2.59. A system has the frequency-response function

$$H(f) = \frac{4\pi + j2\pi f}{8\pi + j2\pi f}$$

Determine and accurately plot the group delay and the phase delay.

2.60. The nonlinear system defined by

$$y(t) = x(t) + 0.1x^2(t)$$

has an input signal with the bandpass spectrum

$$X(f) = 4\Pi\left(\frac{f-20}{6}\right) + 4\Pi\left(\frac{f+20}{6}\right)$$

Sketch the spectrum of the output, labeling all important frequencies and amplitudes.

2.61.

a. Consider a nonlinear device with the transfer characteristic $y(t) = x(t) + 0.1x^3(t)$. The frequency of the input signal $x(t) = \cos(2000\pi t)$ is to be tripled by passing the signal through the nonlinearity and then through a second-order BPF with a frequency response function approximated by

$$H(f) = \frac{1}{1+j2Q(f-3000)} + \frac{1}{1+j2Q(f+3000)}$$

Neglecting negative frequency contributions, compute, in terms of the parameter Q, the *total harmonic distortion* (THD) at the tripler output, defined as

$$\text{THD} = \frac{\text{total power in all output distortion terms}}{\text{power in desired output component}} \times 100\%$$

Note that the desired output component in this case is the third harmonic of the input frequency.

b. Find the minimum value of Q that will result in THD $\leq 0.001\%$.

2.62. A nonlinear device has $y(t) = a_0 + a_1x(t) + a_2x^2(t) + a_3x^3(t)$. If $x(t) = \cos(\omega_1 t) + \cos(\omega_2)t$, list all the frequency components present in $y(t)$. Discuss the use of this device as a frequency multiplier.

2.63. Find the impulse response of an ideal highpass filter with the frequency response function

$$H_{\text{HP}}(f) = H_0\left[1 - \Pi\left(\frac{f}{2W}\right)\right]e^{-j2\pi f t_0}$$

2.64. Verify the pulsewidth–bandwidth relationship of (2.250) for the following signals. Sketch each signal and its spectrum.

a. $x(t) = A\exp(-t^2/2\tau^2)$ (Gaussian pulse)

b. $x(t) = A\exp(-\alpha|t|)$, $\alpha > 0$ (double-sided exponential).

2.65.

a. Show that the frequency response function of a second-order Butterworth filter is

$$H(f) = \frac{f_3^2}{f_3^2 + j\sqrt{2}f_3 f - f^2}$$

where f_3 is the 3-dB frequency in hertz.

b. Find an expression for the group delay of this filter. Plot the group delay as a function of f/f_3.

c. Given that the step response for a second-order Butterworth filter is

$$y_s(t) = \left\{1 - \exp\left(-\frac{2\pi f_3 t}{\sqrt{2}}\right)\left[\cos\left(\frac{2\pi f_3 t}{\sqrt{2}}\right) + \sin\left(\frac{2\pi f_3 t}{\sqrt{2}}\right)\right]\right\}u(t)$$

where $u(t)$ is the unit step function, find the 10% to 90% risetime in terms of f_3.

Section 2.8

2.66. A sinusoidal signal of frequency 1 Hz is to be sampled periodically.

a. Find the maximum allowable time interval between samples.

b. Samples are taken at $\frac{1}{3}$-s intervals (i.e., at a rate of $f_s = 3$ sps). Construct a plot of the sampled signal spectrum that illustrates that this is an acceptable sampling rate to allow recovery of the original sinusoid.

c. The samples are spaced $\frac{2}{3}$ s apart. Construct a plot of the sampled signal spectrum that shows what the recovered signal will be if the samples are passed through a lowpass filter such that only the lowest frequency spectral lines are passed.

2.67. A flat-top sampler can be represented as the block diagram of Figure 2.41.

a. Assuming $T \ll T_s$, sketch the output for a typical $x(t)$.

b. Find the spectrum of the output, $Y(f)$, in terms of the spectrum of the input, $X(f)$. Determine relationship between τ and T_s required to minimize distortion in the recovered waveform?

