# Homework 4: Graph

Code due: Tue., Nov. 5 at 11:59 PM (via GSC)
Self-eval due: Thu., Nov. 7 at 11:59 PM (on GSC)

**You may work on your own or with one (1) partner.**

For this assignment you will implement an API for weighted, undirected graphs; then you will use this API to implement a depth-first search.

In `graph.rkt`[1], I've supplied headers for the methods and function that you'll need to write, along with a few suggested helpers and some code to help you with testing.

## Orientation

The graph for this assignment is a weighted, undirected graph whose vertices are natural numbers. In particular, a graph of $n$ vertices will have vertices numbered 0, 1, …, $n-1$. This makes it straightforward to associate information with each vertex in a vector of size $n$ via direct addressing.

Before defining our signature for weighted, undirected graphs, we define contracts for describing several of the arguments and results involved.

- A vertex is represented as a natural number:

  ```
  let Vertex? = nat?
  ```

- We use singly-linked lists of vertices, made out of a `cons` struct with `car` and `cdr` fields as in the previous homework:

  ```
  let VertexList? = Cons.ListC[Vertex?]
  ```

  The `Cons.ListC` contract (from `import cons`) optionally takes a contract for the list element, which lets us express that a `VertexList?` is indeed a linked list of `Vertex?`es.

- A weight is a real number, and an optional weight is either a weight or `None`:

[1] `https://bit.ly/2P74yWC`

```
let Weight? = AndC(num?, NotC(OrC(inf, -inf, nan)))
let OptWeight? = OrC(Weight?, NoneC)
```

- A weighted edge is represented by a struct containing two vertices and a weight; we use lists of those as well:

```
struct WEdge:
    let u: Vertex?
    let v: Vertex?
    let w: Weight?

let WEdgeList? = Cons.ListC[WEdge?]
```

  Note that `WEdge` is used in the result of one of the graph methods (below), but is not intended, and probably not well-suited, for use internally in your graph representation.

Now we can give our signature for weighted, undirected graphs as a DSSL2 interface with five operations:

```
interface WU_GRAPH:
    def len(self) -> nat?
    def set_edge(self, u: Vertex?,
                       v: Vertex?, w: OptWeight?) -> NoneC
    def get_edge(self, u: Vertex?, v: Vertex?) -> OptWeight?
    def get_adjacent(self, v: Vertex?) -> VertexList?
    def get_all_edges(self) -> WEdgeList?
```

The operations behave as follows:

- The `len` method returns the number of vertices in the graph, that is, $n$.
- The `set_edge` method adds an edge of weight `w` between vertices `v` and `u` when `w` is a number; if the edge already exists, its weight is updated to `w`. If `w` is `None` then the edge, if it exists, is removed, and if absent remains absent.

  Note that because the edges of undirected graphs are symmetric, the order of `u` and `v` mustn't matter; this implies that `set_edge` must maintain an invariant.
- The `get_edge` method returns the weight of the edge between vertices `u` and `v` if it exists, or `None` if it does not.

- The `get_adjacent` method returns a list of all vertices that are directly connected to vertex `v`. The order of the list is unspecified.
- The `get_all_edges` method returns a list of all edges in the graph, in unspecified order[2]. For each edge in the graph, it includes only one direction in the list. For example, if a graph has an edge of weight 10 between vertices 1 and 3, then the resulting list will contain either `WEdge(1, 3, 10)` or `WEdge(3, 1, 10)`, but not both.

  One easy way to avoid redundant edges is to only add an edge `e` to the list when `e.u <= e.v`.

# Your task

## Representation

Your job is to implement the `WuGraph` class, which must satisfy the `WU_GRAPH` interface. To do so, you must choose a representation, as either an adjacency matrix or adjacency lists. Whichever you choose, you will need to add some field(s) to the `WuGraph` class and fill in the `__init__` method to initialize them.

