$$\frac{dy}{dt} + ay = b\frac{dx}{dt} + cx$$

a. Find $H(f)$.

b. Find and plot $|H(f)|$ and $\underline{/H(f)}$ for $c = 0$.

c. Find and plot $|H(f)|$ and $\underline{/H(f)}$ for $b = 0$.

2.47. For each of the following transfer functions, determine the unit impulse response of the system.

a. $H_1(f) = \frac{1}{(5+j2\pi f)}$

b. $H_2(f) = \frac{j2\pi f}{(5+j2\pi f)}$

(*Hint:* Use long division first.)

c. $H_3(f) = \frac{e^{-j6\pi f}}{(5+j2\pi f)}$.

d. $H_4(f) = \frac{1-e^{-j6\pi f}}{(5+j2\pi f)}$.

2.48. A filter has frequency-response function $H(f) = \Pi(f/2B)$ and input $x(t) = 2W\,\text{sinc}(2Wt)$.

a. Find the output $y(t)$ for $W < B$.

b. Find the output $y(t)$ for $W > B$.

c. In which case does the output suffer distortion? What influenced your answer?

2.49. A second-order active bandpass filter (BPF), known as a bandpass Sallen–Key circuit, is shown in Figure 2.38.

a. Show that the frequency-response function of this filter is given by

$$H(j\omega) = \frac{(K\omega_0/\sqrt{2})(j\omega)}{-\omega^2 + (\omega_0/Q)(j\omega) + \omega_0^2}, \quad \omega = 2\pi f$$

where

$$\omega_0 = \sqrt{2}(RC)^{-1}$$

$$Q = \frac{\sqrt{2}}{4-K}$$

$$K = 1 + \frac{R_a}{R_b}$$

b. Plot $|H(f)|$.

c. Show that the 3-dB bandwidth in hertz of the filter can be expressed as $B = f_0/Q$, where $f_0 = \omega_0/2\pi$.

d. Design a BPF using this circuit with center frequency $f_0 = 1000$ Hz and 3-dB bandwidth of 300 Hz. Find values of $R_a, R_b, R,$ and C that will give these desired specifications.

2.50. For the two circuits shown in Figure 2.39, determine $H(f)$ and $h(t)$. Sketch accurately the amplitude and phase responses. Plot the amplitude response in decibels. Use a logarithmic frequency axis.

2.51. Using the Paley–Wiener criterion, show that

$$|H(f)| = \exp(-\beta f^2)$$

is not a suitable amplitude response for a causal, linear time-invariant filter.

2.52. Determine whether the filters with impulse responses given below are BIBO stable.

a. $h_1(t) = \exp(-\alpha t)\cos(2\pi f_0 t)u(t)$.

b. $h_2(t) = \cos(2\pi f_0 t)u(t)$.

c. $h_3(t) = t^{-1}u(t-1)$.

