EECS 310: Discrete Math **Lecture 1**
Overview, Logic

Reading: MIT OpenCourseWare 6.042 Chapter 1

Course Overview

NOTE: if you are not registered and want to be, email `permissions@eecs.northwestern.edu` (not me).

The essentials:

- Prof: Nicole Immorlica, office hours in Ford 3.327
- TAs: office hours in Ford 3.320
  - Manolis Pountourakis
  - Greg Stoddard
- Website: linked to from my homepage, also use blackboard
- Office Hours (starting next week):
  - Monday, 11am-noon: Nicole
  - Monday, 3-4pm: Manolis, Greg
  - Wednesday, 1-2pm: Nicole
  - Wednesday, 2-3pm: Manolis, Greg
- Quizes: in-class on Oct. 11, Nov. 3, and Dec. 1
- Problem Sets: assigned weekly when there's no quiz (should take you 7-8 hours)
- Readings (should take you 1-2 hours):
  - lecture notes (online)
  - textbook Discrete Mathematics, by Rosen (offline, optional)
- Mini-quizes: weekly in-class when there's no quiz

Grading:

- 60% three quizes (20% each)
- 40% eight problem sets (5% each)

  due in lecture. late policy: hand in by following lecture for 25% off, after that we do not accept them.
- 7% seven challenge problems (1% each)
- 8% eight mini-quizes (1% each)

  who has a clicker?

grade on absolute scale

- $\geq$ 94%: A
- $\geq$ 88%: A-
- $\geq$ 84%: B+
- $\geq$ 80%: B
- $\geq$ 76%:B-
- $\geq$ 72%: C+
- $\geq$ 68%: C

- $\geq 64\%$: C-
- $\geq 60\%$: D
- $\geq 0\%$: F

curve quizzes so average is 75%.

[[*MINI-QUIZ time.*]]

Use and Beauty

- Use:

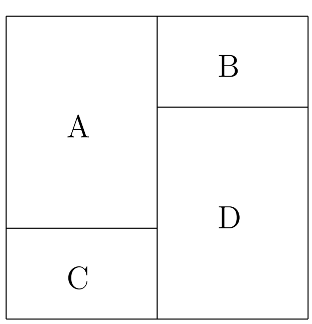
  simple (easy to construct, easy to use), versatile (can be used for a wide variety of situations, abstracts the essence of the problem)

- Beauty:

  inspires me, teaches me something, captures the essence of the problem

Use: graph coloring

Problem 1: want to color a map with smallest number of colors so no two adjacent countries have the same color.

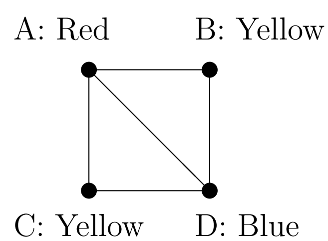


Problem 2: want to schedule exams in smallest number of days so no student has a conflict.

Student	1	2
Course	A	A
Course	B	C
Course	D	D

Abstract relevant information:

- A point for each object (map region, exam).
- Connect points if corresponding objects can't share property (adjacent countries, exams that student must take).
- Find technique to color points so adjacent points don't have same color
- Use this to color map/construct timetable.



[[*Many computer science applications: what computers can do (e.g., complexity, cryptography), how long it takes (e.g., algorithms).*]]

Beauty: puzzle

Tiling:

Can this grid be tiled with dominoes?

- No two dominoes may overlap
- All grid squares must be covered

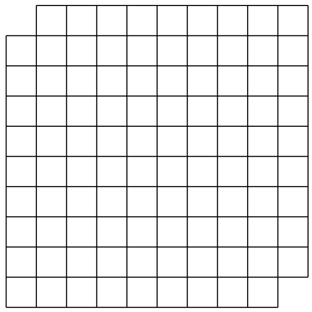


Figure 1: Grid to be tiled

Figure 2: A domino piece

Approach:

1. Start small.
2. Look for patterns.

Answer: Grid can not be tiled because

- Each domino covers one black and one white square.
- In any tiling, number of covered black squares equals number of covered white squares.
- In grid, there are two more black than white squares.

