EECS 395/495: Algorithmic Mechanism Design	**Homework 1**
Assigned: 4/13/10	**Due: 4/20/10**

Note: Work in pairs. Turn in one assignment. You may refer to lecture notes, assigned readings, or the reference text book *Algorithmic Game Theory*. You may not look for help on the Internet or sources outside course materials. Your solutions should easily and concisely convey your complete understanding of each problem. If you cannot solve a problem, come to office hours. If your solution cannot easily be understood then it is wrong.

1. In class we characterized outcomes and payments for BNE in single-dimensional agent settings. This characterization explains what happens when agents behave strategically.

   Suppose instead of strategic interaction, we care about fairness. Consider a valuation profile, $\mathbf{v} = (v_1, \ldots, v_n)$, an allocation vector, $\mathbf{x} = (x_1, \ldots, x_n)$, and payments, $\mathbf{p} = (p_1, \ldots, p_n)$. Here x_i is the probability that i is served and p_i is the expected payment of i regardless of whether i is served or not.

   Allocation $\mathbf{x}$ and payments $\mathbf{p}$ are *envy-free* for valuation profile $\mathbf{v}$ no agent wants to unilaterally swap allocation and payment with another agent. I.e., for all i and j, $v_i x_i - p_i \geq v_i x_j - p_j$.

   Characterize envy-free allocations and payments (and prove your characterization correct). Unlike the BNE characterization, your characterization of payments will not be unique. Instead, characterize the minimum payments that are envy-free. (Hint: you should end up with a very similar characterization to that of BNE.)

2. Consider a simplification of the Google AdWords setting: There are m advertisement slots that appear along side search results and n advertisers. Advertiser i has value v_i for a click. Slot j has *click-through rate* w_j, meaning, if an advertiser is assigned slot j the advertiser will receive a click with probability w_j. Assume that the slots are ordered from highest click-through rate to lowest, i.e., $w_j \geq w_{j+1}$ for all j.

   (a) Find the envy-free outcome and payments with the maximum social surplus. Give a description and formula for the envy-free outcome and payments for each agent. (Feel free to specify your payment formula with a comprehensive picture.)

   (b) In the real AdWords problem, advertisers only pay if they receive a click, where as the payments calculated, i.e., $\mathbf{p}$, are in expected over all outcomes, click or no click. If we are going to charge advertisers only if they are clicked on, give a formula for calculating these payments $\mathbf{p}'$ from $\mathbf{p}$.

   (c) The real AdWords problem is solved by auction. Design an auction that maximizes the social surplus in dominant strategy equilibrium. Give a formula for the payment rule of your auction (again, a comprehensive picture is fine). Compare your DSE payment rule to the envy-free payment rule. Draw some informal conclusions.

3. Consider the AdWords problem with $n = m = 2$. Consider running the following first price auction: The advertisers submit bids $\mathbf{b} = (b_1, b_2)$. The advertisers are assigned to slots in order of their bids. Advertisers pay their bid when clicked. Use revenue equivalence to solve for BNE strategies $\mathbf{s}$ when the values of the agents are drawn independent and identically from $U[0, 1]$. (Hint 1: Use your calculation in the last part of the last question. Hint 2: This can be done in a few simple lines.)