Binary Search Trees

CS 214, Fall 2019

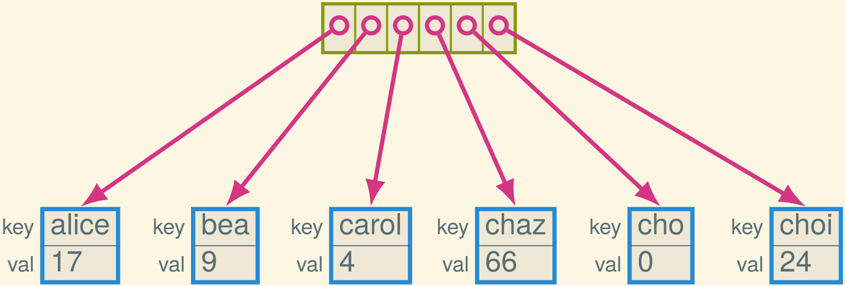
A data structure for dictionaries

There are several data structures that we can use to represent dictionaries:

- A list of keys-value pairs
- A hash table
- An array of key-value pairs
- A sorted array of key-value pairs

Let's consider the last one

A sorted array dictionary



Easy to lookup

Input: a dictionary array *array* and a key *key*
Output: a value, or nothing

```
start ← 0;
limit ← the length of array;
while start < limit do
    mid ← the average of start and limit;
    if key < array[mid].key then
        limit ← mid
    else if key > array[mid].key then
        start ← mid + 1
    else
        return array[mid].val
    end
end
return null
```

Complexity of lookup

(Simplify by dropping values and changing keys to numbers. Same algorithm works, but it fits on less screen space.)

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- Suppose we want 75. We'll track where it might be with a area

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- Initially, the element could be anywhere in the array
- At each iteration, $\mathit{limit} - \mathit{start}$ is cut in half
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- Hence, $\mathcal{O}(\log n)$

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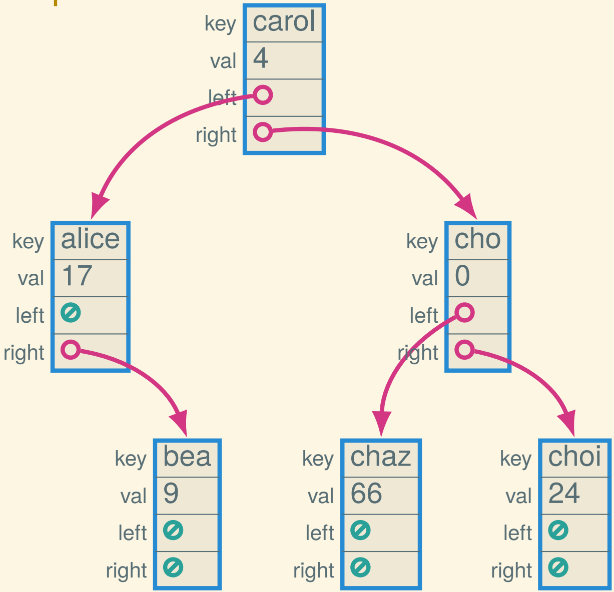
- There may be as many as n elements to move
- Hence insertion is $\mathcal{O}(n)$

Enter the BST

A *binary search tree* stores elements in order in a linked data structure

- In order means we can binary search
- Linked means we can easily insert new elements

BST example


key
carol
val
4
left
right
key
alice
val
17
left
right
key
cho
val
0
left
right
key
bea
val
9
left
right
key
chaz
val
66
left
right
key
choi
val
24
left
right

BST lookup algorithm

Input: a BST root *root* and a key *key*
Output: a value, or nothing

```
curr ← root;
while curr is not null do
    if key < curr.key then
        curr ← curr.left
    else if key > curr.key then
        curr ← curr.right
    else
        return curr.val
    end
end
return null
```

Complexity of BST lookup

It’s binary search, right?

