

1 Reading.

Chapter 5 and the supplemental reading on hash tables.

2 Problems.

This homework is shorter than usual and will count for 10 points (instead of the usual 20).

1. Problem 5.1.
2. Two keys collide when they hash to the same location in the hash table. This problem concerns counting the total number of collisions. If three distinct keys a , b , and c , collide at the same location it counts as $\binom{3}{2} = 3$ collisions (as a collides with b , b collides with c , and a collides with c ; if four keys collide at the same location it counts as $\binom{4}{2}$ collisions, etc. Consider a hash table with size s and $n < s$ distinct keys. Express your answers to the questions below terms of s and n , as needed.
 - (a) For an arbitrary (not random) hash function h , what is the worst case number of collisions? Give an example that achieves this worst case.
 - (b) Let \mathcal{H} be a *universal class of hash functions*. We wish to count the total number of collisions. Draw hash function h uniformly at random from \mathcal{H} . What is the (worst case) expected number of collisions? Prove your bound. (Hint: define the appropriate set of indicator variables and use linearity of expectation.)