Multi-Carrier Transmission with Limited Feedback: Power Loading over Sub-Channel Groups

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***Abstract*— Feedback of channel state information (CSI) enables a multi-carrier transmitter to optimize the power allocation across sub-channels. We consider a single user feedback scheme in which the entire set of sub-channels is evenly divided into smaller groups of sub-channels, and the receiver requests the use of a particular group if the gain of every sub-channel in the group is above a threshold. The transmit power is then uniformly spread across the requested sub-channel groups. The amount of feedback is therefore controlled by the group size and the threshold. For this scheme, given a total power constraint, we characterize how the channel capacity scales with the number of sub-channels N as a function of the feedback rate. We then consider transmission over a block fading channel, assuming that each coherence block contains both feedback and data transmission. We optimize the fraction of feedback overhead as a function of the number of feedback bits per channel use and coherence time. Numerical results show that the asymptotic (large-N) analysis accurately predicts the behavior of finite-size systems of interest.**

I. Introduction

Multi-carrier transmission techniques, including orthogonal frequency division multiplexing (OFDM), provide an effective way to exploit the frequency-diversity present in a multipath fading channel. A substantial increase in achievable rates is possible if the power allocated across the sub-channels can be adapted to the channel variations [1]. However, the optimal (water-filling) power allocation may require a prohibitive amount of feedback of channel state information (CSI).

The problem of power allocation with limited feedback has been previously considered in [2]–[5]. In particular, [2] proposes a codebook of power loading vectors, which maximizes an objective such as achievable rate. In that scheme the size of the codebook, and hence the search complexity, grows exponentially with the amount of feedback. Other schemes, which feed back one bit per sub-channel, have been considered in [3], [5], [6]. It is shown in [3] that with this type of feedback scheme and N independent block Rayleigh fading sub-channels, $O(\log^3 N)$ bits of feedback per coherence block can achieve the optimal growth rate of $O(\log N)$ bits/channel use.

In this work we consider a feedback scheme for a single-user, multi-carrier channel in which the entire set of sub-channels is evenly divided into equal-size groups of sub-channels. The feedback then indicates the particular (active) groups, which the transmitter should use to transmit the data. We assume that the transmitter uniformly spreads the available power across the active sub-channel groups. This feedback scheme has been previously studied for downlink (multi-user) OFDM in [7]. Grouping, or clustering, of sub-channels to reduce feedback overhead, again in a multi-user setting, has also been studied in [8]. Clustering sub-channels to reduce the training overhead and peak-to-average power ratio was previously studied in [9].

As in [7], we assume that the channel is known at the receiver, and that the receiver instructs the transmitter to activate a particular sub-channel group if the gain of every sub-channel in the group exceeds a threshold. Based on this criteria, the receiver forms a binary vector, which designates the set of active sub-channel groups. This vector is compressed (entropy-coded) and relayed to the transmitter. The feedback is therefore represented by a variable-length code. The group size and threshold can be adjusted to maximize the forward achievable rate, subject to a constraint on the average feedback rate.

We analyze the performance (achievable rate) of this feedback scheme in the asymptotic regime where the number of sub-channels N is large. Specifically, we characterize the growth in achievable rate with N as a function of the amount of feedback (which can also scale with N). Numerical analysis shows that these results are quite accurate for a finite size system of interest, namely, when N is a few hundred. The optimal group size and threshold are also computed as a function of the amount of feedback. Let B denote the number of feedback bits per coherence block and S denote the Signal-to-Noise Ratio (SNR). It is known that without feedback the capacity of this channel converges to S as N become large [1]. In the other extreme, with the water-filling power allocation, which is optimal given unlimited feedback, the achievable rate grows as $S \log N$ as N increases. The limited feedback scheme studied in this paper can provide order optimal achievable rate (i.e., $S \log N$) where we assume one sub-channel per group. In this case, the threshold is optimized without regard to the feedback rate, and the amount of feedback grows as $S(\log N)^{2+\epsilon_2}$, where $\epsilon_2 \in (0,1)$ can be characterized as a function of N.

In addition, our results state that for large enough N,

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when B is less than $\frac{S}{u^\star}(\log N)^{2-\epsilon_1}$, where $u^\star$ is a constant (approximately 3.92), and $\epsilon_1 \in (0,2)$ can be characterized as a function of N, the achievable rate in bits/channel use is proportional to $\sqrt{SB}$. If B scales faster than $\frac{S}{u^\star}(\log N)^{2-\epsilon_1}$, then the optimal group size is one sub-channel per group, and the achievable rate scales slower than $\sqrt{SB}$.

