

Channel State and Receiver State Feedback for Frequency-Selective Block Fading Channels

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Abstract—In a fading communication channel, it is often beneficial to feedback *Channel State Information (CSI)* and *Receiver State Information (RSI)* to the transmitter. The CSI generally refers to information about the channel condition available at the receiver, while the RSI in this work is defined as information about the receiver's estimate of the message. The RSI can be used to improve reliability, e.g., through retransmission. This paper considers multi-carrier transmission through a doubly-selective Rayleigh fading channel, and studies the trade-off between the feedback of CSI and RSI under total feedback constraint. In particular, the CSI feedback specifies which groups of sub-channels to activate with equal power, and the RSI feedback determines retransmissions of a codeword. The problem is how to allocate feedback bits between CSI and RSI in order to maximize the error probability exponent. It is found that the optimal trade-off exhibits phase transitions which depends critically on the coherence time and the total amount of feedback. Specifically, the first feedback bits should be CSI up to a critical amount. Additional feedback bits, if available, should be allocated to RSI first, and then to both CSI and RSI. For the model considered, as the amount of feedback exceeds a certain threshold, additional RSI feedback is not beneficial unless CSI feedback increases accordingly.

I. INTRODUCTION

A feedback link from the receiver to the transmitter can be utilized in several different ways. For example, the receiver can inform the transmitter about its estimate of the message, based on which the transmitter may choose to retransmit part or all of the original codeword. These schemes include automatic repeat request (ARQ) protocols [1]. The main benefit is improvement in data reliability, or equivalently, a reduction in average decoding delay for a target probability of error. Several generalizations, including full feedback of received symbols [2], [3] and limited feedback schemes [4], [5] have been studied. We refer to this type of feedback as *Receiver State Information (RSI)* feedback.

Alternatively, in a fading channel with unknown state at the transmitter, feedback can convey *Channel State Information (CSI)* acquired at the receiver. Such feedback can in general increase the capacity of the channel through power and rate adaptation [6]. Limited CSI feedback schemes have been recently studied in [7]–[9] for multi-carrier transmission.

Given a constraint on the total feedback rate, there is in general a trade-off between the two types of feedback. In this paper we formulate a model and study the relative benefits of independent CSI and RSI feedback. Specifically, given fixed forward and feedback rates, our problem is to find the optimal split between CSI and RSI in order to minimize the probability of decoding error. To ease the analysis, we maximize the error exponent instead of minimizing the error probability directly. Note that CSI improves the error exponent by increasing the achievable rate while RSI improves the error exponent by reducing error events [2], [4].

This work assumes a doubly-selective block Rayleigh fading channel, which consists of a bank of independent block fading sub-channels with the same bandwidth and coherence time. The channel realizations are independent across all time-frequency coherence blocks.

The CSI feedback scheme is described as follows. To accommodate a continuous range of feedback rates, which can be very small, the set of sub-channels is partitioned into equal-size *groups*. The CSI feedback occurs at the beginning of each coherence block, and indicates which groups to activate [7], [10]. For simplicity, a group of sub-channels is activated if all their gains exceed a threshold. The transmitter uniformly spreads its power over all active sub-channels. The feedback rate is therefore controlled by the group size and threshold.

The RSI feedback scheme follows [4]. A codeword spans multiple coherence blocks and feedback of limited RSI occurs at the end of each codeword transmission. Although RSI feedback provides some additional information about the channel state during the transmission of each codeword, the CSI in our model pertains to the channel state corresponding to the *next* coherence block. In other words, the RSI is about the *past* and the CSI is about the independent *future*. Conditioned on the CSI, the block fading channel is memoryless.

Given the constraint of total CSI and RSI feedback as well as a target transmission rate (R nats per channel use), we study the optimal allocation of feedback bits to CSI and RSI for maximizing the error exponent. For simplicity, we consider the regime where the number of sub-channels becomes large, and the transmission rate and the feedback rate scale accordingly. The key findings are as follows: The first few feedback bits, in the amount of a critical rate R^{crit} , should be allocated to CSI. Additional feedback bits, if available,

should first be allocated to RSI only, and then allocated to both CSI and RSI. This is because as the amount of feedback increases, allocating additional feedback bits to RSI may not be beneficial unless a certain proportion is allocated to CSI. The results also show that the relative benefits of CSI and RSI depend critically on the coherence time (T channel uses). In particular, the critical rate R^{crit} depends on the coherence time. Given N sub-channels and signal-to-noise ratio (SNR) S , R^{crit} grows as $ST/4$ for $T < \log N$ with large enough N . As T increases beyond $\log N$, R^{crit} grows more slowly and eventually decreases.

