2.59. A system has the frequency-response function

$$H(f) = \frac{4\pi + j2\pi f}{8\pi + j2\pi f}$$

Determine and accurately plot the group delay and the phase delay.

2.60. The nonlinear system defined by

$$y(t) = x(t) + 0.1x^2(t)$$

has an input signal with the bandpass spectrum

$$X(f) = 4\Pi\left(\frac{f-20}{6}\right) + 4\Pi\left(\frac{f+20}{6}\right)$$

Sketch the spectrum of the output, labeling all important frequencies and amplitudes.

2.61.

a. Consider a nonlinear device with the transfer characteristic $y(t) = x(t) + 0.1x^3(t)$. The frequency of the input signal $x(t) = \cos(2000\pi t)$ is to be tripled by passing the signal through the nonlinearity and then through a second-order BPF with a frequency response function approximated by

$$H(f) = \frac{1}{1 + j2Q(f-3000)} + \frac{1}{1 + j2Q(f+3000)}$$

Neglecting negative frequency contributions, compute, in terms of the parameter Q, the *total harmonic distortion* (THD) at the tripler output, defined as

$$\text{THD} = \frac{\text{total power in all output distortion terms}}{\text{power in desired output component}} \times 100\%$$

Note that the desired output component in this case is the third harmonic of the input frequency.

b. Find the minimum value of Q that will result in $\text{THD} \leq 0.001\%$.

2.62. A nonlinear device has $y(t) = a_0 + a_1 x(t) + a_2 x^2(t) + a_3 x^3(t)$. If $x(t) = \cos(\omega_1 t) + \cos(\omega_2)t$, list all the frequency components present in $y(t)$. Discuss the use of this device as a frequency multiplier.

2.63. Find the impulse response of an ideal highpass filter with the frequency response function

$$H_{\text{HP}}(f) = H_0\left[1 - \Pi\left(\frac{f}{2W}\right)\right]e^{-j2\pi f t_0}$$

2.64. Verify the pulsewidth–bandwidth relationship of (2.250) for the following signals. Sketch each signal and its spectrum.

a. $x(t) = A\exp(-t^2/2\tau^2)$ (Gaussian pulse)

b. $x(t) = A\exp(-\alpha|t|)$, $\alpha > 0$ (double-sided exponential).

2.65.

a. Show that the frequency response function of a second-order Butterworth filter is

$$H(f) = \frac{f_3^2}{f_3^2 + j\sqrt{2}f_3 f - f^2}$$

where f_3 is the 3-dB frequency in hertz.

b. Find an expression for the group delay of this filter. Plot the group delay as a function of f/f_3.

c. Given that the step response for a second-order Butterworth filter is

$$y_s(t) = \left\{1 - \exp\left(-\frac{2\pi f_3 t}{\sqrt{2}}\right)\left[\cos\left(\frac{2\pi f_3 t}{\sqrt{2}}\right) + \sin\left(\frac{2\pi f_3 t}{\sqrt{2}}\right)\right]\right\}u(t)$$

where $u(t)$ is the unit step function, find the 10% to 90% risetime in terms of f_3.

Section 2.8

2.66. A sinusoidal signal of frequency 1 Hz is to be sampled periodically.

a. Find the maximum allowable time interval between samples.

b. Samples are taken at $\frac{1}{3}$-s intervals (i.e., at a rate of $f_s = 3$ sps). Construct a plot of the sampled signal spectrum that illustrates that this is an acceptable sampling rate to allow recovery of the original sinusoid.

c. The samples are spaced $\frac{2}{3}$ s apart. Construct a plot of the sampled signal spectrum that shows what the recovered signal will be if the samples are passed through a lowpass filter such that only the lowest frequency spectral lines are passed.

2.67. A flat-top sampler can be represented as the block diagram of Figure 2.41.

a. Assuming $T \ll T_s$, sketch the output for a typical $x(t)$.

b. Find the spectrum of the output, $Y(f)$, in terms of the spectrum of the input, $X(f)$. Determine relationship between τ and T_s required to minimize distortion in the recovered waveform?

