distribution for the independent random variables $\xi_1, \ldots, \xi_n$, and Gaussian switching windows in evaluating the delay pushout D from its definition in (8). Using the add (sub) and max (min) operations from [8] and [9], the overlap O_{ab} between two Gaussian switching windows x_a and x_b is approximated to a Gaussian with mean μ and variance σ^2. From the definition of the probability of an overlap between statistical switching windows in (5), we have the following[1]:

$$\Pr(O_{ab} > 0) = 1 - \Phi\left(\frac{0-\mu}{\sigma}\right) = \Phi\left(\frac{\mu}{\sigma}\right). \tag{21}$$

Consequently, the probability of no overlap between the Gaussian switching windows is given by

$$\Pr(O_{ab} \leq 0) = \Phi\left(-\frac{\mu}{\sigma}\right). \tag{22}$$

Evaluating the coefficients $d_1, \ldots, d_n$ of the delay pushout D in (13) is nontrivial. The conditional expectations and consequently the coefficients d_ks are functions of $\xi_1, \ldots \xi_n$, and not constants. D is therefore not a linear weighted sum of the ξ_is in reality.

For simplicity, we attain to approximate D as a linear weighted sum of the ξ_is. One approach to achieving this is by approximating each random variable d_k to its mean value. We denote the corresponding approximated coefficient as d_k^*. This approach is employed by Wu *et al.* [23] in dynamic range estimations of multiplexing operations in nonlinear systems using polynomial chaos expansion. Monte Carlo simulations are used to generate samples of the random variables and are classified as to whether they produce the condition in the conditional expectation or not. The probabilities of the two outcomes are thus computed, and within each value, the mean of the expectation is computed in the usual Monte Carlo fashion. However, Monte Carlo simulations are expensive and not suitable for statistical timing analysis.

From the theory of conditional expectation [24], we know the following for any random variables X and Y.

$$E\left[E[X|\,Y]\right] = E[X].$$

[1]It is known that $\Phi(-x) = 1 - \Phi(x)$.

Consequently, the mean or the expectation of the conditional expectations in (12), (14), and (15) are expressed as

$$E\left[E[\xi_k|\,O > 0]\right] = E\left[E[\xi_k|\,O \leq 0]\right] = E[\xi_k] \tag{23}$$

$$E\left[E[\xi_i\xi_k|\,O > 0]\right] = E\left[E[\xi_i\xi_k|\,O \leq 0]\right] = E[\xi_i\xi_k]. \tag{24}$$

For $i \neq j$, it is known that $E[\xi_i] = E[\xi_i\xi_j] = 0$ and $E[\xi_i^2] = 1$. The simplified coefficient d_0^* in the representation of the effective delay pushout D from (11) is now expressed as

$$\begin{aligned} d_0^* &= E[d_0] \\ &= d_0^O \Pr(O > 0) + d_0^N \Pr(O \leq 0) + E\left[\sum_{i=1}^{n} E[\mathbf{d}_i\xi_i]\right] \\ &= d_0^O \Pr(O > 0) + d_0^N \Pr(O \leq 0) + \sum_{i=1}^{n} E\left[E[\mathbf{d}_i\xi_i]\right]. \end{aligned} \tag{25}$$

From (12), (23), and given that $E[\xi_i] = 0$, we have

$$\begin{aligned} &E\left[E[\mathbf{d}_i\xi_i]\right] = 0, \\ \text{and thus }\; &d_0^* = d_0^O \Pr(O > 0) + d_0^N \Pr(O \leq 0) \\ &\quad\;\; = d_0^O \Phi\left(\frac{\mu}{\sigma}\right) + d_0^N \Phi\left(-\frac{\mu}{\sigma}\right). \end{aligned} \tag{26}$$

Similarly, from (13)–(15), (23), (24), and given that each ξ_k is a unit normal Gaussian, each of the coefficients $d_1^*, \ldots, d_n^*$ in the representation of the effective delay pushout D are consequently simplified to

$$d_k^* = E[d_k] = d_k^O \Phi\left(\frac{\mu}{\sigma}\right) + d_k^N \Phi\left(-\frac{\mu}{\sigma}\right) \qquad k = 1, \ldots, n. \tag{27}$$

An alternate approach to reducing D to a weighted linear sum of random variables is by introducing new random variables that approximate the product of the ξ_is and true d_ks (as a function of the ξ_is). Fang *et al.* [25] employ this approach for approximating the product of two affine terms into another affine term in static analysis for fixed-point finite-precision effects in DSP designs.

