AVL Trees

EECS 214, Fall 2017

A self-balancing BST

Random binary search trees are *very likely* to be balanced

Self-balancing trees are *guaranteed* to be balanced

Balanced search tree survey

Red–black trees (tomorrow)

Main idea: Every node has an extra bit marking it “red” or “black”

Local invariant: No red node has a red parent

Global invariant: Equal number of black nodes from root to every leaf

2-3 trees

Main idea: 2-nodes have one element and two children;
3-nodes have two elements and three children

Local invariant: All subtrees of a node have the same height

Global invariant: Every leaf is at the same depth

2-4 trees

Main idea: Like 2-3 trees, but also has 4-nodes with three elements and four children

Local invariant: All subtrees of a node have the same height

Global invariant: Every leaf is at the same depth

B-trees

Main idea: Generalizaton of 2–4 trees to 2–k trees

Local invariant: Like 2–4 trees, but allow some number of missing subtrees

Global invariant: Every leaf is at the same depth

Splay trees

Main idea: Cache recently accessed elements near the root of the tree

Local invariant: *Complicated; required amortized analysis*

Global invariant: Paths are *very likely* to be $\mathcal{O}(\log n)$

AVL trees

Main idea: Maintain a *balance factor* giving the difference between each node's subtrees' heights

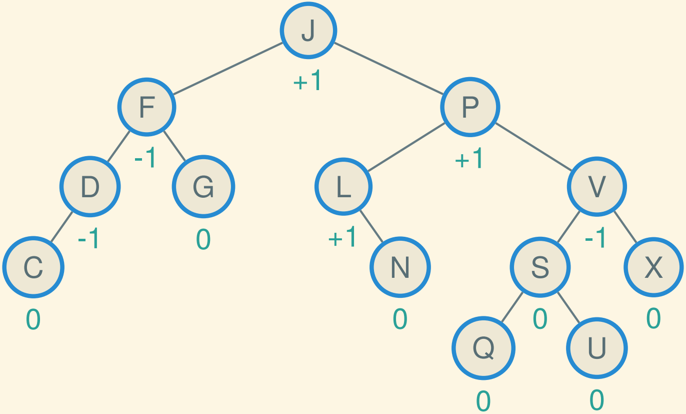
Local invariant: Balance factor between -1 and 1, maintained via rotations

Global invariant: Tree is approximately height-balanced

(AVL stands for Georgy Adelson-Velsky and Evgenii Landis)

AVL trees

Example of an AVL tree



Local invariant maintains global property

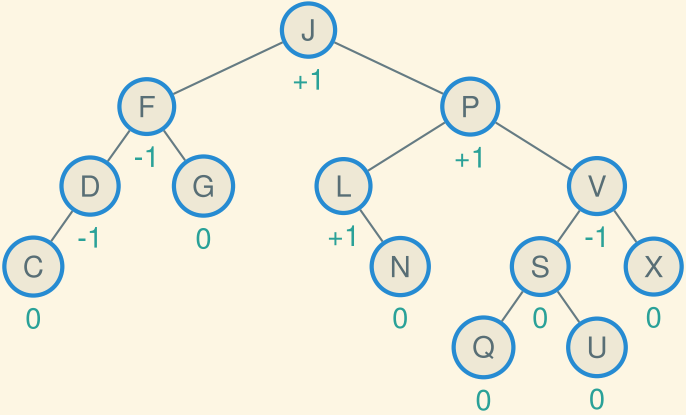
- Balance factors are maintained locally
- Never recompute them from scratch
- Yet the whole tree stays reasonably balanced

AVL insertion

- First do a normal leaf insertion
- Track balance factors on the way back up to the root
- Adjust with rotations as necessary

AVL insertion example

Let's insert H:



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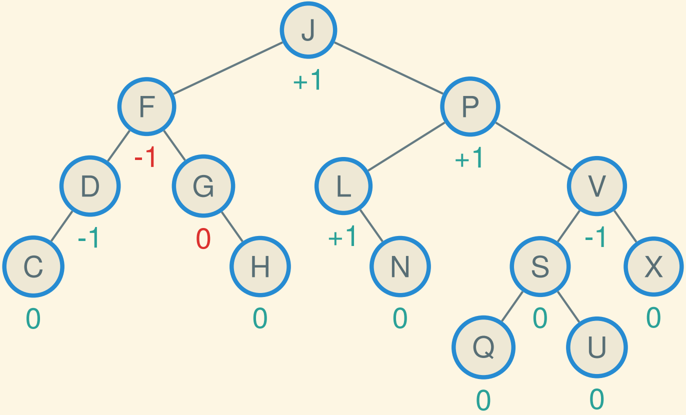

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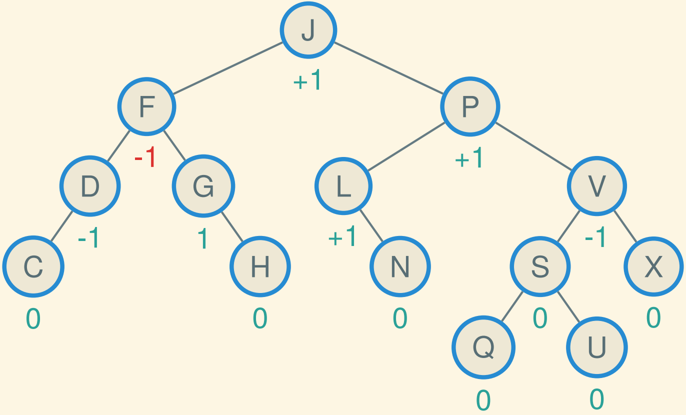

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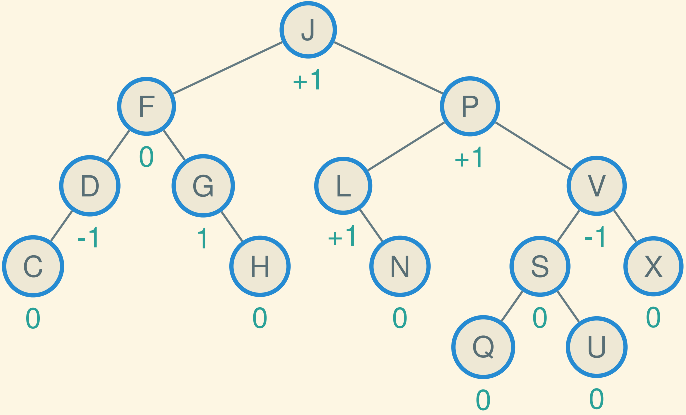
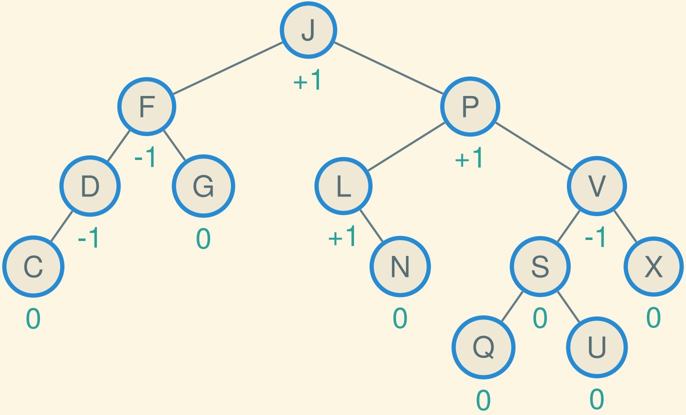


Another AVL insertion example

Let's insert B:



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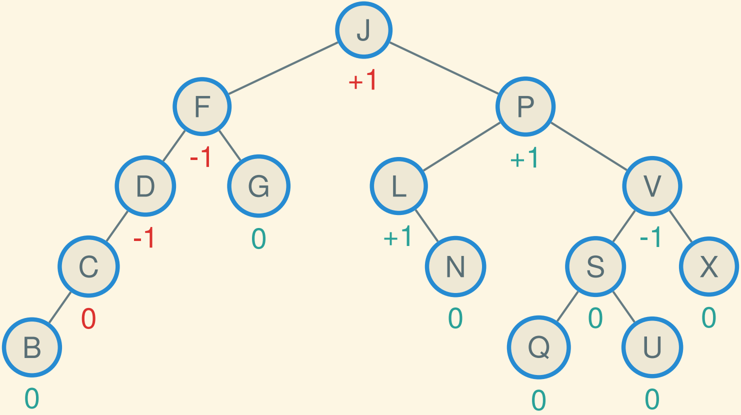

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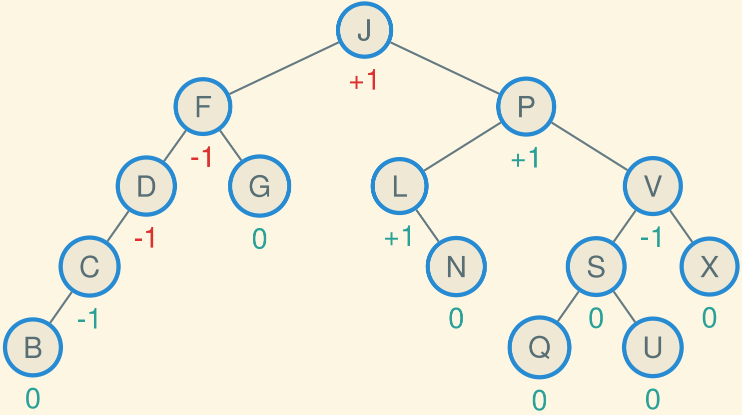

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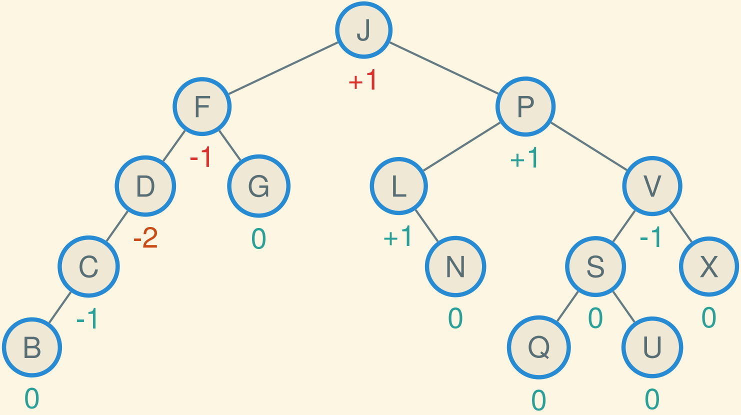

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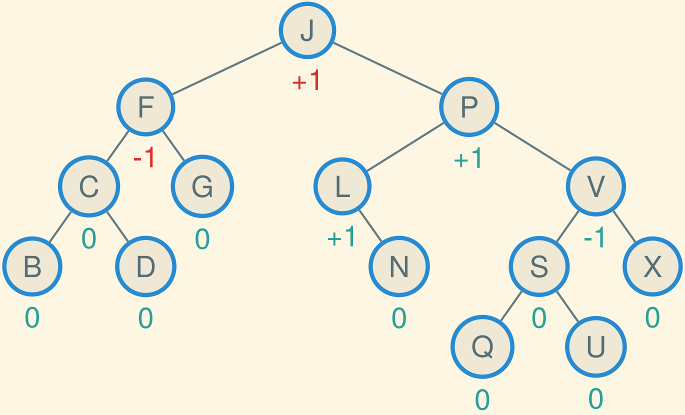

