EECS 310, Fall 2011
Instructor: Nicole Immorlica
Problem Set #1
Due: Sept 27, 2011

1. (15 points) Graph coloring abstraction.

   Describe a problem (other than the two mentioned in lecture) that can be solved using a graph coloring abstraction. Give a (small) instance of the problem, convert it to a graph, describe a coloring of your graph, and explain the implied solution.

2. (15 points) Logic.

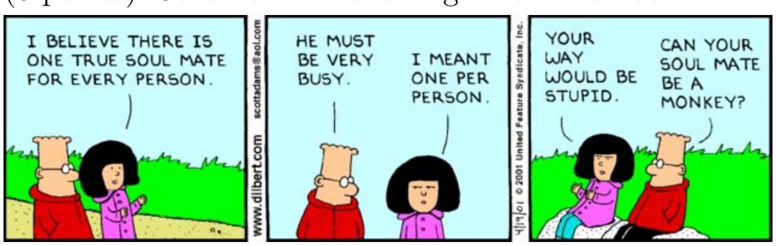
   (a) (5 points) The *exclusive or* operation is defined as: $P \oplus Q$ is true when exactly one of P or Q is true. Find an expression equivalent to $P \oplus Q$ using only *not* ($\neg$), *and* ($\wedge$), and *or* ($\vee$).

   (b) (5 points) The operators *not* ($\neg$) and *and* ($\wedge$) are complete in the sense that any logical statement can be expressed using only these two operators. Find an expression equivalent to $P \iff Q$ using only *not* ($\neg$) and *and* ($\wedge$).

   (c) (5 points) Rewrite $\neg(P \to Q)$ using only *not* ($\neg$) and *and* ($\wedge$), distributing the negation throughout the parathenses (so that the symbol $\neg$ is followed directly by P or Q in your final statement).

3. (20 points) Quantifiers.

   (a) (6 points) Consider the following Dilbert cartoon:



   Let P be the set of all people and $S(n, m)$ be the predicate that person n and person m are soulmates.

   1. Using the existential and universal quantifiers, the universe of discourse defined above, and the predicate defined above, write a mathematical expression for the sentence in the first panel as the speaker intends it.

   2. Using the existential and universal quantifiers, the universe of discourse defined above, and the predicate defined above, write a mathematical expression for the sentence in the first panel as interpretted by Dilbert in the second panel.

   (b) (14 points) Let $Vehicles$ be the set of all transportation vehicles and $Rocket(n)$, $Train(n)$, $Bike(n)$, $Red(n)$, $Expensive(n)$, $Faster(n, m)$ be the predicates "n is a rocketship," "n is a train," "n is a bicycle," "n is red," "n is expensive," and "n is faster than m" respectively. Express the following statements using quantifiers, logical operators, and the above predicates. Do so in the most simple form (i.e., do not use parentheses).

1. Some bicycles are red.
2. Not all bicycles are red.
3. No rocketships are red.
4. All rocketships are expensive.
5. All rocketships are faster than all trains.
6. Some trains are faster than some bicycles.
7. Not all trains are faster than all bicycles.

4. (50 points) Proofs

   Prove each of the following statements using one of the proof methods presented in class (direct proof, contrapositive, contradiction, proof by cases). Be sure to specify what proof technique you decide to apply.

   (a) (10 points) For any real numbers x and y, $|x - y| \geq |x| - |y|$.

   (b) (10 points) For any integer n, if n^2 is odd, then n is odd.

   (c) (10 points) For any rational number x and irrational number y, their sum $(x + y)$ is irrational.

   (d) (10 points) For any two integers n and m, $n + m$ is odd if and only if exactly one of n and m is odd.

   (e) (10 points) There are no positive integers solutions to the equation $n^2 - m^2 = 1$ (a positive integer solution is a pair of integers (n, m) both of which are strictly positive, i.e., $n > 0$ and $m > 0$).

5. **Challenge Problem!**

   Alice and Bob are sharing a loaf of banana bread. The loaf has been pre-cut with slices of varying thickness. Alice takes the first slice from one of the two ends. Thereafter, Alice and Bob alternate taking slices (always from one of the ends). Argue that if there are an even number of slices, then Alice can guarantee that she gets at least as much banana bread as Bob.