We use the previous results to characterize the optimal feedback overhead with a block fading channel. Namely, time division duplexing (TDD) is considered in which the receiver relays the CSI back to the transmitter at the beginning of each coherence block having length T symbols. That is, the first F symbols are devoted to feedback, assuming a fixed rate R_f nats/channel use, and the remaining symbols are devoted to forward data transmission. Increasing the feedback overhead F increases the forward data rate, but decreases the time available for data transmission. The optimal F depends only on the product TR_f. Interestingly, we find that for large enough N, if TR_f is less than $\frac{3S}{u^\star}(\log N)^{2-\epsilon_1}$, then the optimal F is $T/3$, independent of the SNR. For larger amounts of feedback, the optimal overhead is smaller.

Numerical examples are presented, which show that the analytical results are quite accurate given at least a few hundred sub-channels. The numerical results also show that the achievable rate becomes more sensitive to the amount of feedback overhead as the SNR increases.

II. System Model and Feedback Constraint

Consider a multi-carrier channel with N independent and identically distributed Rayleigh fading sub-channels, so that the $N \times 1$ vector of channel outputs across sub-channels is given by

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{z} \tag{1}$$

where $\mathbf{H} = \operatorname{diag}[h_1, h_2, \ldots, h_N]$ is the channel matrix in which the diagonal entries are independent, circularly symmetric complex Gaussian (CSCG) random variables with mean zero and variance σ_h^2. The $N \times 1$ input vector $\mathbf{x}$ satisfies the average power constraint

$$E\left[\mathbf{x}^H \mathbf{x}\right] \le P, \tag{2}$$

and the $N \times 1$ noise vector $\mathbf{z}$ has CSCG entries with mean zero and variance σ_z^2. The time dependence is suppressed to simplify notation. The channel $\mathbf{H}$ is assumed to be known perfectly at the receiver. We assume a block fading model so that $\mathbf{H}$ remains constant for T channel uses and then changes to a new independent value.

Now suppose that an average feedback rate of B nats per coherence block is available to communicate CSI to the transmitter at the beginning of each coherence block. The feedback is assumed to be error-free. We therefore need a power allocation strategy that can be easily adapted to match the amount of feedback. For this we divide the total set of sub-carriers into G nonoverlapping groups each containing $N_G = N/G$ consecutive sub-channels. Let the $N_G \times 1$ vector of sub-channel gains corresponding to the g^{th} group be denoted as $\mathbf{h}_g = [h_{g1}, h_{g2}, \ldots, h_{gN_G}]^T$. Given a threshold t_o, the receiver informs the transmitter to use this group if $|h_{gi}|^2 \ge t_o$ for all $i = 1, 2, \ldots, N_G$. The probability of this event is $p = e^{-N_G t_o/\sigma_h^2}$, so that for large N the average amount of feedback required per coherence block for this CSI scheme can be compressed to the entropy rate $GH(p)$, where $H(p) = -p\log(p) - (1-p)\log(1-p)$.[1]. The feedback constraint is therefore

$$GH(p) \le B. \tag{3}$$

Clearly, the larger the coherence time, the less CSI is required per channel use to achieve a target rate.

Given the feedback, the transmitter allocates power uniformly over the set of active sub-channel groups. To maximize the achievable rate, we take the entries of the input vector $\mathbf{x}_g$ for sub-channel group g to be zero-mean CSCG random variables with variance P_o if $|h_{gi}|^2 \ge t_o$, $i = 1, \cdots, N_G$, and zero otherwise. To satisfy the input power constraint (2), we have $P_o = P/(Np)$.

III. Asymptotic Rate Versus Feedback

Assuming that the transmitter codes across coherence blocks in frequency and time, the achievable rate is given by the averaged mutual information (ergodic capacity),[2]

$$\begin{aligned} C(B) &= G\, E_{\mathbf{h}_g}\left[\mathbf{1}_{\{|h_{gi}|^2 \ge t_o, \forall i\}} \sum_{i=1}^{N_G} \log\left(1 + \frac{P_o}{\sigma_z^2}|h_{gi}|^2\right)\right] \\ &= N e^{-(N_G - 1)t_o/\sigma_h^2} \int_{t_o}^{\infty} \log\left(1 + \frac{P_o}{\sigma_z^2}\, t\right) \frac{e^{-t/\sigma_h^2}}{\sigma_h^2} dt \end{aligned} \tag{4}$$