2.68. Figure 2.42 illustrates so-called zero-order-hold reconstruction.

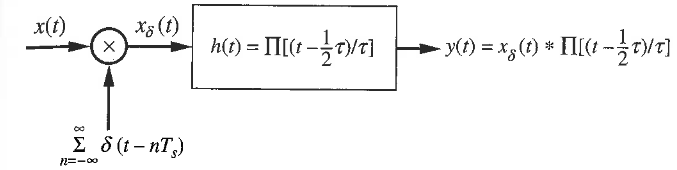

Figure 2.41

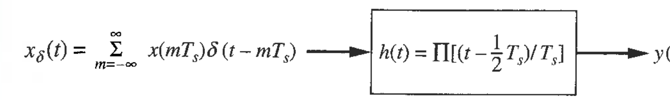

Figure 2.42

a. Sketch $y(t)$ for a typical $x(t)$. Under what conditions is $y(t)$ a good approximation to $x(t)$?

b. Find the spectrum of $y(t)$ in terms of the spectrum of $x(t)$. Discuss the approximation of $y(t)$ to $x(t)$ in terms of frequency-domain arguments.

2.69. Determine the range of permissible cutoff frequencies for the ideal lowpass filter used to reconstruct the signal

$$x(t) = 10\cos(600\pi t)\cos^2(2400\pi t)$$

which is sampled at 6000 samples per second. Sketch $X(f)$ and $X_\delta(f)$. Find the minimum allowable sampling frequency.

2.70. Given the bandpass signal spectrum shown in Figure 2.43, sketch spectra for the following sampling rates f_s and indicate which ones are suitable: (a) $2B$, (b) $2.5B$, (c) $3B$, (d) $4B$, (e) $5B$, (f) $6B$.

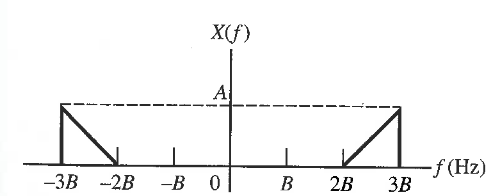

Figure 2.43

Section 2.9

2.71. Using appropriate Fourier transform theorems and pairs, express the spectrum $Y(f)$ of

$$y(t) = x(t)\cos(\omega_0 t) + \widehat{x}(t)\sin(\omega_0 t)$$

in terms of the spectrum $X(f)$ of $x(t)$, where $X(f)$ is lowpass with bandwidth

$$B < f_0 = \frac{\omega_0}{2\pi}$$

Sketch $Y(f)$ for a typical $X(f)$.

2.72. Show that $x(t)$ and $\widehat{x}(t)$ are orthogonal for the following signals ($\omega_0 > 0$):

a. $x_a(t) = \sin(\omega_0 t)$

b. $x_b(t) = 2\cos(\omega_0 t) + \sin(\omega_0 t)\cos(2\omega_0 t)$

c. $x_c(t) = A\exp(j\omega_0 t)$

2.73. Assume that the Fourier transform of $x(t)$ is real and has the shape shown in Figure 2.44. Determine and plot the spectrum of each of the following signals:

a. $x_1(t) = \frac{2}{3}x(t) + \frac{1}{3}j\widehat{x}(t)$.

b. $x_2(t) = \left[\frac{3}{4}x(t) + \frac{3}{4}j\widehat{x}(t)\right]e^{j2\pi f_0 t}$, $f_0 \gg W$.

c. $x_3(t) = \left[\frac{2}{3}x(t) + \frac{1}{3}j\widehat{x}(t)\right]e^{j2\pi Wt}$.

d. $x_4(t) = \left[\frac{2}{3}x(t) - \frac{1}{3}j\widehat{x}(t)\right]e^{j\pi Wt}$.

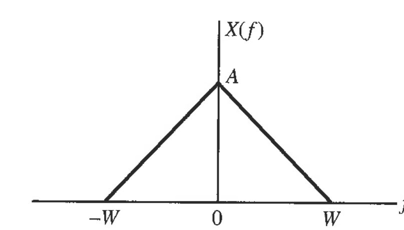

Figure 2.44

2.74. Consider the signal

$$x(t) = 2W\,\text{sinc}(2Wt)\cos(2\pi f_0 t), \quad f_0 > W$$

a. Obtain and sketch the spectrum of $x_p(t) = x(t) + j\widehat{x}(t)$.

b. Obtain and sketch the spectrum of $x_n(t) = x(t) - j\widehat{x}(t)$.

c. Obtain and sketch the spectrum of the complex envelope $\tilde{x}(t)$, where the complex envelope is defined by (2.310).

d. Find the complex envelope $\tilde{x}(t)$.