V. Statistical Timing With MIS

Delay models considering effects due to MIS compute gate delays as functions of the overlap between the switching windows on the fan-in signals of a gate. We consider a simple MIS gate delay model similar to (7), where overlap denotes the overlap between the statistical switching windows on the fan-ins of a gate and is a random variable. Given this delay model, we can compute the effects due to MIS in statistical switching windows. Though analogous to the coupling, MIS does not necessitate iterative timing analysis, and statistical timing with MIS is possible in a single pass of a topologically ordered circuit if coupling induced delay variations are not considered.

We therefore conclude that our framework for statistical timing with coupling is amenable to incorporating the effects of MIS. Our reason for this conclusion is based on the assumption that the MIS effect does not change causality and consequently the timing transformation T remains monotonic under its effects. This implies convergence to a fixpoint in our approach. However, for complex delay models, establishing the delay pushout due to MIS would be nontrivial. We do not consider the computation of these pushouts in this paper.

VI. Experimental Results

In this section, we present statistical timing analysis results for the ISCAS'85 benchmarks [19], realistic parameters for which are generated from a 0.13μ technology library. Two global sources of variation having a truncated Gaussian distribution (at $\mu \pm 4\sigma$) are considered in addition to a local independent component. Cumulative parametric variations are constrained within 10% of their mean values. We use a simple delay model [22] and a delay pushout model (8) for our experiments. Arrival time at all primary inputs are set to zero. For comparisons, we develop the following timing analysis approaches.

1) SSTA: denotes an implementation of a statistical timer based on [9], which does not consider coupling induced delay variation, that is, the delay pushout due to coupling on each wire is assume to be D^N (7).
2) STAC: denotes an implementation of a statistical timer based on the above approach that considers coupling induced delay pushouts assuming a deterministic overlap. In this approach, two statistical switching windows $x_i([x_i^l, x_i^u])$ and $x_j([x_j^l, x_j^u])$ are treated as deterministic switching windows given by $[\mu_{x_i^l} - 3\sigma_{x_i^l}, \mu_{x_i^u} + 3\sigma_{x_i^u}]$ and $[\mu_{x_j^l} - 3\sigma_{x_j^l}, \mu_{x_j^u} + 3\sigma_{x_j^u}]$, respectively, to estimate the deterministic overlap between them. The computed overlap determines the delay pushout. This approach is similar to [16].
3) STAC*: denotes an implementation of a statistical timer based on our proposed approach. It considers a probabilistic approach to the overlap between two statistical switching windows, and the delay pushout due to coupling is computed as proposed in Section III.
4) Monte Carlo (MC): simulations are performed for accuracy comparisons. A static timer is implemented that considers coupling induced delay pushouts and is run for 20 000 samples in the parameter space to capture the statistical behavior of the switching windows on all wires of a given circuit.

We compute the statistical switching window x at the primary output of a given circuit using each of the mentioned approaches. Fig. 2 shows the cdfs of the bounding RVs for the switching windows obtained at the output of the ISCAS'85 benchmark C432. It is seen that SSTA underestimates the switching window as compared to MC. STAC is found to give a conservative estimate, while our proposed approach STAC* accurately estimates the statistical switching window. To give a numerical report, the $(\mu_{x^l} - 3\sigma_{x^l})$ and the $(\mu_{x^u} + 3\sigma_{x^u})$ points obtained from the mentioned approaches are compared. We use the points obtained using MC simulations as our reference for accuracy estimations and formally define the error in any approach A as

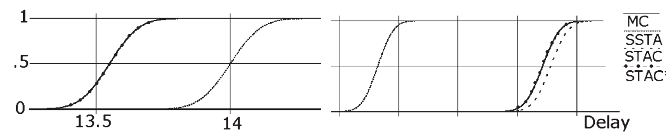

Fig. 2. Bounding RV distributions for the statistical switching windows obtained at the output of ISCAS'85 benchmark C432.

$$\% \, \Xi_l^A \triangleq \frac{\left(\mu_{x_A^l} - 3\sigma_{x_A^l}\right) - \left(\mu_{x_{\mathrm{MC}}^l} - 3\sigma_{x_{\mathrm{MC}}^l}\right)}{\left(\mu_{x_{\mathrm{MC}}^l} - 3\sigma_{x_{\mathrm{MC}}^l}\right)} \times 100$$

$$\% \, \Xi_u^A \triangleq \frac{\left(\mu_{x_A^u} + 3\sigma_{x_A^u}\right) - \left(\mu_{x_{\mathrm{MC}}^u} + 3\sigma_{x_{\mathrm{MC}}^u}\right)}{\left(\mu_{x_{\mathrm{MC}}^u} + 3\sigma_{x_{\mathrm{MC}}^u}\right)} \times 100$$

$$\% \, \Xi_{Av}^A \triangleq \frac{\left|\%\Xi_l^A\right| + \left|\%\Xi_u^A\right|}{2}.$$

Timing analysis accuracy results based on the above metrics are presented in Table I. STAC and STAC* are iterative approaches and compute timing information on some wires multiple times until convergence. We report the average number of Iterations per Node (# I/N) obtained in these approaches. Since SSTA does not consider coupling induced delay pushouts, it is noniterative and is a single pass timer. Consequently, its # I/N value is always 1. The average number of iterations per node is found to be 16 and 15 for STAC and STAC*, respectively. We present run times for all approaches in Table II. STAC* is found to be slower by $1.2X$ in comparison to STAC.