where the indicator function $\mathbf{1}_{\{|h_{gi}|^2 \ge t_o, \forall i\}} = 1$ if $|h_{gi}|^2 \ge t_o$ for all $i = 1, \ldots, N_G$, and is zero otherwise. Note that the rate (4) does not depend upon the coherence block length T. We wish to choose the feedback parameters N_G and t_o to maximize $C(B)$ subject to the feedback constraint (3). Although it appears to be difficult to obtain an analytical characterization of the solution for arbitrary N, the following theorem characterizes the solution for large N and B.

We use the following notation. Suppose

$$\lim_{N\to\infty} \frac{f_1(N)}{f_2(N)} = c \tag{5}$$

for functions $f_1(\cdot)$ and $f_2(\cdot)$. Then if $c = 0$, then we write $f_1 = o(f_2)$, and if $c > 0$ is a constant, then we write $f_1 = O(f_2)$. Also, we define $u^\star$ as the positive solution to

$$\log(1+u) = 2\,u/(1+u) \tag{6}$$

[1]A practical variable-length prefix code typically requires an additional bit per coherence block. We ignore this, since a coherence block is likely to contain several hundred channel uses, so that this extra bit contributes negligible feedback overhead.

[2]A somewhat more conservative rate is obtained by selecting the code rate assuming that all active sub-channel gains $|h_{gi}|^2 = t_0$ [7]. This does not change the following asymptotic results.

i.e., $u^\star \approx 3.92$. Further, define ϵ_1, ϵ_2 as the solutions to

$$\log N - \log\left[\frac{S}{u^\star}(\log N)^{1-\frac{\epsilon_1}{2}}\right] = (\log N)^{1-\frac{\epsilon_1}{2}} \tag{7}$$

and

$$\log N - \log[S(\log N)^{1+\epsilon_2}] = (\log N)^{(1+\epsilon_2)/2} \tag{8}$$

respectively, where $S = P\sigma_h^2/\sigma_z^2$ is the SNR. Note that for large N, $\epsilon_1 \in (0,2)$ and $\epsilon_2 \in (0,1)$. Moreover, as $N \to \infty$, we have $\epsilon_1 \to 0$ and $\epsilon_2 \to 1$.

Proposition 1: Let $u^\star$ be the positive solution to (6). For fixed signal to noise ratio S, as $N \to \infty$, if $B \to \infty$ with N and

$$B \le \frac{S}{u^\star}(\log N)^{2-\epsilon_1}, \tag{9}$$

then the optimal group size, threshold, and corresponding capacity satisfy

$$N_G^\star = \sqrt{\frac{S}{u^\star B}}\log\frac{N}{\sqrt{S\,B/u^\star}} + o(1) \tag{10}$$

$$t_o^\star = \sigma_h^2\sqrt{\frac{u^\star B}{S}} + o(1) \tag{11}$$

$$C(B) = \sqrt{\frac{SB}{u^\star}}\log(1+u^\star) + o(1). \tag{12}$$

If

$$\frac{S}{u^\star}(\log N)^{2-\epsilon_1} \le B \le S(\log N)^{2+\epsilon_2}, \tag{13}$$

then the optimal parameters and capacity satisfy

$$N_G^\star = 1 \tag{14}$$

$$t_o^\star = \sigma_h^2\log\frac{N\log N}{B} + o(1) \tag{15}$$

$$C(B) = \frac{B}{\log N}\log\left(1 + \frac{S\log N}{B}\log\frac{N\log N}{B}\right) + O(1) \tag{16}$$

and finally if

$$S(\log N)^{2+\epsilon_2} \le B, \tag{17}$$

then the corresponding optimal parameters and capacity satisfy,

$$N_G^\star = 1 \tag{18}$$

$$t_o^\star = \sigma_h^2\left[\log N - (1+\epsilon_2)\log\log N - \log S\right] + o(1) \tag{19}$$

$$C(B) = S\left[\log N - (1+\epsilon_2)\log\log N\right] + O(1). \tag{20}$$

The proof is omitted due to the space limit. Proposition 1 can be interpreted as follows. In the range of small to moderate feedback B (specifically, $S/u^\star \ll B \le (S/u^\star)(\log N)^{2-\epsilon_1}$) the group size $N_G > 1$. In this feedback range the capacity is proportional to $\sqrt{B}$.

