

**3.11.** An AM modulator has output

$$x_c(t) = 25 \cos[2\pi(150)t] + 5 \cos[2\pi(160)t] \\ + 5 \cos[2\pi(140)t]$$

Determine the modulation index and the efficiency.

**3.12.** An AM modulator is operating with an index of 0.7. The modulating signal is

$$m(t) = 2 \cos(2\pi f_m t) + \cos(4\pi f_m t) \\ + 2 \cos(10\pi f_m t)$$

a. Sketch the spectrum of the modulator output showing the weights of all impulse functions.

b. What is the efficiency of the modulation process?

**3.13.** Consider the system shown in Figure 3.72. Assume that the average value of  $m(t)$  is zero and that the maximum value of  $|m(t)|$  is  $M$ . Also assume that the square-law device is defined by  $y(t) = 4x(t) + 2x^2(t)$ .

a. Write the equation for  $y(t)$ .

b. Describe the filter that yields an AM signal for  $g(t)$ . Give the necessary filter type and the frequencies of interest.

c. What value of  $M$  yields a modulation index of 0.1?

d. What is an advantage of this method of modulation?

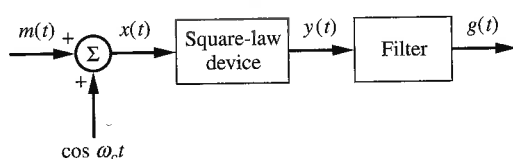


Figure 3.72

**3.14.** Assume that a message signal is given by

$$m(t) = 2 \cos(2\pi f_m t) + \cos(4\pi f_m t)$$

Calculate an expression for

$$x_c(t) = \frac{1}{2} A_c m(t) \cos(2\pi f_c t) \pm \frac{1}{2} A_c \hat{m}(t) \sin(2\pi f_c t)$$

for  $A_c = 4$ . Show that the result is upper-sideband or lower-sideband SSB depending upon the choice of the algebraic sign.

**3.15.** Redraw Figure 3.7 to illustrate the generation of upper-sideband SSB. Give the equation defining the upper-sideband filter. Complete the analysis by deriving the

expression for the output of an upper-sideband SSB modulator.

**3.16.** Prove that carrier reinsertion with envelope detection can be used for demodulation of VSB.

**3.17.** Sketch Figure 3.18 for the case where  $f_{LO} = f_c - f_{IF}$ .

**3.18.** A mixer is used in a short-wave superheterodyne receiver. The receiver is designed to receive transmitted signals between 5 and 25 MHz. High-side tuning is to be used. Determine the tuning range of the local oscillator for IF frequencies varying between 400 kHz and 2 MHz. Plot the ratio defined by the tuning range over this range of IF frequencies as in Table 3.1.

**3.19.** A superheterodyne receiver uses an IF frequency of 455 kHz. The receiver is tuned to a transmitter having a carrier frequency of 1120 kHz. Give two permissible frequencies of the local oscillator and the image frequency for each. Repeat assuming that the IF frequency is 2500 kHz.

### Section 3.2

**3.20.** Let the input to a phase modulator be  $m(t) = u(t - t_0)$ , as shown in Figure 3.20(a). Assume that the unmodulated carrier is  $A_c \cos(2\pi f_c t)$  and that  $f_c t_0 = n$ , where  $n$  is an integer. Sketch accurately the phase modulator output for  $k_p = \pi$  and  $\frac{1}{4}\pi$  as was done in Figure 3.20(c) for  $k_p = \frac{1}{2}\pi$ . Repeat for  $k_p = -\pi$  and  $-\frac{\pi}{4}$ .

**3.21.** We previously computed the spectrum of the FM signal defined by

$$x_{c1}(t) = A_c \cos[2\pi f_c t + \beta \sin(2\pi f_m t)]$$

[see (3.103)]. The amplitude and phase spectra (single sided) was illustrated in Figure 3.24. Now assume that the modulated signal is given by

$$x_{c2}(t) = A_c \cos[2\pi f_c t + \beta \cos(2\pi f_m t)]$$

Show that the amplitude spectrum of  $x_{c1}(t)$  and  $x_{c2}(t)$  are identical. Compute the phase spectrum of  $x_{c2}(t)$  and compare with the phase spectrum of  $x_{c1}(t)$ .

**3.22.** Compute the single-sided amplitude and phase spectra of

$$x_{c3}(t) = A \sin[2\pi f_c t + \beta \sin(2\pi f_m t)]$$

and

$$x_{c4}(t) = A_c \sin[2\pi f_c t + \beta \cos(2\pi f_m t)]$$

Compare the results with Figure 3.24.