It is observed that STAC* improves timing estimation accuracy by as much as 24% in comparison to SSTA and up to 13% in comparison to STAC. On the average, errors in switching window estimation are reduced by 8.5% in comparison to SSTA and by 2.8% in comparison to STAC. All experiments are performed on a Pentium 2.4-GHz machine, with 1-GB RAM, running Red Hat Linux 9.0.

VII. Conclusion

In this paper, we establish a theoretical framework for statistical timing analysis with coupling. We prove the convergence of our proposed iterative approach and discuss implementation issues under the assumption of a Gaussian distribution for the parameters of variation. The framework is not constrained to Gaussian distributions for the parameters of variation, but an arbitrary distribution could have a complicated max operation

TABLE I
% ERRORS IN SWITCHING WINDOW ESTIMATION WITH REFERENCE TO MONTE CARLO SIMULATIONS

Circuit	Nodes	Edges	% Ξ_l			% Ξ_u			% Ξ_{Av}			# I/N	
			SSTA	STAC	STAC*	SSTA	STAC	STAC*	SSTA	STAC	STAC*	STAC	STAC*
C432	198	379	3.30	0.00	0.00	-9.2	0.4	-0.01	6.2	0.2	0.007	9.1	9.1
C499	245	481	0.00	0.00	0.00	-1.3	4.7	0.13	0.7	2.4	0.067	6.6	5.7
C880	445	815	0.01	-3.90	-0.01	-13.0	13.0	0.01	6.3	8.3	0.010	10.8	8.8
C1355	589	1137	0.01	0.01	0.01	-21.0	2.5	0.01	10.0	1.2	0.007	14.8	14.4
C1908	915	1556	6.00	-5.10	-0.01	-23.0	1.6	0.70	14.0	3.4	0.350	14.7	13.9
C2670	1428	2449	0.00	0.00	0.00	-24.0	7.9	0.01	12.0	4.0	0.004	10.3	9.2
C3540	1721	3011	9.60	-2.30	-0.01	-17.0	8.9	0.10	13.0	5.6	0.056	14.9	12.3
C6288	2450	4864	0.01	0.01	0.01	-22.0	0.0	0.00	11.0	0.0	0.005	50.3	50.9
C7552	3721	6459	0.00	0.00	0.00	-7.0	0.6	0.02	3.5	0.3	0.009	12.4	11.4

TABLE II
RUN TIME COMPARISON FOR THE TIMERS

Circuit	Run times(s)			
	SSTA	STAC	STAC*	Monte Carlo
C432	0.00	0.01	0.01	21.4
C499	0.00	0.01	0.01	36.8
C880	0.00	0.03	0.03	184.1
C1355	0.00	0.10	0.15	585.7
C1908	0.00	0.20	0.26	1018.0
C2670	0.01	0.25	0.34	1269.0
C3540	0.02	0.46	0.58	2053.0
C6288	0.02	2.35	2.90	11960.4
C7552	0.03	0.87	1.11	4447.0

and estimation of the conditional expectations would be an important consideration.

We do not discuss approaches to reducing iterations (or speeding up approaches) for our timer. Clustering the circuit into strongly connected components and then topologically performing timing analysis with coupling is suggested in [15]. Breaking up feedback edges in the directed graph representation of the circuit to form clusters is suggested. The feedback edges due to coupling are broken based on a metric of timing proximity. In statistical timing with coupling, we propose to use the probability of an overlap between two windows as the metric for timing proximity, that is, edges having a low probability of overlap are broken. Though we do not discuss details on the approaches to speed up iterations, our formulation of statistical timing analysis with coupling is amenable to each of the mentioned speedup approaches.

Experimental results reveal that our approach improves switching window estimation accuracy by up to 24% in comparison to traditional approaches. Since our statistical timing analysis framework is block based, we are amenable to using the concept of node and edge criticalities introduced in [9]. This makes our approach attractive in guiding timing optimization [26], which is a critical purpose of accurate timing analysis.

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