EECS 395/495: Algorithmic Mechanism Design	Homework 3
Assigned: 5/6/10	**Due: 5/13/10**

Note: Work in pairs. Turn in one assignment. You may refer to lecture notes, assigned readings, or the reference text book *Algorithmic Game Theory*. You may not look for help on the Internet or sources outside course materials. Your solutions should easily and concisely convey your complete understanding of each problem. If you cannot solve a problem, come to office hours. If your solution cannot easily be understood then it is wrong.

1. Prove that the expected social welfare of VCG is always at most a factor of e more than the expected revenue of Myerson for any downward-closed feasibility setting with monotone hazard rate distributions (not necessarily identical). Prove that for non-downward closed feasibility settings the expected social welfare of VCG can be much larger than the expected revenue of Myerson.

2. Recall the Google AdWords setting from Homework 1: There are m advertisement slots that appear along side search results and n advertisers. Advertiser i has value v_i for a click. Slot j has *click-through rate* w_j, meaning, if an advertiser is assigned slot j the advertiser will receive a click with probability w_j. Assume that the slots are ordered from highest click-through rate to lowest, i.e., $w_j \geq w_{j+1}$ for all j.
   (a) Suppose the agents' values are drawn i.i.d. from a regular distribution F. Describe an IC profit maximizing auction. (Your description should be simple and intuitive.)
   (b) Suppose the agents' values are drawn from regular distributions that are not identical. Show that VCG with monopoly reserve prices is a 2-approximation. (Hint: First show that VCG with monopoly reserve prices is a 2-approximation the special case where all $w_j = 1$.)

3. Recall from Homework 1 that we defined an allocation $\mathbf{x}$ and payments $\mathbf{p}$ to be *envy-free* for valuation profile $\mathbf{v}$ no agent wants to unilaterally swap allocation and payment with another agent. I.e., for all i and j,
$$v_i x_i - p_i \geq v_i x_j - p_j.$$
   In that homework you characterized envy-free allocations as being *swap-monotone* in that if an agent swaps with a higher valued agent they should get a higher probability of being served. Assume the agents are sorted in decreasing order, then the allocation probabilities must also be in decreasing order. You also characterized minimum envy-free payments. It is also possible to characterize maximum envy-free payments as:
$$p_i = \sum\nolimits_{j \geq i} v_j (x_j - x_{j+1}) = v_i x_i - \sum\nolimits_{j > i} x_j (v_{j-1} - v_j)$$
   where for convenience we set $x_{n+1} = 0$.
   (a) Characterize profit maximizing envy-free allocations and prices (and prove your characterization correct). Your characterization should reduce envy-free profit maximization to envy-free surplus maximization. (Hint: You should end up with a very similar theory to Myerson's theory of profit maximizing mechanisms.)
   (b) Consider the Google AdWords problem above. Give an algorithm (at a high level) for computing the envy-free optimal allocation and prices.