As the feedback increases, the group size decreases and the threshold increases, so that the CSI is represented with finer granularity. Once B exceeds $(S/u^\star)(\log N)^{2-\epsilon_1}$, each group reduces to a single sub-channel. Interestingly, the threshold then begins to *decrease* with B at the rate given in (15). As the threshold decreases, the fraction of active sub-channels increases so that the feedback needed to specify the number of active sub-channel groups matches the feedback rate. In this range of feedback the capacity grows more slowly than $O(\sqrt{B})$. Note that decreasing the threshold below the value in (19) increases the required feedback rate beyond the upper inequality in (13), and causes the achievable rate to decrease.

For the feedback scheme considered, the maximum amount of CSI is conveyed by setting $N_G = 1$ and optimizing the threshold as a function of N (ignoring the feedback rate constraint). This corresponds to the optimal "on-off" power allocation, in which the power is uniformly spread over active channels [3], [4]. The optimal threshold is given by (19), and the corresponding feedback is given by $B_{max} = S(\log N)^{2+\epsilon_2}$. That is, the achievable rate does not increase when B is increased beyond B_{max}.[3] The corresponding maximum achievable rate is given by (20), which has the optimal order growth of $O(S\log N)$ corresponding to the water-filling power allocation [4]. However, the negative second-order (log log) term in (20) can be substantial, as shown by the subsequent numerical examples.

From (8), as $N \to \infty$, $\epsilon_2 \to 1$, so that (17) and (20) state that $O(\log^3 N)$ feedback can achieve the optimal $O(\log N)$ growth in achievable rate. This result has been previously presented in [3], which considers the same threshold-based feedback scheme considered here, but with $N_G = 1$. Of course, allowing $N_G > 1$ gives more flexibility when the feedback rate is small. Also, subsequent numerical examples show that for reasonable values of N, the amount of feedback needed to achieve the $O(S\log N)$ forward rate may be closer to $S\log^2 N$ than to $S\log^3 N$.

Although the previous theorem does not cover the case of finite B, it is easy to show that as the amount of feedback $B \to 0$, we have $N_G \to N$, $t_o \to 0$ and the achievable rate $C \to S$. This limit is the ergodic capacity of a Rayleigh fading channel without feedback when the bandwidth becomes large, i.e., $N \to \infty$.

A. Numerical Examples

Next we present some numerical examples, which illustrate the preceding asymptotic results. Fig. 1 shows the optimized group size, threshold, and corresponding capacity as a function of feedback B. These results are obtained by optimizing the original capacity expression (4) subject to (3). (Of course, in practice N_G can assume only positive integer values, as opposed to the real values obtained from the optimization, which are shown in the figure.) Unless specified otherwise, for these and subsequent numerical results we choose $N = 1000$, $\sigma_h^2 = 1$ and $\sigma_z^2 = 1$.

Fig. 1 also shows the asymptotic results obtained through analysis. (The values plotted here are refined versions of the expressions presented in Theorem 1). The plot shows that the asymptotic values are close to the values obtained from numerical optimization. As predicted by Theorem 1, the plot

[3] The additional bits could be used to increase the number of quantization levels for the power on active sub-channels. The corresponding increase in rate, relative to the one-bit quantization assumed here, is typically quite small [4].

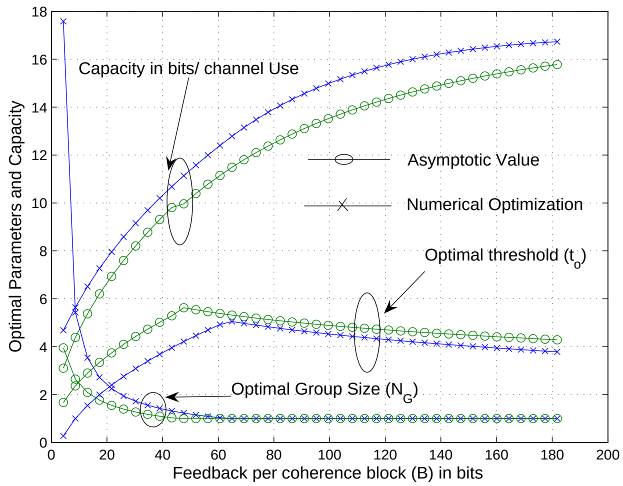

Fig. 1. Comparison of numerically optimized values and asymptotic values versus feedback per coherence block for $N = 1000$ and SNR $S = 5dB$.

shows that as B increases from zero, the group size decreases, and the threshold increases. However, once the group size crosses one (when B is about 60 bits/coherence block), the threshold decreases with B and the capacity increases relatively slowly. Finally, for large amounts of feedback (say, greater than 200 bits/coherence block for this example) the capacity and threshold saturate, and increasing the feedback further does not improve performance. Referring to (7)-(8), these values correspond to $\epsilon_1 \approx \epsilon_2 \approx 0.25$.

