ECE 307: Solution #3

2. $X(f) = \frac{A}{2W} \Pi(f/2W) e^{-j2\pi f t_d}$

$S_x(f) = \left(\frac{A}{2W}\right)^2 \Pi^2(f/2W) = \left(\frac{A}{2W}\right)^2 \Pi(f/2W)$

$\therefore \; R_x(\tau) = \mathcal{F}^{-1}\{S_x(f)\} = \frac{A^2}{2W} \operatorname{sinc} 2W\tau$

Total energy $= R_x(0) = A^2/2W$

3. $X(f) = A/(b + j2\pi f)$

$S_x(f) = \frac{A^2}{b^2 + (2\pi f)^2} \Rightarrow R_x(\tau) = \frac{A^2}{2b} e^{-b|\tau|}$

$E_x = R_x(0) = \frac{A^2}{2b}$

Problem 2.71

The Fourier transform is

$$\begin{aligned} Y(f) &= \frac{1}{2}X(f-f_0)+\frac{1}{2}X(f+f_0) \\ &\quad +[-j\text{sgn}(f)X(f)] * \left[\frac{1}{2}\delta(f-f_0)e^{-j\pi/2}+\frac{1}{2}\delta(f+f_0)e^{j\pi/2}\right] \\ &= \frac{1}{2}X(f-f_0)[1-\text{sgn}(f-f_0)]+\frac{1}{2}X(f+f_0)[1+\text{sgn}(f+f_0)] \end{aligned}$$

Noting that

$$\begin{aligned} \frac{1}{2}[1-\text{sgn}(f-f_0)] &= u(f_0-f) \\ \text{and} \quad \frac{1}{2}[1+\text{sgn}(f+f_0)] &= u(f+f_0) \end{aligned}$$

this may be rewritten as

$$Y(f)=X(f-f_0)u(f_0-f)+X(f+f_0)u(f+f_0)$$

Thus, if $X(f)=\Pi\left(\frac{f}{2}\right)$ (a unit-height rectangle 2 units wide centered at $f=0$) and $f_0=10$ Hz, $Y(f)$ would consist of unit-height rectangles going from -10 to -9 Hz and from 9 to 10 Hz.

Problem 2.73

a. Note that $F[j\widehat{x}(t)]=j[-j\text{sgn}(f)]X(f)$. Hence

$$\begin{aligned} x_1(t) &= \frac{2}{3}x(t)+\frac{1}{3}j\widehat{x}(t)\rightarrow X_1(f)=\frac{2}{3}X(f)+\frac{1}{3}j[-j\text{sgn}(f)]X(f) \\ &= \left[\frac{2}{3}+\frac{1}{3}\text{sgn}(f)\right]X(f) \\ &= \begin{cases} \frac{1}{3}X(f), & f<0 \\ X(f), & f>0 \end{cases} \end{aligned}$$

b. It follows that

$$\begin{aligned} x_2(t) &= \left[\frac{3}{4}x(t)+\frac{3}{4}j\widehat{x}(t)\right]\exp(j2\pi f_0 t) \\ &\Rightarrow X_2(f)=\frac{3}{4}[1+\text{sgn}(f-f_0)]X(f-f_0) \\ &= \begin{cases} 0, & f<f_0 \\ \frac{3}{2}X(f-f_0), & f>f_0 \end{cases} \end{aligned}$$

c. This case has the same spectrum as part (a), except that it is shifted right by W Hz. That is,

$$\begin{aligned} x_3(t) &= \left[\frac{2}{3}x(t)+\frac{1}{3}j\widehat{x}(t)\right]\exp(j2\pi Wt) \\ &\rightarrow X_3(f)=\left[\frac{2}{3}+\frac{1}{3}\text{sgn}(f-W)\right]X(f-W) \end{aligned}$$

d. For this signal

$$\begin{aligned} x_4(t) &= \left[\frac{2}{3}x(t)-\frac{1}{3}j\widehat{x}(t)\right]\exp(j\pi Wt) \\ &\rightarrow X_4(f)=\left[\frac{2}{3}-\frac{1}{3}\text{sgn}(f-W/2)\right]X(f-W/2) \end{aligned}$$