II. MULTI-CARRIER MODEL WITH FEEDBACK

Consider multi-carrier transmission over N independent Rayleigh fading sub-channels with output

$$y_n = h_n x_n + z_n, \quad n = 1, \dots, N \quad (1)$$

where x_n denotes the input to the n -th sub-channel, and the channel coefficients h_1, \dots, h_N are independent, circularly symmetric complex Gaussian (CSCG) random variables with mean zero and variance σ_h^2 . The $N \times 1$ input vector $\mathbf{x} = [x_1, \dots, x_N]^T$ satisfies the average power constraint $\sum_{n=1}^N E[x_n^2] \leq P$ and the noise elements z_1, \dots, z_N are independent CSCG with mean zero and variance σ_z^2 . The channel coefficients are assumed to be known perfectly at the receiver. We assume a block fading model so that the coefficients remains constant for T channel uses and then changes to an independent value. The time dependence is suppressed in (1) to simplify notation.

A. CSI Feedback

Let R_{csi} denote the CSI feedback rate. It is assumed that TR_{csi} nats of feedback arrives at the transmitter at the beginning of each coherence block.¹ The CSI feedback scheme presented in [7], [10] is assumed, in which the total set of sub-carriers is partitioned into G nonoverlapping groups each containing $N_G = N/G$ consecutive sub-channels. Let the $N_G \times 1$ vector of sub-channel gains corresponding to the g^{th} group be $\mathbf{h}_g = [h_{g1}, h_{g2}, \dots, h_{gN_G}]^T$. Given a threshold t_o , the receiver informs the transmitter to use this group if $|h_{gi}|^2 \geq t_o \sigma_h^2$ for all $i = 1, 2, \dots, N_G$. The probability of this event is $p = e^{-N_G t_o}$, so that for large N the average amount of feedback required per coherence block can be compressed to the entropy rate.² The CSI feedback constraint is therefore $GH(p) \leq TR_{\text{csi}}$, where $H(p) = -p \log(p) - (1-p) \log(1-p)$. Clearly, the larger the coherence time, the less CSI is required per channel use to achieve a target rate.

Given the CSI feedback, the transmitter allocates power P_o uniformly over the set of active sub-channel groups. Subject

¹This is an abstraction for the scenario in which the channel is estimated by the receiver at the beginning of the coherence block. Here we ignore the overhead due to training and feedback.

²A practical variable-length prefix code typically requires an additional bit per coherence block. We ignore this, since a coherence block is likely to contain several hundred channel uses, so that this extra bit contributes negligible feedback overhead.

to this constraint, the maximum achievable rate (ergodic capacity) is given by

$$C(R_{\text{csi}}) = GE_{\mathbf{h}_g} \left[\mathbf{1}_{\left\{ \left| \frac{h_{gi}}{\sigma_h} \right|^2 \geq t_o, \forall i \right\}} \sum_{i=1}^{N_G} \log \frac{\sigma_z^2 + P_o |h_{gi}|^2}{\sigma_z^2} \right] \quad (2)$$

$$= \frac{P}{P_o} \int_{t_o}^{\infty} e^{t_o - t} \log \frac{\sigma_z^2 + P_o t \sigma_h^2}{\sigma_z^2} dt \quad (3)$$

where $P_o = P/(N e^{-N_G t_o})$. Note that this rate does not depend upon the coherence block length T , since the transmitter is assumed to code across many coherence blocks in time.³

We wish to choose the feedback parameters N_G and t_o to maximize $C(R_{\text{csi}})$ subject to the feedback constraint $GH(p) \leq TR_{\text{csi}}$. Although it appears to be difficult to obtain an analytical characterization of the solution for arbitrary N , the following proposition, repeated from [10], characterizes the solution for large N and TR_{csi} .

