Further Reading

Papoulis (1991) is a recommended book for random processes. The references given in Chapter 5 also provide further reading on the subject matter of this chapter.

Problems

Section 6.1

6.1. A fair die is thrown. Depending on the number of spots on the up face, the following random processes are generated. Sketch several examples of sample functions for each case. (A is a constant.)

a. $X(t,\zeta) = \begin{cases} 2A, & \text{1 or 2 spots up} \\ 0, & \text{3 or 4 spots up} \\ -2A, & \text{5 or 6 spots up} \end{cases}$

b. $X(t,\zeta) = \begin{cases} 3A, & \text{1 spot up} \\ 2A, & \text{2 spots up} \\ A, & \text{3 spots up} \\ -A, & \text{4 spots up} \\ -2A, & \text{5 spots up} \\ -3A, & \text{6 spots up} \end{cases}$

c. $X(t,\zeta) = \begin{cases} 4A, & \text{1 spot up} \\ 2A, & \text{2 spots up} \\ At, & \text{3 spots up} \\ -At, & \text{4 spots up} \\ -2A, & \text{5 spots up} \\ -4A, & \text{6 spots up} \end{cases}$

Section 6.2

6.2. Referring to Problem 6.1, what are the following probabilities for each case?

a. $F_X(X \leq 2A, t = 4)$

b. $F_X(X \leq 0, t = 4)$

c. $F_X(X \leq 2A, t = 2)$

6.3. A random process is composed of sample functions that are square waves, each with constant amplitude A, period T_0, and random delay τ as sketched in Figure

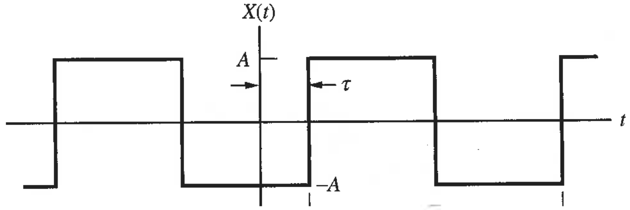

Figure 6.15

6.15. The pdf of τ is

$$f(\tau) = \begin{cases} 1/T_0, & |\tau| \leq T_0/2 \\ 0, & \text{otherwise} \end{cases}$$

a. Sketch several typical sample functions.

b. Write the first-order pdf for this random process at some arbitrary time t_0.
(*Hint*: Because of the random delay τ, the pdf is independent of t_0. Also, it might be easier to deduce the cdf and differentiate it to get the pdf.)

6.4. Let the sample functions of a random process be given by

$$X(t) = A\cos(2\pi f_0 t)$$

where f_0 is fixed and A has the pdf

$$f_A(a) = \frac{e^{-\alpha^2/2\sigma_a^2}}{\sqrt{2\pi}\sigma_a}$$

This random process is passed through an ideal integrator to give a random process $Y(t)$.

a. Find an expression for the sample functions of the output process $Y(t)$.

b. Write down an expression for the pdf of $Y(t)$ at time t_0.
Hint: Note that $\sin 2\pi f_0 t_0$ is just a constant.

c. Is $Y(t)$ stationary? Is it ergodic?

6.5. Consider the random process of Problem 6.3.

a. Find the time-average mean and the autocorrelation function.

b. Find the ensemble-average mean and the autocorrelation function.

c. Is this process wide-sense stationary? Why or why not?

6.6. Consider the random process of Example 6.1 with the pdf of θ given by

$$p(\theta) = \begin{cases} 2/\pi, & \pi/2 \leq \theta \leq \pi \\ 0, & \text{otherwise} \end{cases}$$

a. Find the statistical-average and time-average mean and variance.

b. Find the statistical-average and time-average autocorrelation functions.

c. Is this process ergodic?

6.7. Consider the random process of Problem 6.4.

a. Find the time-average mean and autocorrelation function.

b. Find the ensemble-average mean and autocorrelation function.

c. Is this process wide-sense stationary? Why or why not?

6.8. The voltage of the output of a noise generator whose statistics are known to be closely Gaussian and stationary is measured with a DC voltmeter and a true root-mean-square (rms) voltmeter that is AC coupled. The DC meter reads 6 V, and the true rms meter reads 7 V. Write down an expression for the first-order pdf of the voltage at any time $t = t_0$. Sketch and dimension the pdf.

Section 6.3

6.9. Which of the following functions are suitable autocorrelation functions? Tell why or why not. (ω_0, τ_0, τ_1, A, B, C, and f_0 are positive constants.)

a. $A\cos(\omega_0\tau)$.

b. $A\Lambda(\tau/\tau_0)$, where $\Lambda(x)$ is the unit-area triangular function defined in Chapter 2.

c. $A\Pi(\tau/\tau_0)$, where $\Pi(x)$ is the unit-area pulse function defined in Chapter 2.

d. $A\exp(-\tau/\tau_0)u(\tau)$, where $u(x)$ is the unit step function.

e. $A\exp(-|\tau|/\tau_0)$.

f. $A\,\text{sinc}(f_0\tau) = A\sin(\pi f_0\tau)/\pi f_0\tau$.

6.10. A bandlimited white-noise process has a double-sided power spectral density of 2×10^{-5} W/Hz in the frequency range $|f| \leq 1$ kHz. Find the autocorrelation function of the noise process. Sketch and fully dimension the resulting autocorrelation function.