1. Define the field(s) for your representation at the top of the `WuGraph` class.
2. Complete the definition of the `__init__` method. The `WuGraph` constructor takes one natural number argument, which is the number of vertices desired in the new graph.

## Graph operations

Once you've defined your graph representation, you will have to implement the five graph API methods as specified by the `WU_GRAPH` interface. Their required time complexities depend on your choice of representation.

[2] This means any order you like.

### Adjacency matrix representation

3. Implement the `len` method, which must be $\mathcal{O}(1)$ time.
4. Implement the `set_edge` method, which must be $\mathcal{O}(1)$ time.
5. Implement the `get_edge` method, which must be $\mathcal{O}(1)$ time.
6. Implement the `get_adjacent` method, which must be $\mathcal{O}(V)$ time.
7. Implement the `get_all_edges` method, which must be $\mathcal{O}(V^2)$ time.

### Adjacency lists representation

The running times of several adjacency list operations depend on $d$, the degree of the graph.

3. Implement the `len` method, which must be $\mathcal{O}(1)$ time.
4. Implement the `set_edge` method, which must be $\mathcal{O}(d)$ time.
5. Implement the `get_edge` method, which must be $\mathcal{O}(d)$ time.
6. Implement the `get_adjacent` method, which must be $\mathcal{O}(d)$ time.
7. Implement the `get_all_edges` method, which must be $\mathcal{O}(V + E)$ time.

## Depth-first search

Once you have your graph implementation working, there's one more thing to implement, a depth-first search function:

```
dfs : WU_GRAPH Vertex [Vertex -> None] -> None
```

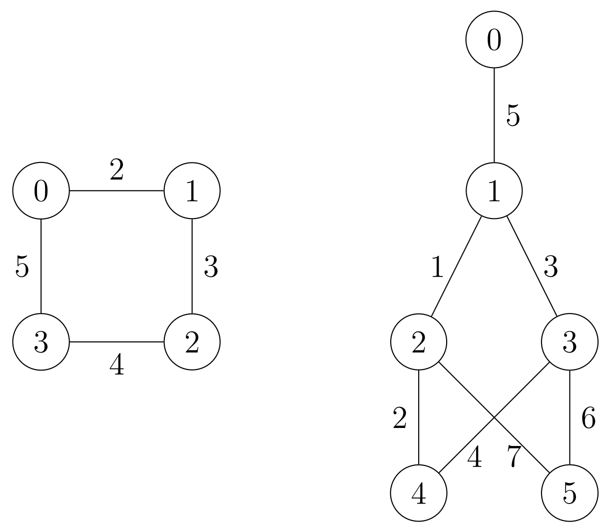
This function takes a graph `g`, a vertex `u`, and a visitor function `f`. It performs a depth-first search starting at `u`. As it encounters each vertex `v` for the first time, it calls `f(v)`. The visitor function is called on each reachable vertex exactly once, in a valid depth-first order.

8. Implement the `dfs` function, which must have the optimal asymptotic time complexity: $\mathcal{O}(V + E)$ if using adjacency lists, or $\mathcal{O}(V^2)$ if using an adjacency matrix.

In order to help you test `dfs`, we have provided a function `dfs_to_list` that uses it to construct a list of vertices in DFS-order. It should be relatively easy to write `assert_eq` tests for `dfs_to_list` once you know in what order your `dfs` function visits vertices.

## Testing

To facilitate testing, we have provided you two example graphs. Function `EX_GRAPH1` returns the four-vertex graph on the left, function `EX_GRAPH2` returns the six-vertex graph on the right:




The starter code also includes functions `sort_vertices` and `sort_edges`, which sort lists of vertices and `WEdges`, respectively. This is useful for testing because several methods produce lists in an unspecified order.

# Deliverables

Your completed "`graph.rkt`," containing

- working definition of the `WuGraph` class,
- a working definition of the `dfs` function, and

- sufficient tests to be confident of your code's correctness.