c. $10\delta(t) + A\, d\delta(t)/dt = B\delta(t) + 5\, d\delta(t)/dt$; find A and B.

d. $\int_2^{11} [e^{-4\pi t} + \tan(10\pi t)]\delta(3t+6)\, dt$.

e. $\int_{-\infty}^{\infty} [\cos(8\pi t) + e^{-2t}][d^2\delta(t-2)/dt^2]\, dt$.

2.7. Which of the following signals are periodic and which are aperiodic? Find the periods of those which are periodic. Sketch all signals.

a. $x_1(t) = \cos(5\pi t) + \sin(7\pi t)$.

b. $x_2(t) = \sum_{n=0}^{\infty} \Lambda(t-2n)$.

c. $x_3(t) = \sum_{n=-\infty}^{\infty} \Lambda(t-2n)$.

d. $x_4(t) = \sin(3t) + \cos(2\pi t)$.

e. $x_5(t) = \sum_{n=-\infty}^{\infty} \Pi(t-3n)$.

f. $x_6(t) = \sum_{n=0}^{\infty} \Pi(t-3n)$.

2.8. Write the signal $x(t) = \sin(6\pi t) + 2\cos(10\pi t)$ as

a. The real part of a sum of rotating phasors.

b. A sum of rotating phasors plus their complex conjugates.

c. From your results in parts (a) and (b), sketch the single-sided and double-sided amplitude and phase spectra of $x(t)$.

Section 2.2

2.9. Find the normalized power for each signal below that is a power signal and the normalized energy for each signal that is an energy signal. If a signal is neither a power signal nor an energy signal, so designate it. Sketch each signal (α is a positive constant).

a. $x_1(t) = 2\cos(4\pi t + 2\pi/3)$.

b. $x_2(t) = e^{-\alpha t}u(t)$.

c. $x_3(t) = e^{\alpha t}u(-t)$.

d. $x_4(t) = (\alpha^2 + t^2)^{-1/2}$.

e. $x_5(t) = e^{-\alpha|t|}$.

f. $x_6 = e^{-\alpha t}u(t) - e^{-\alpha(t-1)}u(t-1)$.

2.10. Classify each of the following signals as an energy signal or a power signal by calculating the energy E or the power P (A, θ, ω, and τ are positive constants).

a. $A|\sin(\omega t + \theta)|$.

b. $A\tau/\sqrt{\tau + jt}$, $j = \sqrt{-1}$.

c. $Ate^{-t/\tau}u(t)$.

d. $\Pi(t/\tau) + \Pi(t/2\tau)$.

2.11. Sketch each of the following periodic waveforms and compute their average powers.

a. $x_1(t) = \sum_{n=-\infty}^{\infty} \Pi[(t-6n)/3]$.

b. $x_2(t) = \sum_{n=-\infty}^{\infty} \Lambda[(t-5n)/2]$.

c. $x_3(t) = \sum_{n=-\infty}^{\infty} \Lambda[(t-3n)/2]u(t-3n)$.

d. $x_4(t) = 2\sin(5\pi t)\cos(5\pi t)$.

(*Hint:* use an appropriate trigonometric identy to simplify.

2.12. For each of the following signals, determine both the normalized energy and power. (Note: 0 and ∞ are possible answers.)

a. $x_1(t) = 6e^{(-3+j4\pi)t}u(t)$.

b. $x_2(t) = \Pi[(t-3)/2] + \Pi[(t-3)/6]$.

c. $x_3(t) = 7e^{j6\pi t}u(t)$.

d. $x_4(t) = 2\cos(4\pi t)$.

2.13. Show that the following are energy signals. Sketch each signal

a. $x_1(t) = \Pi(t/12)\cos(6\pi t)$

b. $x_2(t) = e^{-|t|/3}$

c. $x_3(t) = 2u(t) - 2u(t-8)$

d. $x_4(t) = \int_{-\infty}^{t} u(\lambda)\, d\lambda - 2\int_{-\infty}^{t-10} u(\lambda)\, d\lambda + \int_{-\infty}^{t-20} u(\lambda)\, d\lambda$

(*Hint:* Consider the integral of a step function.)

Section 2.3

2.14.

a. Fill in the steps for obtaining (2.33) from (2.32).

b. Obtain (2.34) from (2.33).

c. Given the set of orthogonal functions

$$\phi_n(t) = \Pi\left(\frac{4[t-(2n-1)T/8]}{T}\right), \quad n = 1, 2, 3, 4$$

sketch and dimension accurately these functions.

d. Approximate the ramp signal

$$x(t) = \frac{t}{T}\,\Pi\left(\frac{t-T/2}{T}\right)$$

by a generalized Fourier series using this set.

e. Do the same for the set

$$\phi_n(t) = \Pi\left\{\frac{2[t-(2n-1)T/4]}{T}\right\}, \quad n = 1, 2$$

f. Compute the integral-squared error for both part (b) and part (c). What do you conclude about the dependence of ϵ_N on N?

