

# Lectures on Approximation in Mechanism Design

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Lectures: March 30 – June 4, 2010

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## About These Notes

These lecture notes cover EECS 395/495 “Algorithmic Mechanism Design” offered at Northwestern University in the Spring of 2010. As mechanism design is a traditional topic in economics this course will review the economics treatment simultaneously as needed for the presentation of recent research on approximation in mechanism design. The formal methods that make the technical basis for the course come from theoretical computer science, machine learning theory, game theory, and economics. Prerequisites for reading these lecture notes are basic understanding of algorithms and complexity, at the level of an undergraduate computer science course in algorithms and elementary calculus and probability theory.

Thanks to the students of EECS 395/495. Comments are welcome.



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