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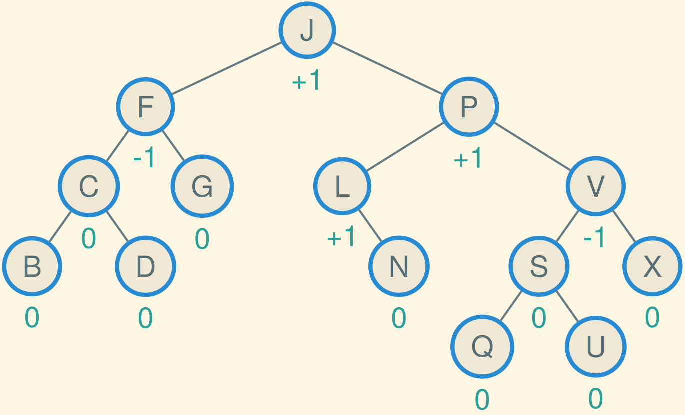
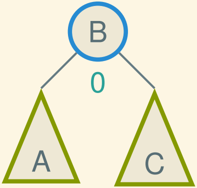


Maintaining the AVL property

Suppose we have an AVL tree:



(Convention: triangles represent equal-height subtrees.)

Maintaining the AVL property

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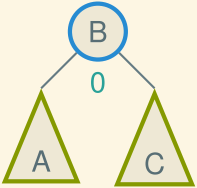
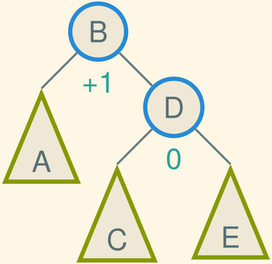


(Convention: triangles represent equal-height subtrees.)

Right now the balance factor is 0. So if we insert into A or C and that subtree grows in height, it becomes -1 or 1.

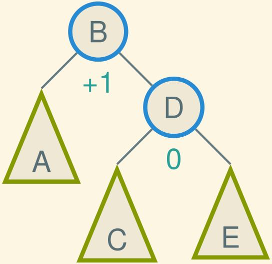
Maintaining the AVL property



Right now the balance factor at B is +1.

Suppose we insert into A. What happens to B's balance factor?

Maintaining the AVL property

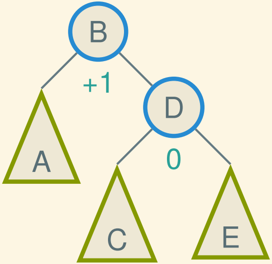


Right now the balance factor at B is +1.

Suppose we insert into A. What happens to B's balance factor?

- If no change in A's height then no change in B's balance
- If A's height grows then B's balance factor goes to 0

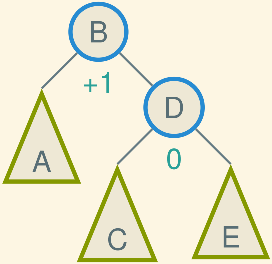
Maintaining the AVL property



Right now the balance factor at B is +1.

Suppose we insert into C. What happens to B's balance factor?

Maintaining the AVL property

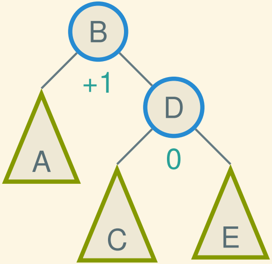


Right now the balance factor at B is +1.

Suppose we insert into C. What happens to B's balance factor?

- If no height change then B's balance doesn't change
- If C grows then B's balance factor becomes +2

Maintaining the AVL property

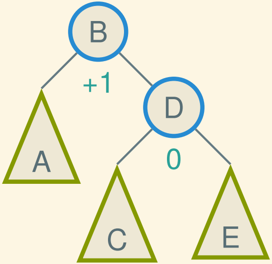


Right now the balance factor at B is +1.

Suppose we insert into C. What happens to B's balance factor?

- If no height change then B's balance doesn't change
- If C grows then B's balance factor becomes +2—not okay!

Maintaining the AVL property



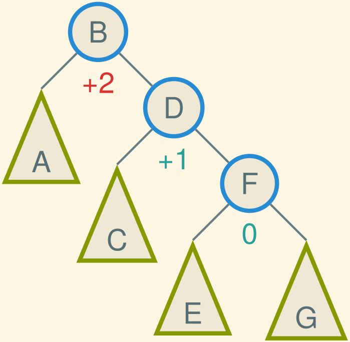
Right now the balance factor at B is +1.

