Network-Based Control of Distributed Real-Time Systems – From Theory to Application

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A Hierarchical Network Game for Internet Pricing

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1. Internet Pricing

- Pricing for best effort service
  Pricing with QoS guarantees
- Single class pricing
  Priority pricing
- Uniform pricing
  Differentiated pricing
- Noncooperative game
  Cooperative game

2. General Network

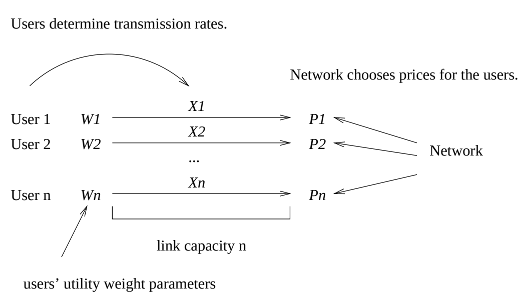
Problem Formulation

1. The network (ISP) and the users play a Stackelberg network game:
   - As the leader, the network announces the prices
   - Accordingly, the users as followers respond with certain transmission rates to maximize their individual utilities
   - Knowing this kind of reaction of users, the network must determine the prices to achieve optimal revenue
2. The users play an n-player noncooperative game.

Result

- The n-player noncooperative game admits a unique Nash equilibrium

3. Single Link Network



3a. Uniform Price (UniPri) [1]

Problem Formulation

- The network charges a uniform price $p_1 = p_2 = \cdots = p_n = p$ for all the users
- User i determines x_i to maximize his utility $F_i = w_i \log(1+x_i) - \frac{1}{n-\sum_{j=1}^n x_j} - p x_i$
- The network must choose p to maximize revenue $R = p\sum_{j=1}^n x_j$

Results

- Positive solution (admission) condition:

$$(\frac{2w_i}{w_{av}} - 1)(n^2 w_{av})^{\frac{1}{3}} > 1, \quad i = 1, 2, \cdots, n, \quad (1)$$

  where $w_{av} = \frac{1}{n}\sum_{j=1}^n w_j$
- If the above condition is not satisfied for all the users, then order the users according to w_i's from the largest to the smallest and the first $\tilde{n}$ users are admitted where $\tilde{n}$ is the largest possible integer such that the admission condition is satisfied

3b. Differentiated Prices (DiffPri)

Problem Formulation

- The network can charge different prices for different users
- User i determines x_i to maximize his utility $F_i = w_i \log(1+x_i) - \frac{1}{n-\sum_{j=1}^n x_j} - p_i x_i$
- The network must choose p_i's to maximize revenue $R = \sum_{j=1}^n p_j x_j$

Results

- Positive solution (admission) condition:

$$(\frac{2\sqrt{w_i}}{v_{av}^{\frac{1}{2}}} - 1)(n v_{av}^{\frac{1}{2}})^{\frac{2}{3}} > 1, \quad i = 1, 2, \cdots, n, \quad (2)$$

  where $v_{av}^{\frac{1}{2}} = \frac{1}{n}\sum_{j=1}^n \sqrt{w_j}$
- If the above condition is not satisfied for all the users, then the solution can be obtained similarly as in UniPri

4. Admission Condition Comparison

(1) $\Rightarrow$ (2)

Hence:

1. If the users are admitted in UniPri, they must be admitted in DiffPri.
2. More users can be admitted in DiffPri than in UniPri.

4a. Same Users Admitted

Results

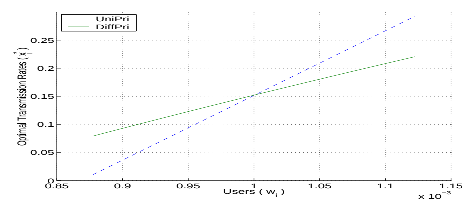
By price differentiation:

- Congestion decreases
- Revenue increases for the network
- Users with smaller w_i's are better off but not the others (see example)

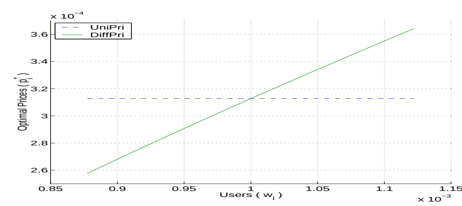
Example

$n = 50$ with users' utility weight parameters evenly distributed around $w_{av} = 0.001$.

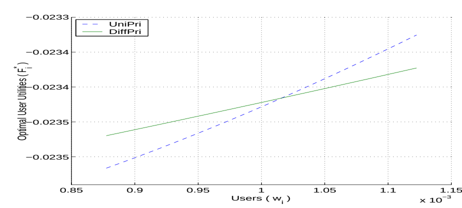
- Individual flows:



- Prices:



- Utilities:

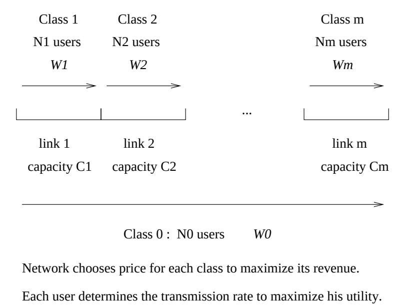


4b. More Users Admitted in DiffPri

Asymptotic Analysis Results

1. By price differentiation, congestion decreases, the network is better off with revenue and those users with smaller w_i's are better off but not the others.
2. Both for UniPri and DiffPri, as the number of admitted users increases, all parties are better off in terms of the throughput, flows, congestion, prices, utilities and revenue. Therefore, the network intends to increase the capacity to accommodate more users as possible, which benefits the users in return.

5. Linear Network



Result

- Asymptotic analysis shows that the revenue increases as the number of users increases. Thus, the network has an incentive to increase link capacities, which also reduces the congestion cost.

6. Extensions

- Stochastically distributed users
- Multiple service providers

Reference

[1]T. Başar and R. Srikant, "Revenue-Maximizing Pricing and Capacity Expansion in a Many-Users Regime," *Proc. IEEE Infocom 2002*, pp. 1556-63, 2002.