IV. Optimal Feedback Overhead

For large N, Theorem 1 states that on the order of $(\log N)^{2+\epsilon_2}$ bits per codeword are needed to obtain an achievable rate, which is on the order of $\log N$ bits per channel use. Given a fixed coherence time of T channel uses, as N increases, this implies that the feedback overhead increases faster than the forward rate, so that the $\log N$ rate is unsustainable. Here we account for the feedback overhead directly by assuming that both the feedback and forward data transmission occur within the coherence time T. We then select the amount of feedback overhead to maximize the forward rate. This problem has been previously studied in [7] for the downlink (multiuser) model in which the number of users scales with the number of channels. Here we show that the optimal feedback overhead and corresponding capacity behave differently when only a single user is present.

Given a coherence block of T channel uses, we assume that the first F channel uses relay CSI from the receiver to the transmitter at the rate of R_f nats per channel use. Accounting for the feedback overhead, the rate is given by

$$R = \left(1 - \frac{F}{T}\right) C(FR_f) \tag{21}$$

where $C(\cdot)$ is given by (4) with $B = FR_f$. Here we assume that the amount of feedback FR_f is large enough so that the entropy encoded feedback codes are approximately of fixed length. Alternatively, we can think of F/T as the average feedback overhead since we are coding across multiple blocks. Given T and R_f, we wish to find the F that maximizes R.

Proposition 2: As $N \to \infty$, if $TR_f \to \infty$ with N, and

$$R_f \le S(\log N)^{2+\epsilon_2} \tag{22}$$

where ϵ_2 is computed from (8), then the F that maximizes R is given by

$$F^\star = \begin{cases} \frac{T}{3} & \text{if } TR_f \le \frac{3S}{u^\star}(\log N)^{2-\epsilon_1} \\ \frac{w^\star \log N}{R_f} & \text{if } \frac{3S}{u^\star}(\log N)^{2-\epsilon_1} \le TR_f \end{cases} \tag{23}$$

where $w^\star$ is the solution to

$$\left(\frac{TR_f}{\log N} - 2w\right) \log\left[1 + \frac{S}{w}\log\frac{N}{w}\right]\left[w + S\log\frac{N}{w}\right] = S\left(\frac{TR_f}{\log N} - w\right)\left[1 + \log\frac{N}{w}\right] \tag{24}$$

and ϵ_1 is given by (7). Moreover, if $S(\log N)^{2+\epsilon_2} \le R_f$, then $F^\star = 1$.

The proof is again omitted due to the space limit. The proposition states that the optimal $F^\star$ depends on the product TR_f. If TR_f is smaller than $\frac{3S}{u^\star}(\log N)^{2-\epsilon_1}$, then for large N the optimal feedback duration is approximately $T/3$. This overhead is relatively large, and is due to the fact that the forward rate increases relatively quickly as a function of feedback in this range (i.e., as $\sqrt{B}$). From (12) the achievable rate in this regime is given by $C = \frac{2}{3}\sqrt{\frac{STR_f}{3u^\star}}\log(1 + u^\star)$, i.e., the rate increases as $\sqrt{TR_f}$.

If TR_f is larger than $\frac{3S}{u^\star}(\log N)^{2-\epsilon_1}$, then $F^\star < T/3$, and decreases with TR_f. For fixed R_f and $\log^3 N \le T$, computing $w^\star$ from (24) and substituting it into (23) gives $F^\star = \frac{S(\log N)^{2+\epsilon_2}}{R_f}$ for large N. That is, for large enough coherence times, the number of feedback symbols is large enough to feed back the maximum amount of CSI required by the power loading scheme. Similarly, for fixed T the feedback scheme does not benefit from increasing the rate R_f beyond $S(\log N)^{2+\epsilon_2}$, in which case $F^\star = 1$.

