2.59. A system has the frequency-response function

$$H(f) = \frac{4\pi + j2\pi f}{8\pi + j2\pi f}$$

Determine and accurately plot the group delay and the phase delay.

2.60. The nonlinear system defined by

$$y(t) = x(t) + 0.1x^2(t)$$

has an input signal with the bandpass spectrum

$$X(f) = 4\Pi\left(\frac{f-20}{6}\right) + 4\Pi\left(\frac{f+20}{6}\right)$$

Sketch the spectrum of the output, labeling all important frequencies and amplitudes.

2.61.

a. Consider a nonlinear device with the transfer characteristic $y(t) = x(t) + 0.1x^3(t)$. The frequency of the input signal $x(t) = \cos(2000\pi t)$ is to be tripled by passing the signal through the nonlinearity and then through a second-order BPF with a frequency response function approximated by

$$H(f) = \frac{1}{1 + j2Q(f-3000)} + \frac{1}{1 + j2Q(f+3000)}$$

Neglecting negative frequency contributions, compute, in terms of the parameter Q, the *total harmonic distortion* (THD) at the tripler output, defined as

$$\text{THD} = \frac{\text{total power in all output distortion terms}}{\text{power in desired output component}} \times 100\%$$

Note that the desired output component in this case is the third harmonic of the input frequency.

b. Find the minimum value of Q that will result in $\text{THD} \le 0.001\%$.

2.62. A nonlinear device has $y(t) = a_0 + a_1 x(t) + a_2 x^2(t) + a_3 x^3(t)$. If $x(t) = \cos(\omega_1 t) + \cos(\omega_2)t$, list all the frequency components present in $y(t)$. Discuss the use of this device as a frequency multiplier.

2.63. Find the impulse response of an ideal highpass filter with the frequency response function

$$H_{\text{HP}}(f) = H_0\left[1 - \Pi\left(\frac{f}{2W}\right)\right]e^{-j2\pi f t_0}$$

2.64. Verify the pulsewidth–bandwidth relationship of (2.250) for the following signals. Sketch each signal and its spectrum.

a. $x(t) = A\exp(-t^2/2\tau^2)$ (Gaussian pulse)

b. $x(t) = A\exp(-\alpha|t|)$, $\alpha > 0$ (double-sided exponential).

2.65.

a. Show that the frequency response function of a second-order Butterworth filter is

$$H(f) = \frac{f_3^2}{f_3^2 + j\sqrt{2}f_3 f - f^2}$$

where f_3 is the 3-dB frequency in hertz.

b. Find an expression for the group delay of this filter. Plot the group delay as a function of f/f_3.

c. Given that the step response for a second-order Butterworth filter is

$$y_s(t) = \left\{1 - \exp\left(-\frac{2\pi f_3 t}{\sqrt{2}}\right)\left[\cos\left(\frac{2\pi f_3 t}{\sqrt{2}}\right) + \sin\left(\frac{2\pi f_3 t}{\sqrt{2}}\right)\right]\right\}u(t)$$

where $u(t)$ is the unit step function, find the 10% to 90% risetime in terms of f_3.

Section 2.8

2.66. A sinusoidal signal of frequency 1 Hz is to be sampled periodically.

a. Find the maximum allowable time interval between samples.

b. Samples are taken at $\frac{1}{3}$-s intervals (i.e., at a rate of $f_s = 3$ sps). Construct a plot of the sampled signal spectrum that illustrates that this is an acceptable sampling rate to allow recovery of the original sinusoid.

c. The samples are spaced $\frac{2}{3}$ s apart. Construct a plot of the sampled signal spectrum that shows what the recovered signal will be if the samples are passed through a lowpass filter such that only the lowest frequency spectral lines are passed.

2.67. A flat-top sampler can be represented as the block diagram of Figure 2.41.

a. Assuming $T \ll T_s$, sketch the output for a typical $x(t)$.

b. Find the spectrum of the output, $Y(f)$, in terms of the spectrum of the input, $X(f)$. Determine relationship between τ and T_s required to minimize distortion in the recovered waveform?

2.68. Figure 2.42 illustrates so-called zero-order-hold reconstruction.