[[*Illustrates beauty in proofs – an unexpected harmony, something that can be generalized, teach us about other objects.*]]

Chomp:

Alice and Bob eat $n \times m$ chocolate bar.

- If you eat (i, j)'th square, must eat all squares north-east as well.
- Alice eats first.

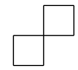


Figure 3: Grid to be tiled

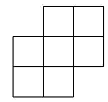


Figure 4: Grid to be tiled

- Bottom-left square is poison.

Question: Prove Alice won't die (assume either $n > 1$, or $m > 1$).

Approach:

- Start small.
  - 1×2 chocolate
  - 2×1 chocolate
  - 2×2 chocolate
- Look for patterns.
  - Alice has a "dummy move".
  - Alice can force Bob to make a board where she wins.

Logic

Formalize statements

1. You can have cake or ice cream.
2. If pigs fly, then you can understand math.
3. If you can solve any problem I pose, then you get an A.
4. Every American has a dream.

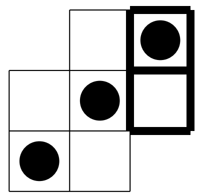


Figure 5: Grid to be tiled

[[*Can you have cake and ice cream? If the second sentence is true, can you understand math?*]]

Def: *Proposition*: a statement that is true or false.

- $2 + 2$. Not a proposition.
- $2 + 2 = 4$. A proposition.
- $2 + 2 = 5$. A proposition.

Famous propositions:

- Goldbach Conjecture (1742): Every even integer greater than two can be written as the sum of two primes. **Unknown.**
- Fermat's Last Theorem (1637): For all $n > 2$, $x^n + y^n = z^n$ has no non-trivial solution. **True. Proved in 1995 by Wiles.**
- Polya Conjecture (1919): For any natural number n, at least half of numbers in $\{1, \ldots, n\}$ have an odd number of prime divisors. **False. Counterexample for** $n \approx 10^{361}$ **found by Haselgrove in 1958.**

Combining propositions:

Let P, Q be propositions.

- Negation, disjunction, conjunction

P	Q	$\neg Q$	$P \vee Q$	$P \wedge Q$
T	T	F	T	T
T	F	T	T	F
F	T	F	T	F
F	F	T	F	F

- Implications (conditionals), equivalence (biconditionals)

P	Q	$P \to Q$	$P \leftrightarrow Q$
T	T	T	T
T	F	F	F
F	T	T	F
F	F	T	T

[[*In fact $\neg$ and one of other above operations are enough to produce all logical statements. I.e., it forms a functionally complete system.*]]

Equivalent Statements:

- Implication/equivalence: $P \leftrightarrow Q \equiv (P \to Q) \wedge (Q \to P)$
- Contrapositive: $P \to Q \equiv \neg Q \to \neg P$
- Implication/Disjunction: $P \to Q \equiv Q \vee \neg P$

NOTE: *Converse* of $P \to Q$ is $Q \to P$. They ARE NOT logically equivalent!!!

Exercise: prove above using truth tables.

[[*For above statements, what are propositions? Write in math? Can you have cake and ice cream? Can you understand math (assuming pigs can't fly)?*]]

More Logic

Formalize statements

1. If you can solve any problem I pose, then you get an A.
2. Every American has a dream.

[[If you solve some but not all problems, do you get an A? How about if you can't solve any problems? Do all Americans have the same dream? Or do some have different dreams?]]

Def: *Predicate*: a proposition whose truth depends on the value of one or more variables drawn from a universe of discourse.

Example:

Let the universe be the set of all integers.

- $P(n) = n$ is divisible by 3: $P(6)$ is true, $P(7)$ is false.
- $P(n,m) = n+m$ is even: $P(3,4)$ is false, $P(3,5)$ is true.

