Solution of Hw 1, 4(e)

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1 Problem 4(e)

Assume the sequence in part (c) was generated by the following process: first, a fair coin generates the first flip. Thereafter, each flip i is the same as the previous flip with probability gamma, and is otherwise the opposite. NOTE: the gamma parameter has a very different role in generating a sequence of flips than does the theta parameter from the lecture notes, so be aware.

1.1 Is there a setting of gamma that corresponds to using a fair coin for the entire sequence?

Yes, $\gamma = 0.5$.

1.2 What is the maximum likelihood estimate of gamma for the sequence in part (c)?

$$\begin{aligned}\hat{\gamma} &= \frac{\text{\# of same-as-previous flip}}{\text{\# of flip}} \\ &= \frac{111}{299} \\ &= 0.371\end{aligned}$$

1.3 Assume the gamma parameter was chosen for the sequence in part (c) to be 0.6 with probability 0.01, or to be 0.4 with probability 0.99. What's the MAP estimate of gamma given the sequence in part (c) and this prior knowledge?

$$p(\gamma|\text{sequence}) \propto p(\text{sequence}|\gamma)p(\gamma) = \gamma^{111}(1-\gamma)^{188}p(\gamma),$$
$$p(\text{sequence}|\gamma = 0.6)p(\gamma = 0.6) = 0.6^{111} \times 0.4^{188} \times 0.01,$$
$$p(\text{sequence}|\gamma = 0.4)p(\gamma = 0.4) = 0.4^{111} \times 0.6^{188} \times 0.99.$$

As
$$0.6^{111} \times 0.4^{188} < 0.4^{111} \times 0.6^{188}$$
and
$$0.01 < 0.99,$$
we obtain
$$p(\gamma = 0.6|\text{sequence}) < p(\gamma = 0.4|\text{sequence}).$$
Therefore, the MAP estimate of γ is 0.4.

1.4 Assume the gamma parameter is Beta(3,3) distributed. What is the MAP estimate for gamma given the sequence in part (c) and this prior knowledge?

$$\gamma_{\text{MAP}} = \frac{111 + 3 - 1}{111 + 3 - 1 + 188 + 3 - 1} = 0.373.$$

1.5 Now the fun part. One might hypothesize that a person's mental randomness is less like a fair coin than it is like the process described above, with a gamma value slightly lower than 0.5. What does this hypothesis mean, in a single sentence of plain English?

A person's mental randomness is less likely to repeat the same coin orientation than a fair coin is.

1.6 To test the above hypothesis, compute the likelihood of your sequence from part (a) under the process with gamma=0.45. Is this higher or lower than that of a fair coin (i.e. $(1/2)^{50}$)? Do your results support or contradict the hypothesis?

From the data of the homework, every student's sequence supports the hypothesis.