**3.23.** The power of an unmodulated carrier signal is 50 W, and the carrier frequency is  $f_c = 50$  Hz. A sinusoidal

message signal is used to FM modulate it with index  $\beta = 10$ . The sinusoidal message signal has a frequency of 5 Hz. Determine the average value of  $x_c(t)$ . By drawing appropriate spectra, explain this apparent contradiction.

**3.24.** Given that  $J_0(3) = -0.2601$  and that  $J_1(3) = 0.3391$ , determine  $J_4(3)$ . Use this result to calculate  $J_5(3)$ .

**3.25.** Determine and sketch the spectrum (amplitude and phase) of an angle-modulated signal assuming that the instantaneous phase deviation is  $\phi(t) = \beta \sin(2\pi f_m t)$ . Also assume  $\beta = 10$ ,  $f_m = 20$  Hz, and  $f_c = 1000$  Hz.

**3.26.** A modulated signal is given by

$$x_c(t) = 6 \cos[2\pi(70)t] + 6 \cos[2\pi(100)t] \\ + 6 \cos[2\pi(130)t]$$

Assuming a carrier frequency of 100 Hz, write this signal in the form of (3.1). Give equations for the envelope  $R(t)$  and the phase deviation  $\phi(t)$ .

**3.27.** A transmitter uses a carrier frequency of 1000 Hz so that the unmodulated carrier is  $A_c \cos(2\pi f_c t)$ . Determine both the phase and frequency deviation for each of the following transmitter outputs:

- a.  $x_c(t) = \cos[2\pi(1000)t + 40t^2]$
- b.  $x_c(t) = \cos[2\pi(500)t^2]$
- c.  $x_c(t) = \cos[2\pi(1200)t]$
- d.  $x_c(t) = \cos[2\pi(900)t + 10\sqrt{t}]$

**3.28.** An FM modulator has output

$$x_c(t) = 100 \cos \left[ 2\pi f_c t + 2\pi f_d \int_0^t m(\alpha) d\alpha \right]$$

where  $f_d = 20$  Hz/V. Assume that  $m(t)$  is the rectangular pulse  $m(t) = 4\Pi[\frac{1}{8}(t-4)]$

- a. Sketch the phase deviation in radians.
- b. Sketch the frequency deviation in hertz.
- c. Determine the peak frequency deviation in hertz.
- d. Determine the peak phase deviation in radians.
- e. Determine the power at the modulator output.

**3.29.** Repeat the preceding problem assuming that  $m(t)$  is the triangular pulse  $4\Lambda[\frac{1}{3}(t-6)]$ .

**3.30.** An FM modulator with  $f_d = 10$  Hz/V. Plot the frequency deviation in hertz and the phase deviation in radians for the three message signals shown in Figure 3.73.

**3.31.** An FM modulator has  $f_c = 2000$  Hz and  $f_d = 14$  Hz/V. The modulator has input  $m(t) = 5 \cos 2\pi(10)t$ .

a. What is the modulation index?

b. Sketch, approximately to scale, the magnitude spectrum of the modulator output. Show all frequencies of interest.

c. Is this narrowband FM? Why?

d. If the same  $m(t)$  is used for a phase modulator, what must  $k_p$  be to yield the index given in (a)?

**3.32.** An audio signal has a bandwidth of 12 kHz. The maximum value of  $|m(t)|$  is 6 V. This signal frequency modulates a carrier. Estimate the peak deviation and the bandwidth of the modulator output, assuming that the deviation constant of the modulator is

- a. 20 Hz/V
- b. 200 Hz/V
- c. 2 kHz/V
- d. 20 kHz/V.

**3.33.** By making use of (3.110) and (3.118), show that

$$\sum_{n=-\infty}^{\infty} J_n^2(\beta) = 1$$

**3.34.** Prove that  $J_n(\beta)$  can be expressed as

$$J_n(\beta) = \frac{1}{\pi} \int_0^\pi \cos(\beta \sin x - nx) dx$$

and use this result to show that

$$J_{-n}(\beta) = (-1)^n J_n(\beta)$$

**3.35.** An FM modulator is followed by an ideal band-pass filter having a center frequency of 500 Hz and a bandwidth of 70 Hz. The gain of the filter is 1 in the passband. The unmodulated carrier is given by  $10 \cos(1000\pi t)$ , and the message signal is  $m(t) = 10 \cos(20\pi t)$ . The transmitter frequency deviation constant  $f_d$  is 8 Hz/V.