Binary search, again

Input: a dictionary array *array* and a key *key*
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start $\leftarrow 0$;
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while start < limit do
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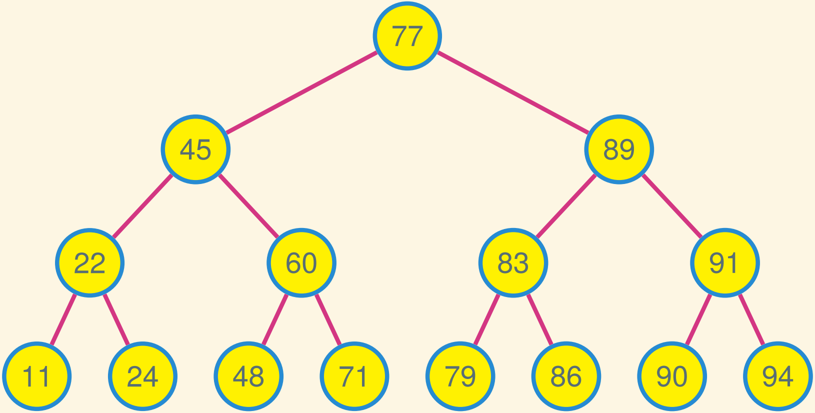
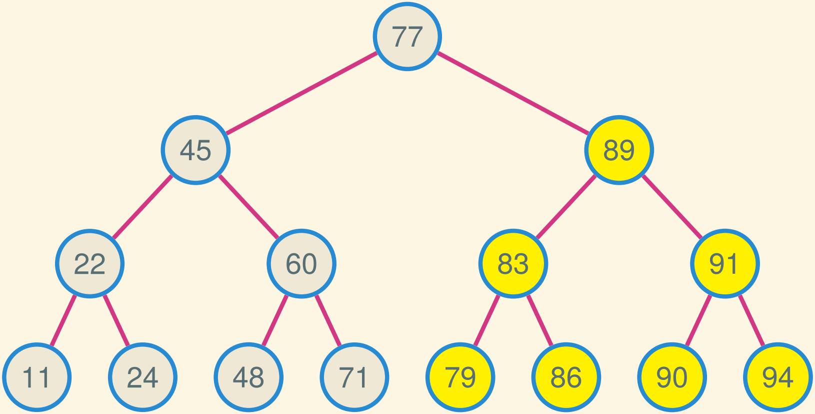
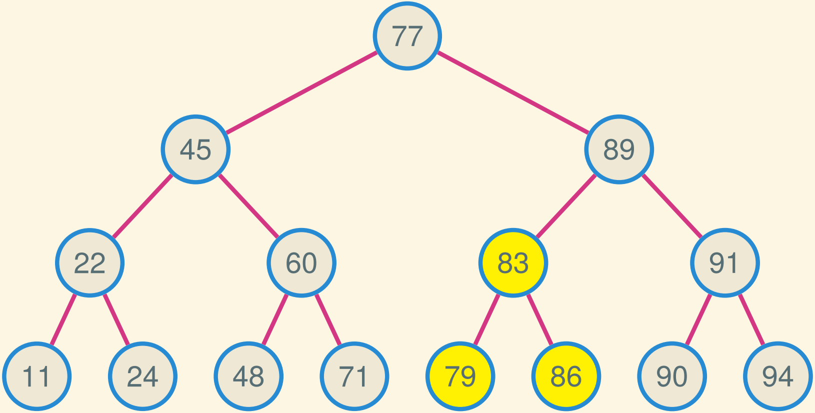