Let u be the positive number satisfying $\log(1+u) = 2u/(1+u)$, i.e., $u \approx 3.92$. Let ϵ_1, ϵ_2 satisfy

$$\log N - \log \left[\frac{S}{u} (\log N)^{1 - \frac{\epsilon_1}{2}} \right] = (\log N)^{1 - \frac{\epsilon_1}{2}} \quad (4)$$

$$\log N - \log [S (\log N)^{1 + \epsilon_2}] = (\log N)^{(1 + \epsilon_2)/2}, \quad (5)$$

respectively, where $S = P\sigma_h^2/\sigma_z^2$ is the SNR. Note that for large N , $\epsilon_1 \in (0, 2)$ and $\epsilon_2 \in (0, 1)$. Moreover, as $N \rightarrow \infty$, we have $\epsilon_1 \rightarrow 0$ and $\epsilon_2 \rightarrow 1$.

Proposition 1: [10] As $N \rightarrow \infty$, if $TR_{\text{csi}} \rightarrow \infty$ with N and

$$TR_{\text{csi}} \leq \frac{S}{u} (\log N)^{2 - \epsilon_1}, \quad (6)$$

then the capacity, optimized over both N_G and t_o , satisfies

$$C(R_{\text{csi}}) = \sqrt{\frac{S}{u}} \log(1+u) \sqrt{TR_{\text{csi}}} + o(1) \quad (7)$$

where $o(1)$ stands for a vanishing term as $N \rightarrow \infty$. If instead the feedback rate is greater than (6) but satisfies

$$\frac{S}{u} (\log N)^{2 - \epsilon_1} \leq TR_{\text{csi}} \leq S (\log N)^{2 + \epsilon_2}, \quad (8)$$

then the optimized capacity satisfies

$$C(R_{\text{csi}}) = \frac{TR_{\text{csi}}}{\log N} \log \left(1 + \frac{S \log N}{TR_{\text{csi}}} \log \frac{N \log N}{TR_{\text{csi}}} \right) + O(1). \quad (9)$$

Finally, if

$$S (\log N)^{2 + \epsilon_2} \leq TR_{\text{csi}}, \quad (10)$$

then the optimized capacity satisfies

$$C(R_{\text{csi}}) = S \log N + o(\log N). \quad (11)$$

A characterization of the optimized threshold t_o^* and the optimized sub-channel group size N_G^* , along with a discussion of those results, is given in [10]. In particular, assuming a large fixed N , N_G^* decreases with TR_{csi} . When $TR_{\text{csi}} > (S/u)(\log N)^{2 - \epsilon_1}$, $N_G^* = 1$ and t_o^* increases as $\log N$.

³Since the transmitter codes across sub-channels as well as coherence blocks in time, for large N the ergodic capacity expression (3) and subsequent results are valid even when each codeword spans only a few coherence blocks in time.

B. RSI Feedback

In addition to CSI feedback, we assume RSI feedback at an average rate of R_{rsi} nats per channel use. The RSI feedback bits are assumed to be transmitted after each codeword. We adopt the approach presented in [4], [5], in which the RSI feedback determines whether or not the transmitter retransmits the codeword. This also requires a synchronization mechanism between the transmitter and receiver via the noisy forward channel.

Here we focus on the scheme given in [4] for which $0 \leq R_{\text{rsi}} \leq R$, i.e., increasing the feedback rate beyond the forward rate does not improve performance. Although the results in [4] assume a discrete memoryless channel, they can be extended to block fading channels by treating each block as one vector channel use. Moreover, the results in [4] can be extended to a wide class of continuous alphabet channels.

It is shown in [4] that, with RSI feedback alone, the probability of decoding error can be upper bounded by

$$P_e \leq P_e^{\text{ub}} = e^{-n E_1(R_{\text{csi}}, R_{\text{rsi}}, \lambda)} + e^{-n E_2(R_{\text{csi}}, \lambda)} \quad (12)$$

where n is the *average* decoding delay measured in number of channel uses, and the fraction $1 - \lambda$ of channel uses is devoted to synchronization where $\lambda \in (0, 1)$. The first exponent E_1 is determined by the probability that the receiver makes a decoding error, which the transmitter is unable to detect due to the limited RSI. The second exponent E_2 is determined by the probability of the event that the transmitter is aware that the receiver's estimate is incorrect, but is unable to synchronize within the time constraint. Note that R_{rsi} only affects the first exponent. In particular, from [4]