- a. Determine the peak frequency deviation in hertz.
  - b. Determine the peak phase deviation in radians.
  - c. Determine the modulation index.
  - d. Determine the power at the filter input and the filter output
  - e. Draw the single-sided spectrum of the signal at the filter input and the filter output. Label the amplitude and frequency of each spectral component.
- 3.36.** A sinusoidal message signal has a frequency of 150 Hz. This signal is the input to an FM modulator with an index of 10. Determine the bandwidth of the modulator

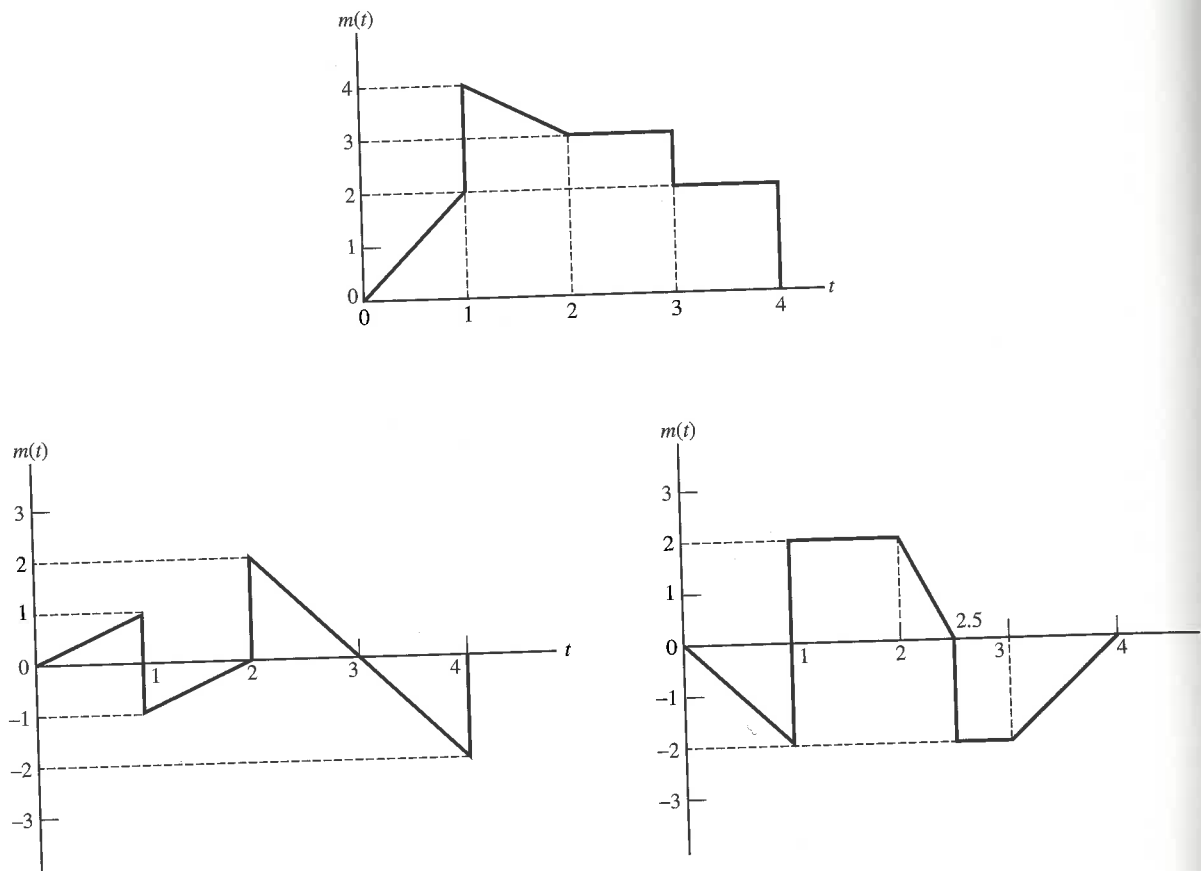


Figure 3.73

output if a power ratio,  $P_r$ , of 0.8 is needed. Repeat for a power ratio of 0.9.

**3.37.** A narrowband FM signal has a carrier frequency of 110 kHz and a deviation ratio of 0.05. The modulation bandwidth is 10 kHz. This signal is used to generate a wideband FM signal with a deviation ratio of 20 and a carrier frequency of 100 MHz. The scheme utilized to accomplish this is illustrated in Figure 3.31. Give the

required value of frequency multiplication,  $n$ . Also, fully define the mixer by giving *two* permissible frequencies for the local oscillator, and define the required bandpass filter (center frequency and bandwidth).

**3.38.** Consider the FM discriminator shown in Figure 3.74. The envelope detector can be considered ideal with an infinite input impedance. Plot the magnitude of the transfer function  $E(f)/X_r(f)$ . From this plot, determine a

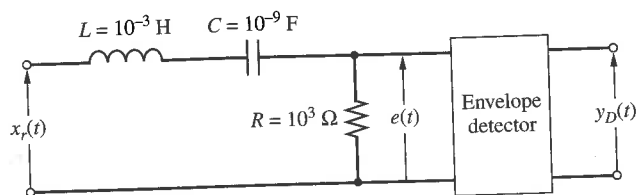


Figure 3.74