2.75. Consider the input

$$x(t) = \Pi(t/\tau)\cos[2\pi(f_0 + \Delta f)t], \quad \Delta f \ll f_0$$

to a filter with impulse response

$$h(t) = \alpha e^{-\alpha t}\cos(2\pi f_0 t)u(t)$$

Find the output using complex envelope techniques.

Computer Exercises[19]

2.1.

a. Write a computer program to obtain the generalized Fourier series for an energy signal using the orthonormal basis set

$$\Phi_n(t) = \Pi(t-0.5-n), \quad n = 0, 1, 2, \cdots, T-1, \quad T \text{ integer}$$

where the signal extent is $(0, T)$ with T assumed to be integer valued. Your program should compute the generalized Fourier coefficients and the integral-squared error and should make a plot of the signal being approximated and the approximating waveform. Test your program with the signal $e^{-2t}u(t)$, $0 \le t \le 5$.

b. Repeat part (a) with the orthonormal basis set

$$\Phi_n(t) = \sqrt{2}\Pi\left(\frac{t-0.5-n}{0.5}\right), \quad n = 0, 1, 2, \cdots, 2T-1, \quad T \text{ integer}$$

What is the ISE now?

c. Can you deduce whether the basis set resulting from repeatedly halving the pulse width and doubling the amplitude is complete?

2.2. Generalize the computer program of Computer Example 2.1 to evaluate the coefficients of the complex exponential Fourier series of several signals. Include a plot of the amplitude and phase spectrum of the signal for which the Fourier series coefficients are evaluated. Check by evaluating the Fourier series coefficients of a square wave. Plot the square-wave approximation by summing the series through the seventh harmonic.

2.3. Write a computer program to evaluate the coefficients of the complex exponential Fourier series of a signal by using the FFT. Check it by evaluating the Fourier series coefficients of a square-wave and comparing your results with Computer Exercise 2.2.

2.4. How would you use the same approach as in Computer Exercise 2.3 to evaluate the Fourier transform of a pulse-type signal. How do the two outputs differ? Compute an approximation to the Fourier transform of a square pulse signal 1 unit wide and compare with the theoretical result.

2.5. Write a computer program to find the bandwidth of a lowpass energy signal that contains a certain specified percentage of its total energy, for example, 95%. In other words, write a program to find W in the equation

$$E_W = \frac{\int_0^W G_x(f)\,df}{\int_0^\infty G_x(f)\,df} \times 100\%$$

with E_W set equal to a specified value, where $G_X(f)$ is the energy spectral density of the signal.

2.6. Write a computer program to find the time duration of a lowpass energy signal that contains a certain specified percentage of its total energy, for example, 95%. In other words, write a program to find T in the equation

$$E_T = \frac{\int_0^T |x(t)|^2\,dt}{\int_0^\infty |x(t)|^2\,dt} \times 100\%$$

with E_T set equal to a specified value, where it is assumed that the signal is zero for $t < 0$.

2.7. Use a MATLAB program like Computer Example 2.2 to investigate the frequency response of the Sallen–Key circuit for various Q-values.

[19]When doing these computer exercises, we suggest that the student make use of a mathematics package such as MATLAB. Considerable time will be saved in being able to use the plotting capability of MATLAB. You should strive to use the vector capability of MATLAB as well.

locations. Demodulation is accomplished coherently using quadrature demodulation carriers. A phase error in a demodulation carrier results in serious distortion of the demodulated signal. This distortion has two components: a time-varying attenuation of the desired output signal and crosstalk from the quadrature channel.

36. Time-division multiplexing results when samples from two or more data sources are interlaced, using commutation, to form a baseband signal. Demultiplexing is accomplished by using a second commutator, which must be synchronous with the multiplexing commutator.

Further Reading

One can find basic treatments of modulation theory at about the same technical level of this text in a wide variety of books. Examples are Carlson et al. (2001), Haykin (2000), Lathi (1998), and Couch (2007). Taub and Schilling (1986) have an excellent treatment of PLLs. The performance of the PLL in the absence of noise is discussed by Viterbi (1966, Chapters 2 and 3) and Gardner (1979). The simulation of a PLL is treated by Tranter et al. (2004).