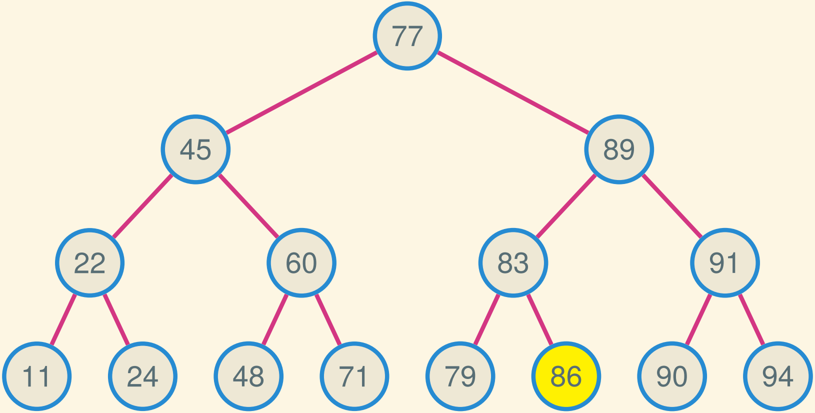
Complexity of BST lookup



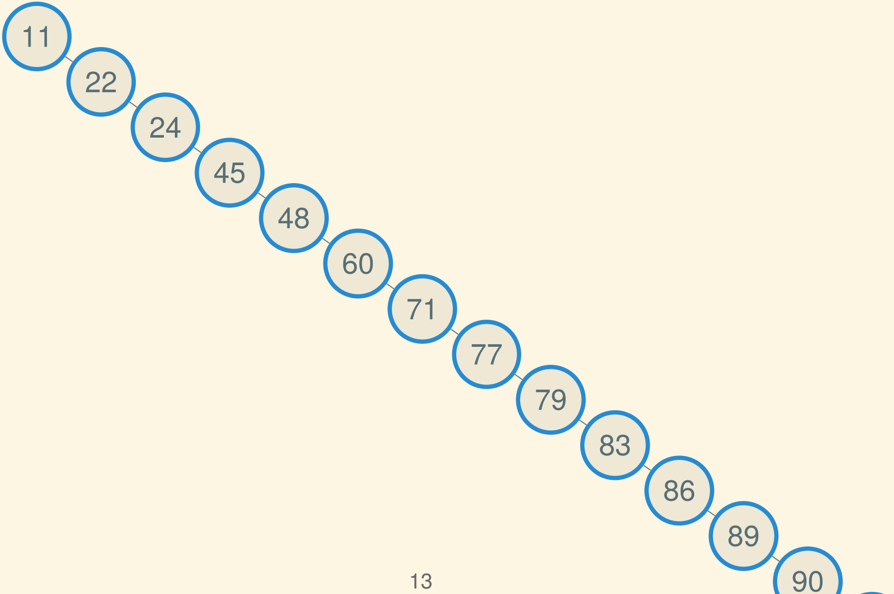
Complexity of BST lookup



Complexity of BST lookup



Complexity of BST lookup, take 2



BST insert algorithm (recursive)

```
Function BstInsert(node, key, value) is
    Output: the updated BST

    if node is null then
        node ← a new node with key, value, and no children
    else if key < node.key then
        node.left ← BstInsert(node.left, key, value)
    else if key > node.key then
        node.right ← BstInsert(node.right, key, value)
    else
        node.val = value
    end

    return node
end
```

BST insert algorithm (with pointers)

Input: a BST root *root*, a key *key*, and a value *value*
Output: the updated BST

```
curr ← the address of root;
while the value addressed by curr is not null do
    if key < curr.key then
        curr ← the address of curr.left
    else if key > curr.key then
        curr ← the address of curr.right
    else
        curr.val ← value;
        return
    end
end
newNode ←
  a new node with key, value, and null for both children;
the value addressed by curr ← newNode
```

Complexity of BST insert

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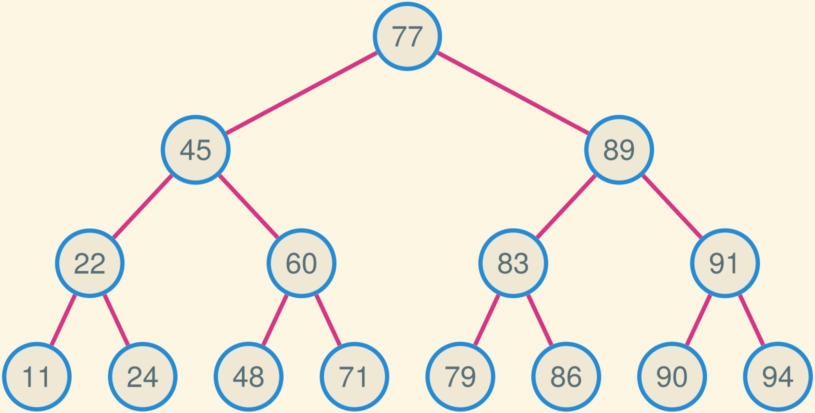
- Hence, $\mathcal{O}(\log n)$

BST delete



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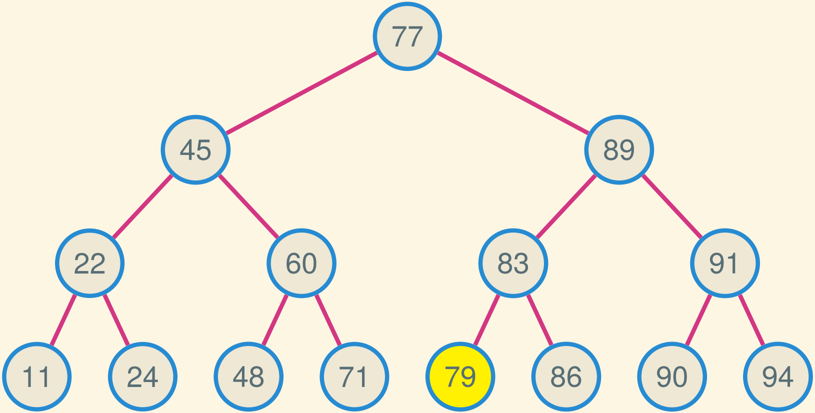