Likewise, suppose we insert into E. What happens to B's balance factor?

- If no height change then B's balance doesn't change
- If E grows then B's balance factor becomes +2—not okay!

The right-right case

If the height of the right-right subtree (formerly E) increases, we get a situation like this:



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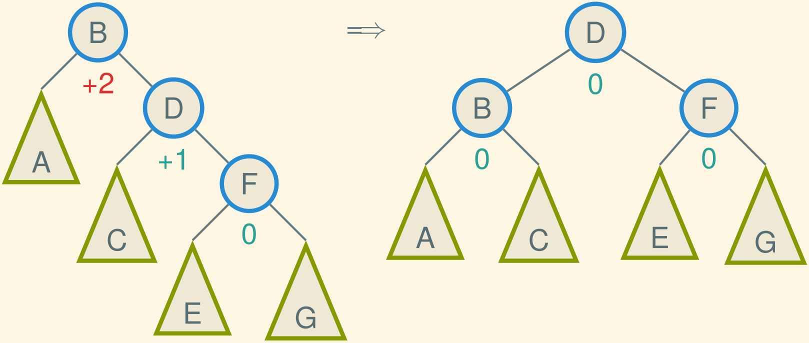
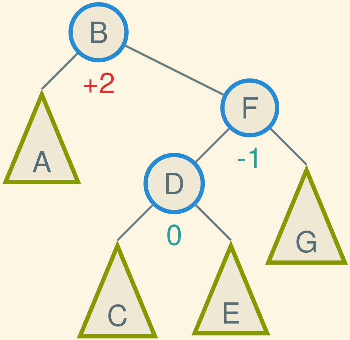


The right-left case

If the height of the right-left subtree (formerly C) increases, we get a situation like this:



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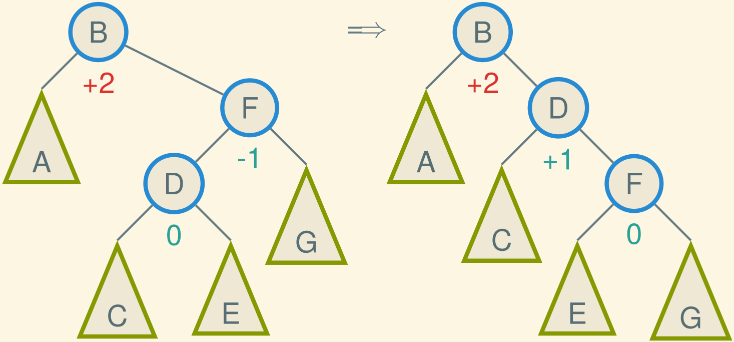

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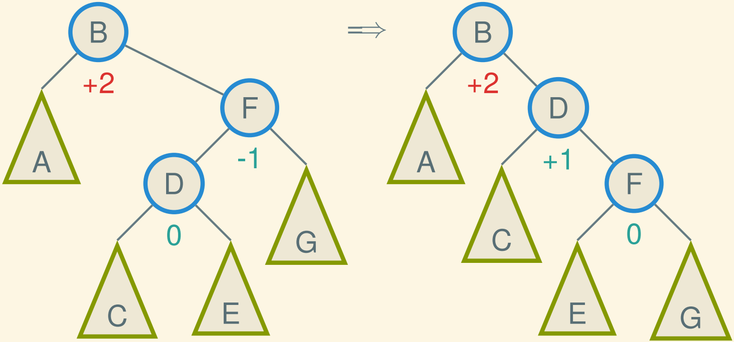

But this is now the right-right case, which we know how to handle!

Maintaining the AVL property

- We've seen the right-right and right-left cases
- The left-left and left-right cases are symmetrical
- Deletion is like ordinary BST deletion, with the same rebalancing cases

Next time: red–black trees