Problems

Section 3.1

3.1. Assume that a DSB signal

$$x_c(t) = A_c m(t)\cos(2\pi f_c t + \phi_0)$$

is demodulated using the demodulation carrier $2\cos[2\pi f_c t + \theta(t)]$. Determine, in general, the demodulated output $y_D(t)$. Let $A_c = 1$ and $\theta(t) = \theta_0$, where θ_0 is a constant, and determine the mean-square error between $m(t)$ and the demodulated output as a function of ϕ_0 and θ_0. Now let $\theta(t) = 2\pi f_0 t$ and compute the mean-square error between $m(t)$ and the demodulated output.

3.2. Show that an AM signal can be demodulated using coherent demodulation by assuming a demodulation carrier of the form

$$2\cos[2\pi f_c t + \theta(t)]$$

where $\theta(t)$ is the demodulation phase error.

3.3. Design an envelope detector that uses a full-wave rectifier rather than the half-wave rectifier shown in Figure 3.3. Sketch the resulting waveforms, as was done in Figure 3.3(b) for a half-wave rectifier. What are the advantages of the full-wave rectifier?

3.4. Three message signals are periodic with period T, as shown in Figure 3.69. Each of the three message signals is applied to an AM modulator. For each message signal, determine the modulation efficiency for $a = 0.2$, $a = 0.4$, $a = 0.7$, and $a = 1$.

3.5. The positive portion of the envelope of the output of an AM modulator is shown in Figure 3.70. The message signal is a waveform having zero DC value. Determine the modulation index, the carrier power, the efficiency, and the power in the sidebands.

3.6. In this problem we examine the efficiency of AM for the case in which the message signal does not have symmetrical maximum and minimum values. Two message signals are shown in Figure 3.71. Each is periodic with period T, and τ is chosen such that the DC value of $m(t)$ is zero. Calculate the efficiency for each $m(t)$ for $a = 1$.

3.7. An AM modulator operates with the message signal

$$m(t) = 9\cos(20\pi t) - 8\cos(60\pi t)$$

The unmodulated carrier is given by $110\cos(200\pi t)$, and the system operates with an index of $\frac{1}{2}$.

a. Write the equation for $m_n(t)$, the normalized signal with a minimum value of -1.

b. Determine $\langle m_n^2(t)\rangle$, the power in $m_n(t)$.

c. Determine the efficiency of the modulator.

locations. Demodulation is accomplished coherently using quadrature demodulation carriers. A phase error in a demodulation carrier results in serious distortion of the demodulated signal. This distortion has two components: a time-varying attenuation of the desired output signal and crosstalk from the quadrature channel.

36. Time-division multiplexing results when samples from two or more data sources are interlaced, using commutation, to form a baseband signal. Demultiplexing is accomplished by using a second commutator, which must be synchronous with the multiplexing commutator.

Further Reading

One can find basic treatments of modulation theory at about the same technical level of this text in a wide variety of books. Examples are Carlson et al. (2001), Haykin (2000), Lathi (1998), and Couch (2007). Taub and Schilling (1986) have an excellent treatment of PLLs. The performance of the PLL in the absence of noise is discussed by Viterbi (1966, Chapters 2 and 3) and Gardner (1979). The simulation of a PLL is treated by Tranter et al. (2004).

Problems

Section 3.1

3.1. Assume that a DSB signal

$$x_c(t) = A_c m(t)\cos(2\pi f_c t + \phi_0)$$

is demodulated using the demodulation carrier $2\cos[2\pi f_c t + \theta(t)]$. Determine, in general, the demodulated output $y_D(t)$. Let $A_c = 1$ and $\theta(t) = \theta_0$, where θ_0 is a constant, and determine the mean-square error between $m(t)$ and the demodulated output as a function of ϕ_0 and θ_0. Now let $\theta(t) = 2\pi f_0 t$ and compute the mean-square error between $m(t)$ and the demodulated output.

3.2. Show that an AM signal can be demodulated using coherent demodulation by assuming a demodulation carrier of the form

$$2\cos[2\pi f_c t + \theta(t)]$$

where $\theta(t)$ is the demodulation phase error.

3.3. Design an envelope detector that uses a full-wave rectifier rather than the half-wave rectifier shown in Figure 3.3. Sketch the resulting waveforms, as was done in Figure 3.3(b) for a half-wave rectifier. What are the advantages of the full-wave rectifier?

3.4. Three message signals are periodic with period T, as shown in Figure 3.69. Each of the three message signals is applied to an AM modulator. For each message signal, determine the modulation efficiency for $a = 0.2$, $a = 0.4$, $a = 0.7$, and $a = 1$.

