Z&T (Ed. 6): 5.14, 5.15, 5.21, 5.28

5a. Suppose that you are given a random variable X, and are asked to guess the value of another random variable Y. Assume that your estimate has the linear form $\hat{Y} = a(X - \bar{X})$ where a is a constant (and you know $\bar{X}$). Show that the best linear estimate is obtained by taking $a = \mu_{xy}/\sigma_x^2$, in the sense that this minimizes the mean squared error $E[(\hat{Y} - Y)^2]$.

b. Note that if we scale the estimate as $\hat{Y} = a(\sigma_y/\sigma_x)(X - \bar{X})$, then the optimal value of a is the correlation coefficient ρ_{xy}. Sketch the mean squared estimation error as a function of ρ_{xy}.

6. Find the p.d.f. of $W = X - 3Y + Z$ where X, Y, and Z are Gaussian r.v.'s with $\bar{X} = 6$, $\bar{Y} = -2$, $\bar{Z} = 0$, $\sigma_x = \sigma_y = \sigma_z = 4$, $E[XY] = -22$, $E[XZ] = 10$, and $E[YZ] = 4$.