2.68. Figure 2.42 illustrates so-called zero-order-hold reconstruction.

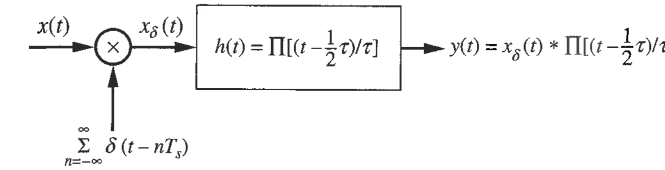

Figure 2.41

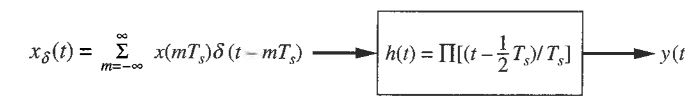

Figure 2.42

a. Sketch $y(t)$ for a typical $x(t)$. Under what conditions is $y(t)$ a good approximation to $x(t)$?

b. Find the spectrum of $y(t)$ in terms of the spectrum of $x(t)$. Discuss the approximation of $y(t)$ to $x(t)$ in terms of frequency-domain arguments.

2.69. Determine the range of permissible cutoff frequencies for the ideal lowpass filter used to reconstruct the signal

$$x(t) = 10\cos(600\pi t)\cos^2(2400\pi t)$$

which is sampled at 6000 samples per second. Sketch $X(f)$ and $X_\delta(f)$. Find the minimum allowable sampling frequency.

2.70. Given the bandpass signal spectrum shown in Figure 2.43, sketch spectra for the following sampling rates f_s and indicate which ones are suitable: (a) $2B$, (b) $2.5B$, (c) $3B$, (d) $4B$, (e) $5B$, (f) $6B$.

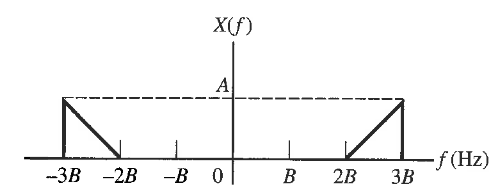

Figure 2.43

Section 2.9

2.71. Using appropriate Fourier transform theorems and pairs, express the spectrum $Y(f)$ of

$$y(t) = x(t)\cos(\omega_0 t) + \widehat{x}(t)\sin(\omega_0 t)$$

in terms of the spectrum $X(f)$ of $x(t)$, where $X(f)$ is lowpass with bandwidth

$$B < f_0 = \frac{\omega_0}{2\pi}$$

Sketch $Y(f)$ for a typical $X(f)$.

2.72. Show that $x(t)$ and $\widehat{x}(t)$ are orthogonal for the following signals ($\omega_0 > 0$):

a. $x_a(t) = \sin(\omega_0 t)$

b. $x_b(t) = 2\cos(\omega_0 t) + \sin(\omega_0 t)\cos(2\omega_0 t)$

c. $x_c(t) = A\exp(j\omega_0 t)$

2.73. Assume that the Fourier transform of $x(t)$ is real and has the shape shown in Figure 2.44. Determine and plot the spectrum of each of the following signals:

a. $x_1(t) = \frac{2}{3}x(t) + \frac{1}{3}j\widehat{x}(t)$.

b. $x_2(t) = \left[\frac{3}{4}x(t) + \frac{3}{4}j\widehat{x}(t)\right]e^{j2\pi f_0 t}$, $f_0 \gg W$.

c. $x_3(t) = \left[\frac{2}{3}x(t) + \frac{1}{3}j\widehat{x}(t)\right]e^{j2\pi W t}$.

d. $x_4(t) = \left[\frac{2}{3}x(t) - \frac{1}{3}j\widehat{x}(t)\right]e^{j\pi W t}$.

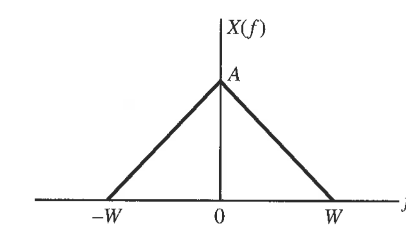

Figure 2.44

2.74. Consider the signal

$$x(t) = 2W\,\text{sinc}(2Wt)\cos(2\pi f_0 t), \quad f_0 > W$$

a. Obtain and sketch the spectrum of $x_p(t) = x(t) + j\widehat{x}(t)$.

b. Obtain and sketch the spectrum of $x_n(t) = x(t) - j\widehat{x}(t)$.

c. Obtain and sketch the spectrum of the complex envelope $\tilde{x}(t)$, where the complex envelope is defined by (2.310).

d. Find the complex envelope $\tilde{x}(t)$.