3.5. The positive portion of the envelope of the output of an AM modulator is shown in Figure 3.70. The message signal is a waveform having zero DC value. Determine the modulation index, the carrier power, the efficiency, and the power in the sidebands.

3.6. In this problem we examine the efficiency of AM for the case in which the message signal does not have symmetrical maximum and minimum values. Two message signals are shown in Figure 3.71. Each is periodic with period T, and τ is chosen such that the DC value of $m(t)$ is zero. Calculate the efficiency for each $m(t)$ for $a = 1$.

3.7. An AM modulator operates with the message signal

$$m(t) = 9\cos(20\pi t) - 8\cos(60\pi t)$$

The unmodulated carrier is given by $110\cos(200\pi t)$, and the system operates with an index of $\frac{1}{2}$.

a. Write the equation for $m_n(t)$, the normalized signal with a minimum value of -1.

b. Determine $\langle m_n^2(t)\rangle$, the power in $m_n(t)$.

c. Determine the efficiency of the modulator.

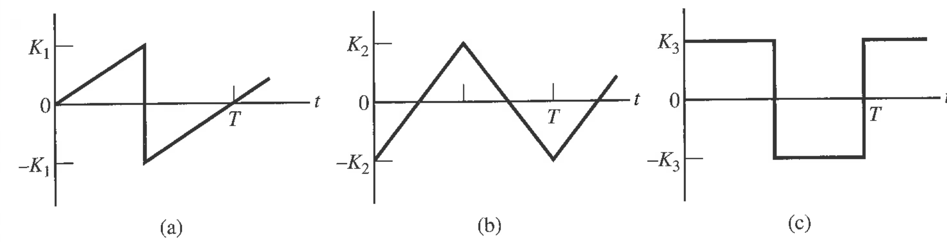

Figure 3.69

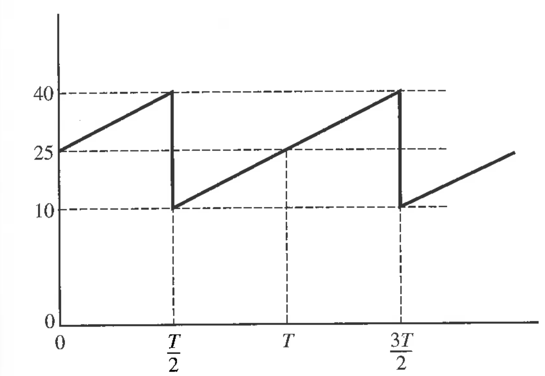

Figure 3.70

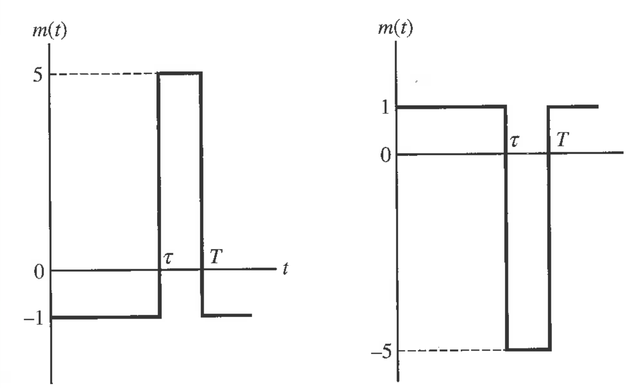

Figure 3.71

d. Sketch the double-sided spectrum of $x_c(t)$, the modulator output, giving the weights and frequencies of all components.

3.8. Rework Problem 3.7 for the message signal

$$m(t) = 9\cos(20\pi t) + 8\cos(60\pi t)$$

3.9. An AM modulator has output

$$x_c(t) = 30\cos[2\pi(200)t] + 4\cos[2\pi(180)t] + 4\cos[2\pi(220)t]$$

Determine the modulation index and the efficiency.

3.10. An AM modulator has output

$$x_c(t) = A\cos[2\pi(200)t] + B\cos[2\pi(180)t] + B\cos[2\pi(220)t]$$

The carrier power is P_0 and the efficiency is E_{ff}. Derive an expression for E_{ff} in terms of P_0, A, and B. Determine A, B, and the modulation index for $P_0 = 100\ W$ and $E_{ff} = 40\,\%$.