Note: *Predicates* are true/false, *numerical functions* take values:

- $P(n) = n^2$ equals 4, is TRUE for $n = 2, -2$
- $f(n) = n^2$, is FOUR for $n = 2, -2$

Quantifiers

Somtimes true vs. always true.

Sometimes true: existential
$\exists \rightsquigarrow$ "There exists"

- There exists an $n \in \mathbb{N}$ such that $n^2 = 4$.

  $\exists n \in \mathbb{N} : n^2 = 4$
- There exists an n such that $P(n)$ is true. $P(n)$ is true for some n, at least one n.

  $\exists n \in D : P(n)$

Always true: universal:
$\forall \rightsquigarrow$ "For all"

- For all $n \in \mathbb{N}$, $n^2 \geq 0$.

  $\forall n \in \mathbb{N} : n^2 \geq 0$
- For all n, $P(n)$ is true. $P(n)$ is true for every n.

  $\forall n \in D : P(n)$

Note: quantified statements are propositions. Can combine, take multiple variables.

$Even(n) = n$ is even
$P(n,k) = n = 2k$

IF $[\exists k \in \mathbb{N} : P(n,k)]$ THEN $Even(n)$.

Mixing quantifiers:

- $\forall n \in D \forall m \in D : P(n,m)$ – $P(n,m)$ true for every pair in D.
- $\exists n \in D \exists m \in D : P(n,m)$ – There is a pair n,m for which $P(n,m)$ is true.
- $\forall n \in D \exists m \in D : P(n,m)$ – For every n there is some m for which $P(n,m)$ is true.
- $\exists n \in D \forall m \in D : P(n,m)$ – There is an n for which $P(n,m)$ is true for every m.

Example: Give pairs. $P(n,m) = n$ divides m

- $\forall n \in \mathbb{N}, \forall m \in \mathbb{N} : P(n, nm)$
  Answer: any pair
- $\exists n \in \mathbb{N}, \exists m \in \mathbb{N} : P(n,m) \wedge P(m,n)$
  Answer: any pair where n=m.
- $\forall n \in \mathbb{N}, \exists m \in \mathbb{N} : P(n,m)$
  Answer: e.g., $m = 10n$
- $\exists n \in \mathbb{N}, \forall m \in \mathbb{N} : P(n,m)$
  Answer: any pair where $n = 1$

Order matters! (e.g., third statement false if switch order.)

[[For above statements, what are predicates? Write in math? Must you solve all problems to get an A? Do all Americans have the same dream?]]

Negating quantifiers:

It is not the case that all integers are even.
There exists an integer that is not even.

$\neg \forall n \in \mathbb{N} : Even(n)$
$\exists n \in \mathbb{N} : \neg Even(n)$

It is not the case that there exists an integer n with $n^2 < 0$.
For all integers, $n^2 \geq 0$.

$\neg \exists n \in \mathbb{N} : n^2 < 0$
$\forall n \in \mathbb{N} : n^2 \geq 0$

General rule: $\neg \forall n \in D : P(n)$ is same as $\exists n \in D : \neg P(n)$
and $\neg \exists n \in D : P(n)$ is same as $\forall n \in D : \neg P(n)$

To negate mixed quantifiers, move negation inside sequentially, making sure to negate *each statement*.

Example: Goldbach conjecture: let Even be even integers greater than 2, Primes be set of prime numbers.

- Every even integer greater than 2 can be written as the sum of two primes.
- $\forall n \in Even, \exists p \in Primes, \exists q \in Primes : n = p + q$

Negate Goldbach's Conjecture:

- $\neg[\forall n \in Even, \exists p \in Primes, \exists q \in Primes : n = p + q]$
- $\exists n \in Even, \neg[\exists p \in Primes, \exists q \in Primes : n = p + q]$
- $\exists n \in Even, \forall p \in Primes, \neg[\exists q \in Primes : n = p + q]$
- $\exists n \in Even, \forall p \in Primes, \forall q \in Primes : \neg[n = p + q]$
- $\exists n \in Even, \forall p \in Primes, \forall q \in Primes : n \neq p + q$