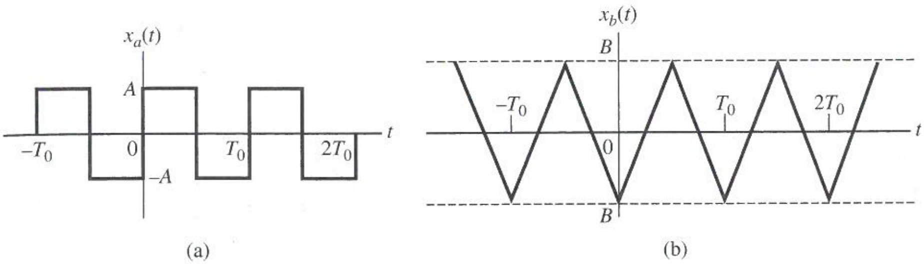

Figure 2.36

waveform shown in Figure 2.36(b) and those of $x_a(t)$ shown in Figure 2.35(a).
Hint: Note that $x_a(t) = K[dx_b(t)/dt]$, where K is an appropriate scale change.

c. Plot the double-sided amplitude and phase spectra for $x_b(t)$.

Section 2.5

2.24. Sketch each signal given below and find its Fourier transform. Plot the amplitude and phase spectra of each signal (A and τ are positive constants).

a. $x_1(t) = A\exp(-t/\tau)u(t)$.

b. $x_2(t) = A\exp(t/\tau)u(-t)$.

c. $x_3(t) = x_1(t) - x_2(t)$.

d. $x_4(t) = x_1(t) + x_2(t)$. Does it check with the answer found using Fourier transform tables?

2.25.

a. Use the Fourier transform of

$$x(t) = \exp(-\alpha t)u(t) - \exp(\alpha t)u(-t)$$

where $\alpha > 0$ to find the Fourier transform of the signum function defined as

$$\text{sgn}\, t = \begin{cases} 1, & t > 0 \\ -1, & t < 0 \end{cases}$$

(*Hint:* Take the limit as $\alpha \to 0$ of the Fourier transform found.)

b. Use the result above and the relation $u(t) = \frac{1}{2}[\text{sgn}\, t + 1]$ to find the Fourier transform of the unit step.

c. Use the integration theorem and the Fourier transform of the unit impulse function to find the Fourier transform of the unit step. Compare the result with part (b).

2.26. Using only the Fourier transform of the unit impulse function and the differentiation theorem, find the Fourier transforms of the signals shown in Figure 2.37.

2.27.

a. Write the signals of Figure 2.37 as the linear combination of two delayed triangular functions. That is, write $x_a(t) = a_1\Lambda((t-t_1)/T_1) + a_2\Lambda((t-t_2)/T_2)$ by finding appropriate values for a_1, a_2, t_1, t_2, T_1, and T_2. Do similar expressions for all four signals shown in Figure 2.37.

b. Given the Fourier transform pair $\Lambda(t) \leftrightarrow \text{sinc}^2 f$, find their Fourier transforms using the superposition, scale change, and time delay theorems. Compare your results with the answers obtained in Problem 2.26.

2.28.

a. Given $\Pi(t) \leftrightarrow \text{sinc} f$, find the Fourier transforms of the following signals using the frequency translation followed by the time delay theorem.

i. $x_1(t) = \Pi(t-1)\exp[j4\pi(t-1)]$.
ii. $x_2(t) = \Pi(t+1)\exp[j4\pi(t+1)]$.

b. Repeat the above, but now applying the time delay followed by the frequency translation theorem.

2.29. By applying appropriate theorems and using the signals defined in Problem 2.28, find Fourier transforms of the following signals:

a. $x_a(t) = \frac{1}{2}x_1(t) + \frac{1}{2}x_1(-t)$.
b. $x_b(t) = \frac{1}{2}x_2(t) + \frac{1}{2}x_2(-t)$.

2.30. Use the scale change and time delay theorems along with the transform pairs $\Pi(t) \leftrightarrow \text{sinc} f$, $\text{sinc}\, t \leftrightarrow \Pi(f)$, $\Lambda(t) \leftrightarrow \text{sinc}^2 f$, and $\text{sinc}^2 t \leftrightarrow \Lambda(f)$ to find Fourier transforms of the following:

a. $x_a(t) = \Pi[(t-1)/2]$.

b. $x_b(t) = 2\,\text{sinc}[2(t-1)]$.

c. $x_c(t) = \Lambda[(t-2)/8]$.

d. $x_d(t) = \text{sinc}^2[(t-3)/4]$.