$$E_1 = \lambda E_g\left(\frac{R - R_{\text{rsi}}}{\lambda}, R_{\text{csi}}\right) + \lambda \left| C(R_{\text{csi}}) - \frac{R - R_{\text{rsi}}}{\lambda} \right|^+ \quad (13)$$

and

$$E_2 = \lambda E_g\left(\frac{R}{\lambda}, R_{\text{csi}}\right) + \lambda \left| C(R_{\text{csi}}) - \frac{R}{\lambda} \right|^+ + (1 - \lambda) C_1(R_{\text{csi}}) \quad (14)$$

where $E_g(R, R_{\text{csi}})$ is the random coding error exponent⁴ [11] and $C_1(R_{\text{csi}})$ is the distance between the two most distinguishable channel inputs.

Let $f(\bar{\mathbf{y}}, \mathbf{s}|\bar{\mathbf{x}})$ denote the joint probability density function of the length T channel output vector $\bar{\mathbf{y}}$ and the channel state \mathbf{s} , given the length T input vector $\bar{\mathbf{x}}$ in one coherence block. Let $f(\bar{\mathbf{x}}|\hat{\mathbf{s}})$ denote the probability density function of the input given the estimate of the channel state $\hat{\mathbf{s}}$. Then we have

$$E_g(R, R_{\text{csi}}) = \max_{0 \leq \rho \leq 1} \left\{ \frac{1}{T} E_o(\rho) - \rho R \right\} \quad (15)$$

where

$$E_o(\rho) = -\log \int \left(\int_{\bar{\mathbf{x}}} (f(\bar{\mathbf{y}}, \mathbf{s}|\bar{\mathbf{x}}))^{1/(1+\rho)} f(\bar{\mathbf{x}}|\hat{\mathbf{s}}) d\bar{\mathbf{x}} \right)^{(1+\rho)} d\bar{\mathbf{y}} ds \quad (16)$$

⁴We use the random coding exponent E_g instead of the sphere packing exponent, as in [4], to simplify the analysis. Although this gives a slightly looser upper bound on P_e , it is unlikely to change our results.

and

$$C_1(R_{\text{csi}}) = \max_{\bar{\mathbf{x}}_1(\hat{\mathbf{s}}), \bar{\mathbf{x}}_2(\hat{\mathbf{s}})} \frac{1}{T} \int f(\bar{\mathbf{y}}, \mathbf{s}|\bar{\mathbf{x}}_1(\hat{\mathbf{s}})) \log \frac{f(\bar{\mathbf{y}}, \mathbf{s}|\bar{\mathbf{x}}_1(\hat{\mathbf{s}}))}{f(\bar{\mathbf{y}}, \mathbf{s}|\bar{\mathbf{x}}_2(\hat{\mathbf{s}}))} d\bar{\mathbf{y}} ds \quad (17)$$

where the maximization is over all codewords $\bar{\mathbf{x}}_1(\hat{\mathbf{s}})$ and $\bar{\mathbf{x}}_2(\hat{\mathbf{s}})$. Both E_g and C_1 are assumed to be continuous and increasing functions of R_{csi} .

For the block Rayleigh fading channel considered, we can evaluate (16) and (17) and apply Jensen's inequality to obtain the upper bound

$$E_o(\rho) \leq N_G \rho e^{-N_G t_o} \log \left(1 + \frac{P_o(t_o + 1)\sigma_h^2}{\sigma_z^2(1 + \rho)} \right) \quad (18)$$

Using (18), it is easy to show that the random coding exponent (15) satisfies

$$E_g(R, R_{\text{csi}}) \leq C(R_{\text{csi}}) - R. \quad (19)$$

Numerical results indicate that this upper bound is fairly tight. We will therefore assume that the bound is the exact value when solving for the optimal CSI and RSI trade-off. Also,

$$E_g(R - \Delta R, R_{\text{csi}}) - E_g(R, R_{\text{csi}}) \leq \Delta R \quad (20)$$

with equality if R is sufficiently below the capacity.