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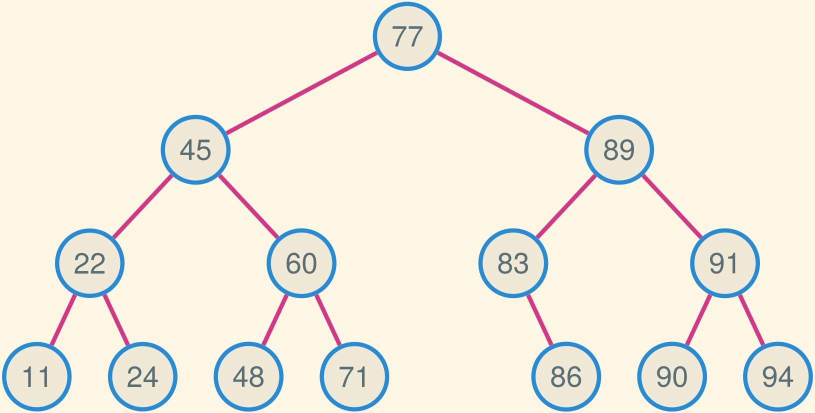

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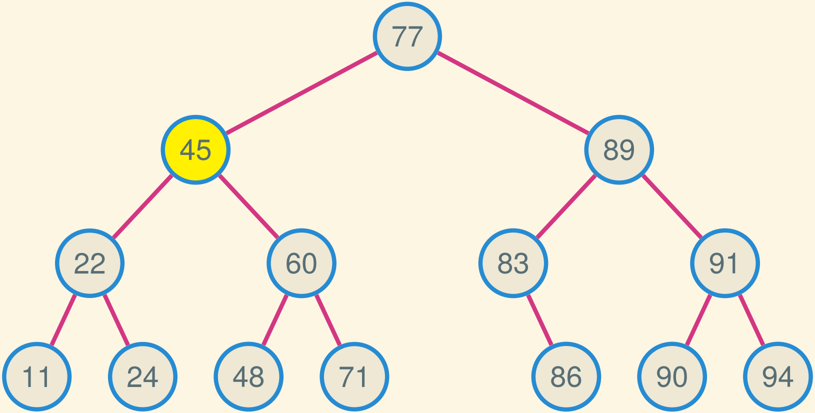

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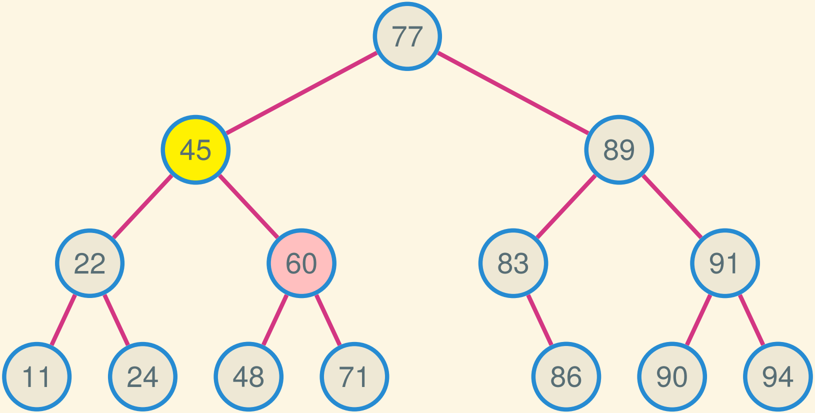

