EECS 395/495: Algorithmic Mechanism Design **Homework 4**
Assigned: 5/27/10 **Due: 6/3/10 (accepted until 6/8)**

Note: Work in pairs. Turn in one assignment. You may refer to lecture notes, assigned readings, or the reference text book *Algorithmic Game Theory*. You may not look for help on the Internet or sources outside course materials. Your solutions should easily and concisely convey your complete understanding of each problem. If you cannot solve a problem, come to office hours. If your solution cannot easily be understood then it is wrong.

1. Prove that single-minded combinatorial auctions are not matroids.

2. Consider the following single-agent prior-free pricing game. There is a value $v \in [1, h]$. If you offer a price $p \leq v$ you get p otherwise you get zero.
   (a) Design a randomized pricing strategy to maximize the ratio of the expected revenue to the value.
   (b) Prove that your randomized pricing strategy is optimal. Hint: use the lower-bounding technique for digital-goods auctions from class.
   (c) Discuss the connection between your above results and the claim from class that it is impossible for a digital-goods auction to approximate the prior-free benchmark $\mathcal{G}(\mathbf{v}) = \max_i i v_{(i)}$.

3. Give an IC mechanism that achieves an expected revenue that is within a $\Theta(\log h)$ factor of the optimal social surplus for any downward-closed setting when all values v_i are on the interval $[1, h]$.

   Extra credit: adapt your mechanism to general (not single-minded) combinatorial auctions. I.e., m items, each agent i has a value $v_i(S')$ for each subset $S' \subseteq S = \{1, \ldots, m\}$ of the m items.