To evaluate C_1 in (17) we use two input sequences with the maximum distance subject to the input power constraint. We can then evaluate (17) as

$$C_1(R_{\text{csi}}) = \frac{4P(t_o + 1)\sigma_h^2}{\sigma_z^2}. \quad (21)$$

III. PROBLEM STATEMENT

Assuming that on average R_f nats per channel use are available for feedback, the problem is to determine the optimal split between CSI and RSI. The objective is the asymptotic decay rate, or exponent, of the probability of decoding error. More precisely, we wish to maximize

$$\lim_{n \rightarrow \infty} -\frac{1}{n} \log P_e^{\text{ub}} \quad (22)$$

over $(R_{\text{csi}}, R_{\text{rsi}}, \lambda)$ subject to

$$\lambda C(R_{\text{csi}}) \geq R \quad (23)$$

$$R_{\text{csi}} + R_{\text{rsi}} \leq R_f \quad (24)$$

where (23) ensures that the forward rate constraint is met with the RSI feedback scheme.

Let R^{min} denote the minimum CSI feedback rate such that $C(R_{\text{csi}}) \geq R$. We assume that $R_f \geq R^{\text{min}}$ so that the problem is feasible. Note that the objective (22) with finite n corresponds to minimizing the probability of decoding error assuming a decoding delay of n channel uses. An equivalent objective to (22) is to maximize the smaller one of the two exponents E_1 and E_2 given by (13) and (14) respectively.

Although our objective is to maximize (22), to simplify the analysis, we will assume that the CSI parameters N_G and t_o are chosen to maximize the achievable rate, as in Section

II. Although those parameters may change according to the different objectives, we do not expect that using the capacity objective will affect our asymptotic results.

IV. OPTIMAL ALLOCATION OF FEEDBACK

In this section we investigate the trade-off between CSI and RSI. To simplify the analysis we make the following approximations: The bounds (19) and (20) are the exact values and the ratio $C(R_{\text{csi}})/C_1(R_{\text{csi}}) \approx \phi$ where ϕ is a constant. When R_{csi} satisfies (6), using (7) and the optimal value of t_o , we approximate $\phi = (\log(1+u))/(4u) \approx 0.1$, and similarly for the range (8), we take $\phi = 1/4 = 0.25$. The numerical results in the next section show that the corresponding analysis provides accurate insight into the optimized CSI and RSI trade-off.

Recall that the objective is to maximize the minimum of E_1 and E_2 , which are increasing and decreasing functions of λ , respectively. If $E_2 > E_1$, then λ can be increased until $E_1 = E_2$. This is possible since $E_1 \geq E_2$ for $\lambda = 1$. On the other hand, if $E_2 < E_1$, then we wish to decrease λ until $E_1 = E_2$. Equating (13) and (14), assuming that the bound (20) is exact, we get the optimum time-sharing parameter

$$\lambda^* = 1 - \frac{2R_{\text{rsi}}}{C_1(R_{\text{csi}})}. \quad (25)$$

This assumes that sufficient CSI is available to support the forward rate, i.e.,

$$R \leq \lambda^* C(R_{\text{csi}}) = C(R_{\text{csi}}) - 2\phi R_{\text{rsi}}. \quad (26)$$

Since λ can be chosen to equate E_1 and E_2 , we define $E_1^*(R_{\text{csi}}, R_{\text{rsi}}) = E_1(R, R_{\text{csi}}, R_{\text{rsi}}, \lambda^*)$, and rewrite the optimization problem (22)–(24) as

$$\text{maximize } E_1^*(R_{\text{csi}}, R_{\text{rsi}}) \quad (27)$$

$$\text{subject to } R_{\text{csi}} + R_{\text{rsi}} \leq R_f \quad (28)$$

$$2\phi R_{\text{rsi}} \leq C(R_{\text{csi}}) - R \quad (29)$$

$$0 \leq R_{\text{rsi}} \leq R \quad (30)$$

where (29) is a rearrangement of (26) which guarantees the balance of the two error exponents E_1 and E_2 . Assuming the bounds (19) and (20) are tight, we have

$$E_1^*(R_{\text{csi}}, R_{\text{rsi}}) \approx 2[\lambda^* C(R_{\text{csi}}) - R + R_{\text{rsi}}] \quad (31)$$