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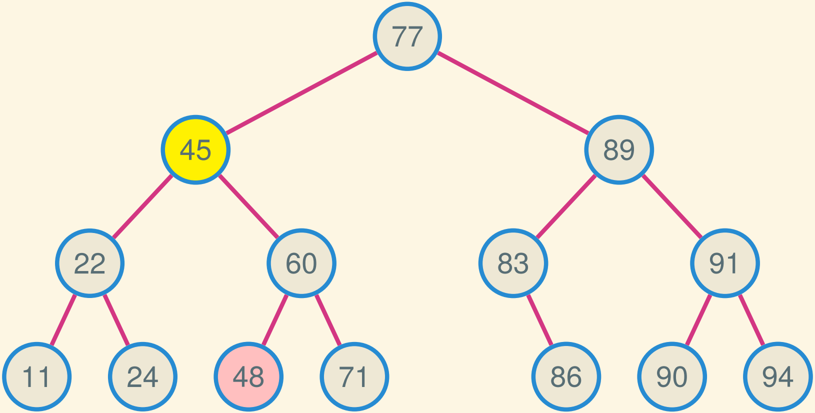

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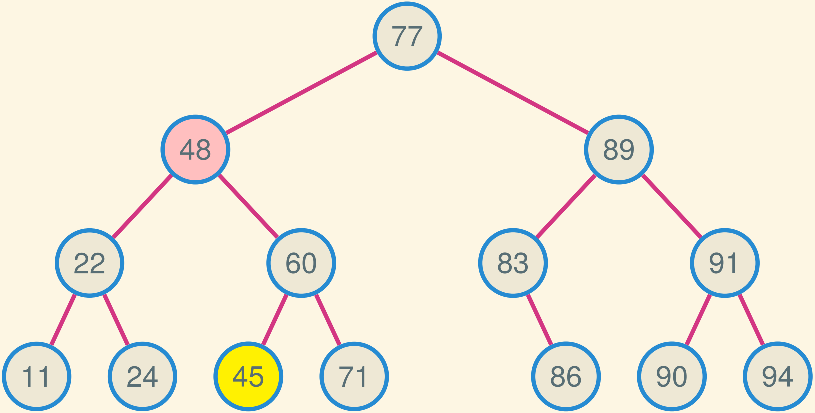

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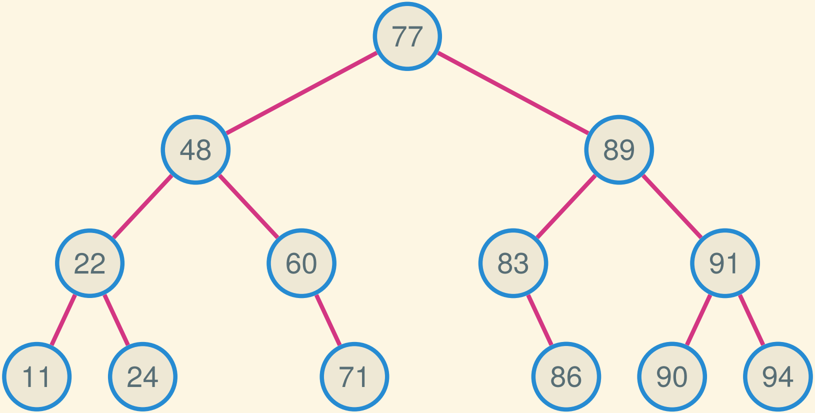

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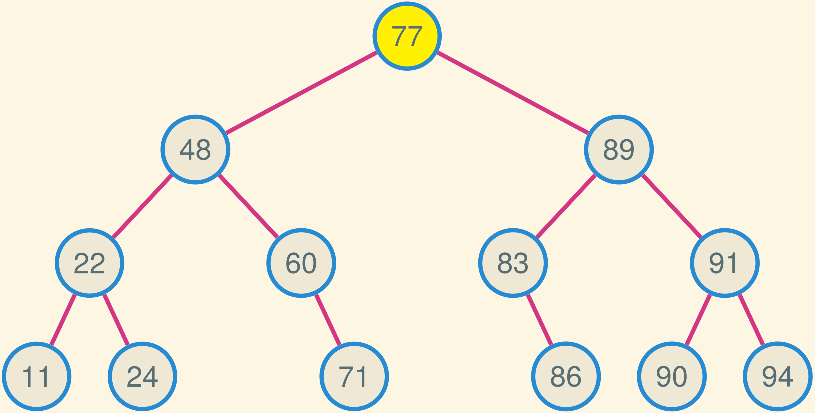