Fig. 2 shows the optimized feedback overhead F/T versus feedback rate R_f with fixed coherence time $T = 100$. These results are obtained by optimizing the expression (21). Also shown are the asymptotic values given by Proposition 2. The figure shows that for the given SNR values, the analysis accurately predicts the results of the numerical optimization over a wide range of feedback values. (The results become less accurate for very small B as the SNR increases.) For example, with SNR $S = 5$ dB (23) implies that $F^\star \approx T/3$ if $R_f \le 1.6$ bits per channel use, which closely matches the numerical results shown in the figure.

Fig. 3 shows achievable rate R versus normalized feedback overhead with different SNRs. This plot shows that the achievable rate is insensitive to the choice of F at low SNRs, and that the sensitivity increases with the SNR.

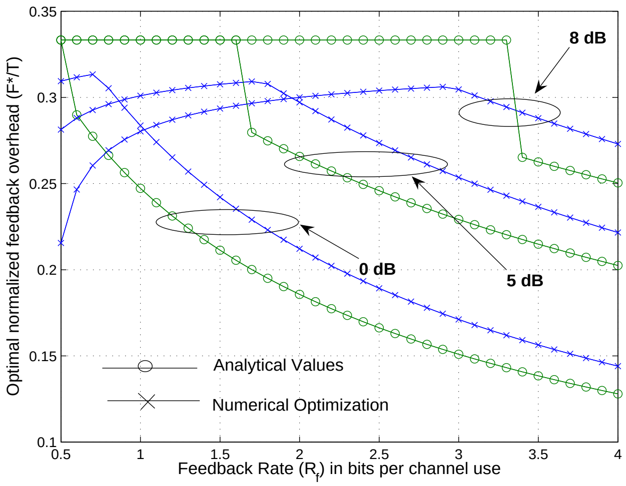

Fig. 2. Optimal fractional feedback overhead versus R_f for fixed $T = 100$ and $N = 1000$, at different SNRs.

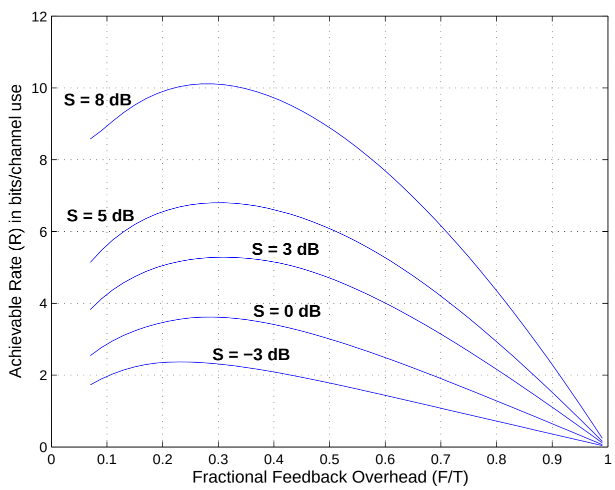

Fig. 3. Achievable rate versus fractional feedback overhead for fixed $T = 100$, $N = 1000$ and $R_f = 1$ bit/channel use.

V. Conclusions

We have analyzed the performance of a limited feedback scheme for multi-carrier transmission, which is based on grouping the sub-channels. With this scheme the forward rate is an increasing function of the amount of feedback, which is determined by the sub-channel group size and activation threshold. For large N this scheme achieves optimal order growth for the capacity (attained by the waterfilling power allocation) with $S(\log N)^{2+\epsilon_2}$ feedback bits per coherence block, where $\epsilon_2 \in (0,1)$ depends on N. We have also shown that for this scheme, when the feedback rate is relatively small, the optimal feedback overhead is one third of the coherence time (again for large N).

The model and results presented here can be extended in several directions. For example, we have assumed that a sub-channel group is activated only when all sub-channel gains exceed the threshold. In practice, different metrics may be used to activate a sub-channel group (e.g., the average of the sub-channel gains [8]). Whether or not the scaling results given here apply with those other metrics is an open question. We have also assumed perfect channel estimation at the receiver. Typically the channel is estimated by means of a training sequence, which introduces additional overhead. A model, which accounts for both training and feedback overhead in the context of beamforming, has been studied in [10]. That approach may also be appropriate for the multi-carrier scenario considered here. Finally, it may be possible to relay the feedback over different frequencies (frequency-division duplex), as opposed to the time division duplex scheme assumed here. Comparative advantages and disadvantages remain to be studied.

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