$$= 2C(R_{\text{csi}}) + 2(1 - 2\phi)R_{\text{rsi}} - 2R. \quad (32)$$

Evidently, $\partial E_1^*/\partial R_{\text{rsi}} = 2(1 - 2\phi)$ is approximately constant, and $\partial E_1^*/\partial R_{\text{csi}} = 2C'(R_{\text{csi}})$. The derivative $C'(R_{\text{csi}})$ is very large for small R_{csi} and decreases monotonically to zero as R_{csi} becomes large. As R_f increases from zero, we conclude that the benefit from CSI dominates that of RSI until the two derivatives are equal, and then the benefit from RSI becomes larger until $R_{\text{rsi}} = R$ or that the condition (29) is met with equality. From then on, more CSI is needed along with RSI to achieve the optimal allocation.

Let R^{crit} be the solution to

$$C'(R_{\text{csi}}) = 1 - 2\phi \quad (33)$$

and $R_m = \max\{R^{\text{min}}, R^{\text{crit}}\}$. The optimal CSI-RSI split can now be specified as follows,

$$R_{\text{csi}}^* = \begin{cases} R_f & \text{if } R^{\text{min}} \leq R_f \leq R_m \\ R_m & \text{if } R_m < R_f \leq R_m + (C(R_m) - R)/(2\phi) \\ R_c & \text{otherwise} \end{cases} \quad (34)$$

where R_c is the solution to

$$R_f = R_c + (C(R_c) - R)/(2\phi). \quad (35)$$

Clearly, the trade-off exhibits three phases depending on the total amount of feedback R_f .

Proposition 1 allows us to determine R^{crit} . Namely, if $T \leq (\log N)^{1-\frac{\epsilon_1}{2}}$ for large N , then R^{crit} satisfies (6) and we can substitute the capacity expression (7) into (33). Taking $\phi = 0.1$ and also using the approximation $2(\log(1+u^*))/\sqrt{u^*} \approx 1.6$ gives

$$R^{\text{crit}} \approx \frac{ST}{4}. \quad (36)$$

If $(\log N)^{1-\frac{\epsilon_1}{2}} < T$, then R^{crit} satisfies (8). Substituting the capacity expression (9) into (33) and taking $\phi = 0.25$ gives the condition

$$\log\left(1 + \frac{S \log N}{TR_{\text{csi}}} a(N)\right) = S \frac{1 + a(N)}{\frac{TR_{\text{csi}}}{\log N} + Sa(N)} + \frac{\log N}{2T}. \quad (37)$$

where $a(N) = \log[(N \log N)/TR_{\text{csi}}]$. For larger values of T , R^{crit} can be shown to increase sublinearly with T (in contrast to (36)), and it is easy to show that $TR^{\text{crit}} \leq S(\log N)^{2+\epsilon_2}$.

V. NUMERICAL RESULTS

This section presents numerical solutions to the constrained maximization of (22), and thereby demonstrates that the analytical results in this paper provide accurate insights into the optimized CSI and RSI trade-off for finite-size systems. Exact expressions for E_g and $C(R_{\text{csi}})$ are used and N_G and t_o are chosen to maximize the exponent (22) as opposed to the capacity $C(R_{\text{csi}})$. Unless stated otherwise, the SNR is normalized to $S = 1$, the number of sub-channels is $N = 1000$ and the transmission rate is assumed to be $R = 1.6$ nats per channel use.

Figure 1 plots optimal amounts of CSI and RSI versus total feedback rate R_f for different coherence times. Due to the choice of R , here we have $R^{\text{crit}} > R^{\text{min}}$. Specifically, $R_m = R^{\text{crit}} \approx 0.7$ and 0.82 for $T = 6$ and 8 , respectively. As anticipated, the first R_m bits of feedback go to CSI, and zero is allocated to RSI. Thereafter, the CSI remains almost unchanged (decreases slightly) while the RSI increases. For $T = 6$, both CSI and RSI begin to increase simultaneously after the RSI reaches a certain value while for $T = 8$ the CSI remains constant (decreases slightly) as the RSI continues to increase.