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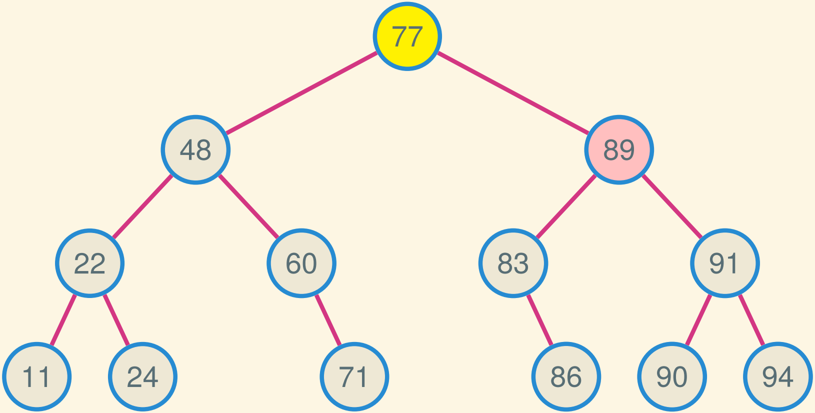

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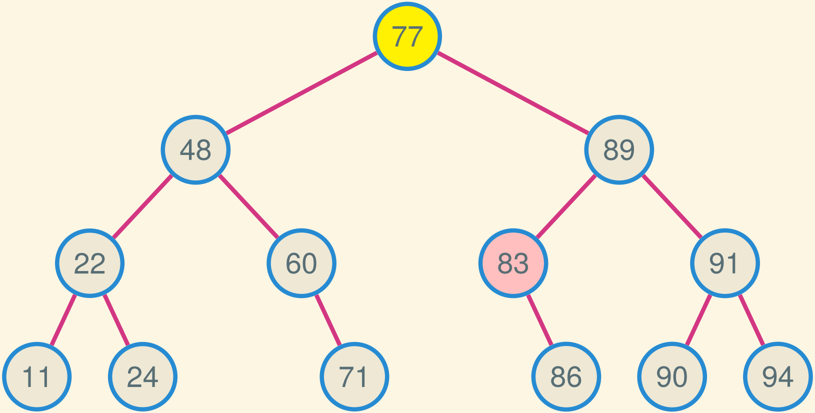

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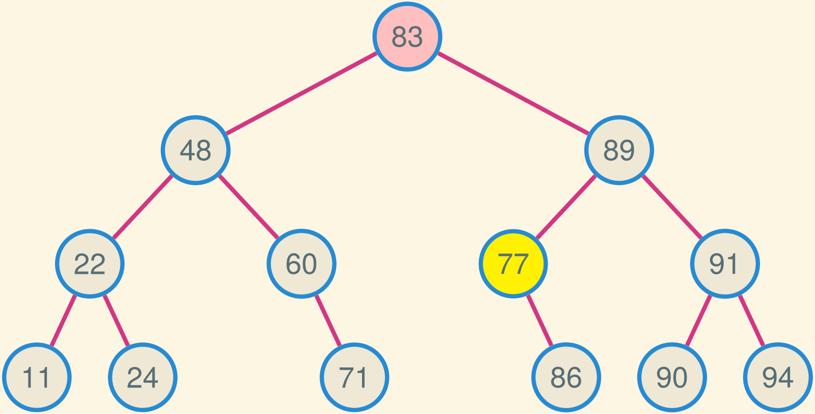

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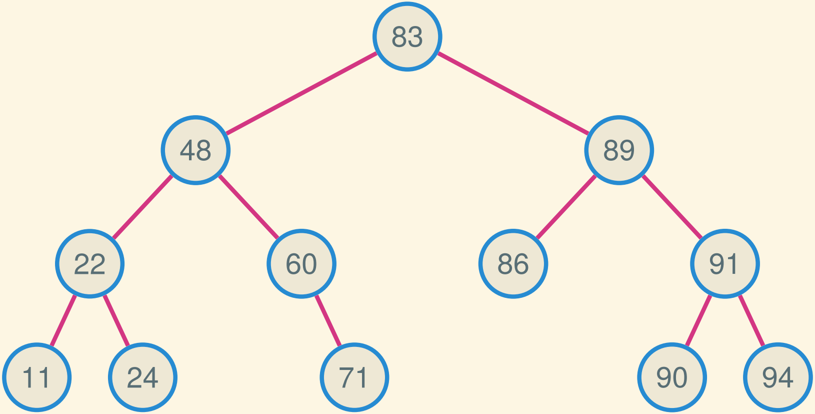

Next time: hashing and hash tables