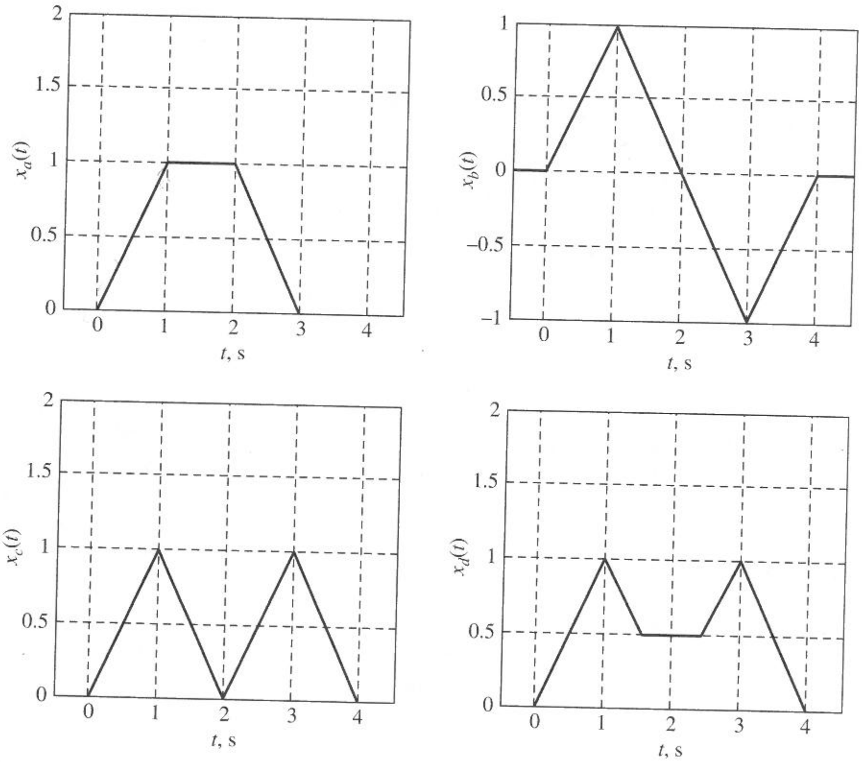

Figure 2.37

2.31. Without actually computing them, but using appropriate sketches, tell if the Fourier transforms of the signals given below are real, imaginary, or neither; even, odd, or neither. Give your reasoning in each case.

a. $x_1(t) = \Pi(t+1/2) - \Pi(t-1/2)$.

b. $x_2(t) = \Pi(t/2) + \Pi(t)$.

c. $x_3(t) = \sin(2\pi t)\Pi(t)$.

d. $x_4(t) = \sin(2\pi t + \pi/4)\Pi(t)$.

e. $x_5(t) = \cos(2\pi t)\Pi(t)$.

f. $x_6(t) = 1/[1 + (t/5)^4]$.

2.32. Using the sifting property of the delta function, find the Fourier transforms of the signals given below. Discuss how any symmetry properties a given signal may have affect its Fourier transform in terms of being real or purely imaginary.

a. $x_1(t) = \delta(t+4) + 3\delta(t) + \delta(t-4)$.

b. $x_2(t) = 2\delta(t+8) - 2\delta(t-8)$.

c. $x_3(t) = \sum_{n=0}^{4} (n^2+1)\delta(t-2n)$.

(*Hint:* Write out the terms for this signal.)

d. $x_4(t) = \sum_{n=-2}^{2} n^2\delta(t-2n)$

(*Hint:* Write out the terms for this signal.)

2.33. Find and plot the energy spectral densities of the following signals. Dimension your plots fully. Use appropriate Fourier transform pairs and theorems.

a. $x_1(t) = 2e^{-3|t|}$.

b. $x_2(t) = 20\,\mathrm{sinc}(30t)$.

c. $x_3(t) = 4\Pi(5t)$.

d. $x_4(t) = 4\Pi(5t)\cos(40\pi t)$.

2.34. Evaluate the following integrals using Rayleigh's energy theorem (Parseval's theorem for Fourier transforms).

a. $I_1 = \int_{-\infty}^{\infty} \frac{df}{\left[\alpha^2 + (2\pi f)^2\right]}$.

(*Hint:* Consider the Fourier transform of $\exp(-\alpha t)u(t)$).

b. $I_2 = \int_{-\infty}^{\infty} \mathrm{sinc}^2(\tau f)\,df$.

c. $I_3 = \int_{-\infty}^{\infty} \frac{df}{\left[\alpha^2 + (2\pi f)^2\right]^2}$.

d. $I_4 = \int_{-\infty}^{\infty} \mathrm{sinc}^4(\tau f)\,df$.

2.35. Obtain and sketch the convolutions of the following signals.