Reading: Chapter 6 (except 6.4–6.7); Chapter 11, Sections 2 and 4.

Binomal Heaps and Queues

binary heaps based queues may be improvable.

1. perform some operations faster?
2. binomial queue
   - $O(\log n)$ merge.
3. redundant binomial queue
   - $O(1)$ insert
   - $\Theta(\log n)$ delete-min, reduce-key.
4. Fibonacci heaps
   - $O(1)$ insert, reduce-key
   - $O(\log n)$ delete-min
   - Dijkstra's shortest paths + ficonacci heap $\Rightarrow \Theta(n \log n + m)$.

Binomial Heaps

Def: a **binomial heaps** is a tree that satisfies

heap order: key at node is smaller than keys of children.

binomial tree structure:

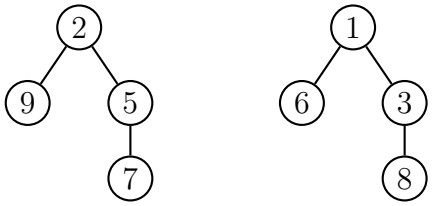
- n is power of 2.
- $n = 1$: binomial tree is a single node.
- $n = 2^i$ two 2^{i-1} node binomial trees, make one a child of root of other.

Example:

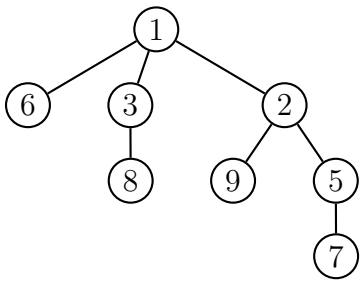
- One node trees:

- Two node trees:

- Four node trees:



- Eight node tree:



Def: a **binomial queue** is

- a list of binomial heaps.
- for each i, at most one heap of size 2^i.

Algorithm: binomial heap merge
Input: H_1 and H_2, two n node binomial heaps.

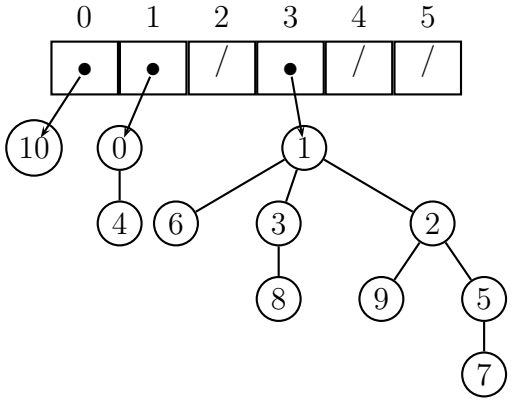
- Make tree with greater root-key child of root of other tree.

Runtime: $O(1)$

Analogy:

- size n binomial queue $\Leftrightarrow$ number n
- heap of size 2^i in list $\Leftrightarrow$ ith binary digit of n is 1.

Example:



Algorithm: merge queue
Input: Q_1, Q_2

1. treat queues like numbers
2. add numbers.

Runtime: $\Theta(\log n)$

Algorithm: insert
Input: queue Q, key k

1. make k into single-node queue Q'.
2. merge Q and Q'.

Runtime: merge $= \Theta(\log n)$

Algorithm: delete-min
Input: Q

1. find heap with min key $\Rightarrow H_i$, size 2^i
2. remove heap $H_i \Rightarrow Q'$
3. delete root of $H_i \Rightarrow i$ heaps sizes $\{1, \ldots, i\}$
4. treats heaps as queue Q''.
5. merge Q' and Q''.

Runtime: search + merge $= \Theta(\log n)$

Algorithm: reduce-key
Input: Q, key k, value v.

1. find (k', v) in queue (assume we have pointer)
2. reduce k' to k and percolate-up.

Runtime: percolate-up $= \Theta(\log n)$.

Claim: from empty queue, n inserts costs $\Theta(n)$.

Proof:

- merge $\Leftrightarrow$ add
- insert $\Leftrightarrow$ increment
- n increments from zero costs $\Theta(n)$.

□

Algorithm: build-heap
Input: unordered list

1. initialize empty queue.
2. insert each object.

Redundant Binomial Queue

"$\Theta(1)$ worst-case insert"

Recall: *redundant binary counter*

- digits are 0, 1, or 2.
- **regularity** property: 0s and 2s alternate.

Def: a **redundant binomial queue** is

- a list of binomial heaps.
- for each i, at most two heaps of size 2^i.
- heaps (as digits) satisfy redundant counter regularity.

Claim: all queue operations can be implemented to maintain regularity.

Proof: all queue operations are like arithmetic operations

Claim: redundant binary counter has $\Theta(1)$ insert, and $\Theta(\log n)$ merge, delete-min, reduce-key.