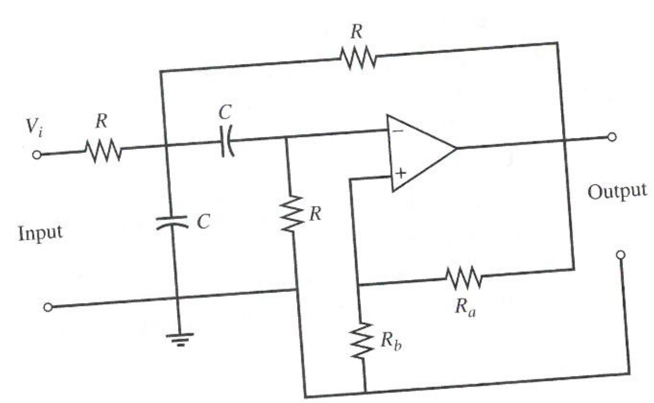

Figure 2.38

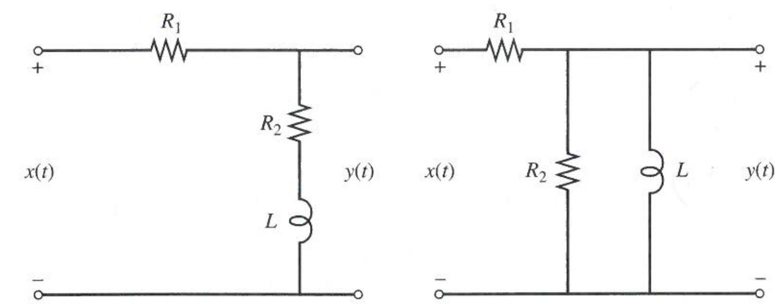

Figure 2.39

2.53. Given a filter with frequency-response function

$$H(f) = \frac{5}{4 + j(2\pi f)}$$

and input $x(t) = e^{-3t}u(t)$, obtain and plot accurately the energy spectral densities of the input and output.

2.54. A filter with frequency-response function

$$H(f) = 3\Pi\left(\frac{f}{26}\right)$$

has, as an input, a half-rectified cosine waveform of fundamental frequency 10 Hz. Determine the output of the filter.

2.55. Another definition of bandwidth for a signal is the 90% energy containment bandwidth. For a signal with energy spectral density $G(f) = |X(f)|^2$, it is given by B_{90} in the relation

$$0.9E_{\text{Total}} = \int_{-B_{90}}^{B_{90}} G(f)\,df = 2\int_0^{B_{90}} G(f)\,df$$

$$E_{\text{Total}} = \int_{-\infty}^{\infty} G(f)\,df = 2\int_0^{\infty} G(f)\,df$$

Obtain B_{90} for the following signals if it is defined. If it is not defined for a particular signal, state why it is not.

a. $x_1(t) = e^{-\alpha t}u(t)$, where α is a positive constant.

b. $x_2(t) = 2W\,\text{sinc}(2Wt)$.

c. $x_3(t) = \Pi(t/\tau)$ (requires numerical integration).

2.56. An ideal quadrature phase shifter has

$$H(f) = \begin{cases} e^{-j\pi/2}, & f > 0 \\ e^{+j\pi/2}, & f < 0 \end{cases}$$

Find the outputs for the following inputs:

a. $x_1(t) = \exp(j100\pi t)$.

b. $x_2(t) = \cos(100\pi t)$.

c. $x_3(t) = \sin(100\pi t)$.

d. $x_4(t) = \Pi(t/2)$.

2.57. A filter has amplitude response and phase shift shown in Figure 2.40. Find the output for each of the inputs given below. For which cases is the transmission distortionless? Tell what type of distortion is imposed for the others.

a. $x_1(t) = \cos(48\pi t) + 5\cos(126\pi t)$.

b. $x_2(t) = \cos(126\pi t) + 0.5\cos(170\pi t)$.

c. $x_3(t) = \cos(126\pi t) + 3\cos(144\pi t)$.

d. $x_4(t) = \cos(10\pi t) + 4\cos(50\pi t)$.

2.58. Determine and accurately plot, on the same set of axes, the group delay and the phase delay for the systems with unit impulse responses:

a. $h_1(t) = 3e^{-5t}u(t)$.

b. $h_2(t) = 5e^{-3t}u(t) - 2e^{-5t}u(t)$.

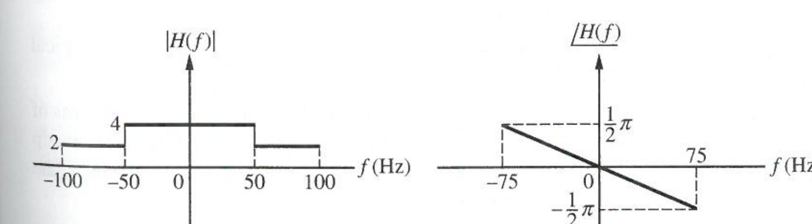

Figure 2.40