Figure 2 plots R_m (the first few bits that go into CSI before feedback bits are assigned to RSI) for different values of T corresponding to $S = 0$ dB and 5 dB. For $S = 0$ dB and approximately $T \leq 4$, $R^{\text{crit}} < R^{\text{min}}$, so that $R_m = R^{\text{min}}$. Note

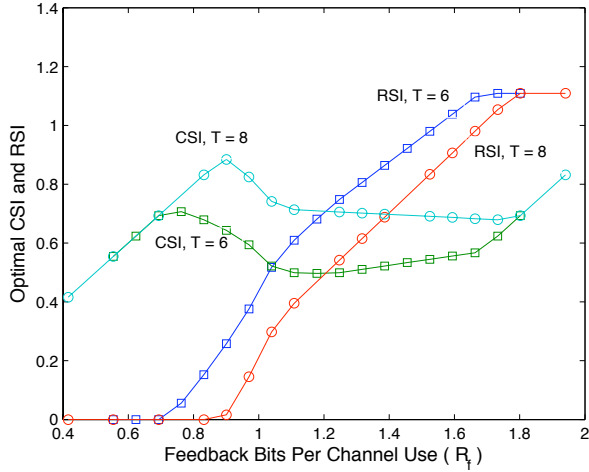


Fig. 1. Optimal CSI and RSI feedback versus total feedback rate.

that R_m^{\min} is a decreasing function of T and hence the trend in the plot. For $4 \leq T \leq 8$, R_m increases approximately linearly and eventually decreases with T as predicted by the analytical results. Similar trends are observed for $S = 5$ dB.

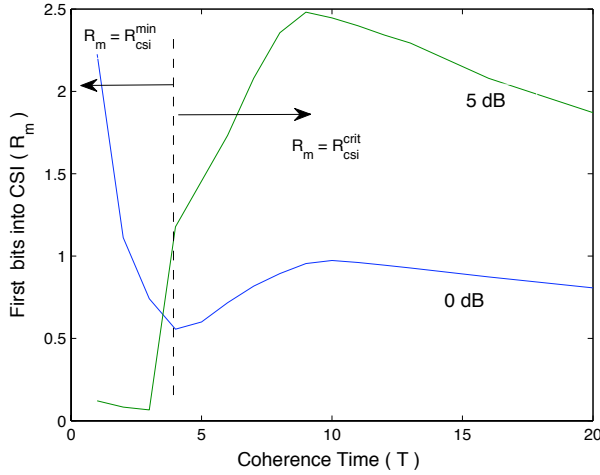


Fig. 2. R_m versus T .

Figure 3 compares the error exponent with the optimal allocation of feedback bits to the exponent achieved when all the feedback is allocated to CSI. The range of the horizontal axis is roughly the minimum to maximum values of CSI per coherence block. Clearly, allocating bits to RSI can provide substantial gains in the exponent for the smaller coherence times. For the larger coherence time it is optimal to allocate all bits to CSI.

VI. CONCLUSIONS

The trade-off between CSI and RSI has been studied in the context of block-fading multicarrier channels. The optimal

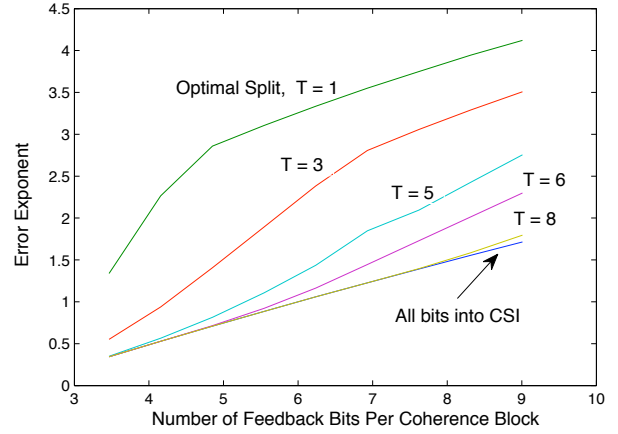


Fig. 3. Error Exponent versus feedback per coherence block (TR_f) for different values of T .

allocation of feedback bits between these two types of feedback depends critically on the coherence time. The first few feedback bits should be allocated to CSI, and the transition point (feedback rate), at which feedback switches to RSI depends on the coherence time. Of course, the results in this paper correspond to the particular channel model and feedback schemes assumed. It would be interesting to see whether the preceding trends are generally valid for other channel models and feedback schemes.

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