EECS 310, Fall 2010
Instructor: Nicole Immorlica
Problem Set #1
Due: Sept 30, 2010

1. (20 points) Logic

When asked to write statements in mathematical notation in this question, please do so in the most simple form (i.e., do not use paranthesis).

(a) (8 points) Consider the following statement:

It is a truth universally acknowledged, that "a single man in possession of a good fortune must be in want of a wife."

Let P be the proposition "A single man is in possession of a good fortune" and Q be the proposition "A single man wants a wife".

1. Rewrite this statement (i.e., the portion of the statement in quotes) in mathematical notation using the defined propositions P and Q and the logical operators $\wedge, \vee, \rightarrow, \leftrightarrow$, and/or $\neg$.

2. Describe someone that disproves the statement. Be sure to explain why it's a counterexample.

3. Write the converse of the statement in both mathematical notation using the defined proposition P and Q and the logical operators $\wedge, \vee, \rightarrow, \leftrightarrow$, and/or $\neg$ and English.

4. Describe someone that disproves the converse of the statement (i.e., the sentence you wrote in the previous part). Be sure to explain why it's a counterexample.

(b) (8 points) Consider the following statemet.

"If it isn't fresh, it isn't legal."

Let P be the proposition "It is fresh" and Q be the proposition "It is legal".

1. Rewrite this statement in mathematical notation using the defined proposition P and Q and the logical operators $\rightarrow$ and $\neg$.

2. Write the contrapositive of the statement in both mathematical notation and English.

3. Write an equivalent statement in both mathematical notation and English using the defined proposition P and Q and the logical operators $\vee$ and $\neg$.

4. Prove the equivalence of the statements in the preceeding three parts using truth tables.

(c) (4 points) Write an equivalence statement (i.e., a $P \leftrightarrow Q$ statement) in English of your own choosing, define associated propositions, and write the negation of your statement in both mathematical notation and English.

BONUS: Where are the above quotes from?

2. (20 points) Predicates

   Let $C(x)$, $D(x)$, $U(x)$, $F(x)$, and $H(x,y)$ be the statments "x is a cat", "x is a dog", "x is ugly", "x is fluffy", and x hates y respectively. Express the following statements using quantifiers, logical operaters, and $P(x), Q(x), R(x)$, where x is from the universe of all animals. Do so in the most simple form (i.e., do not use paranthesis).

   (a) (2 points) No cats are ugly.

   (b) (2 points) All dogs are ugly.

   (c) (2 points) Some cats are fluffy.

   (d) (2 points) Not all cats are fluffy.

   (e) (4 points) All cats hate some dog.

   (f) (4 points) Some cats hate all dogs.

   (g) (4 points) Not all cats hate all dogs.

   JOKE: Why are all epsilons happy? Because for every epsilon there is a delta.

3. (50 points) Proofs

   Prove each of the following statements using one of the proof methods presented in class (direct proof, contrapositive, contradiction, proof by cases). Be sure to specify what proof technique you decide to apply.

   (a) (10 points) For any real numbers x and y, either x or y is smaller than or equal to the average of x and y.

   (b) (10 points) For any two integers a and b, the product of a and b is odd if and only if both a and b are odd.

   (c) (10 points) For any integer a, if a is divisible by 4, then a is the difference of two perfect squares (i.e., $a = b^2 - c^2$ for two integers b and c).

   (d) (10 points) For any positive integer a and prime number p, if $a^2 - 1$ is divisible by p, then either $a + 1$ or $a - 1$ is divisible by p.

   (e) (10 points) There are no rational solutions to the equation $x^5 + x^4 + x^3 + x^2 + 1 = 0$.

4. (10 points) Puzzles

   Imagine an apartment complex with 3 floors, each of which contains 9 apartments arranged in a 3×3 grid. In each apartment there is a child who has lost a tooth. The tooth fairy must collect all the teeth, starting with the apartment on the ground floor in the southwest corner. Passing only through walls, ceilings, and floors, is it possible for her to collect the tooth of the child in the middle apartment last without leaving the building and while visiting each apartment just once?