Throughput Optimal Control of Cooperative Relay Networks

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Abstract

In cooperative relaying, multiple nodes cooperate to forward a packet within a network. To date, such schemes have been primarily investigated at the physical layer with the focus on communication of a single end-to-end flow. This paper considers cooperative relay networks with multiple stochastically varying flows, which may be queued within the network. *Throughput optimal* network control policies are studied that take into account queue dynamics to jointly optimize routing, scheduling and resource allocation. To this end, a generalization of the *Maximum Differential Backlog* algorithm is given, which takes into account the cooperative gains in the network. Several structural characteristics of this policy are discussed for the special case of parallel relay networks.

I. Introduction

Given stochastically varying traffic, there is a growing body of work on *throughput optimal* control schemes for wireless networks that jointly address issues such as routing, scheduling and physical-layer resource allocation, e.g. [1]–[6]. By "throughput optimal" we mean that a control scheme stabilizes all the queues within the network whenever it is possible to do so. In other words, such a scheme stabilizes the network for any rate in the network's *stability region*. Many of these schemes utilize some version of a *maximum differential backlog (MDB)* policy (also

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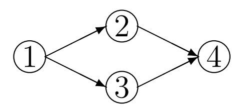

Fig. 1. A four node parallel relay network model.

sometimes called the Backpressure Algorithm) [4], which has the desirable property of requiring no *a priori* knowledge of the traffic statistics.

A feature of all the above models is that each packet is forwarded along a single route of point-to-point links. At any time a packet resides at a single location in the network, and the resources needed for the next transmission do not depend on the previous transmissions of the packet. Recently, there has been much interest in various *cooperative relaying* techniques (e.g. [7]–[12]) that do not satisfy these assumptions. With such techniques, multiple nodes cooperate to relay a packet. For example, consider the four node "parallel relay" network from [7], in Fig. 1. Suppose that node 1 has traffic to send to node 4. The arrows in Fig. 1 indicate the feasible links for this traffic using traditional point-to-point forwarding.[1] If node 1 broadcasts the same packet to *both* nodes 2 and 3, then these nodes can cooperatively forward this packet to node 4 by, for example, forming a distributed antenna array. In certain cases, this cooperative rate will be greater than the rate achieved by point-to-point forwarding.

To date, cooperative relaying has mainly been addressed at the physical-layer, i.e. by studying the achievable rates or diversity gains of given cooperative schemes, and often just focusing on a single end-to-end backlogged session. A goal of this paper is to study models of cooperative relaying that incorporate the stochastic arrival of traffic for multiple sessions and the related network queueing dynamics. For example, in Fig. 1, suppose node 2 has its own traffic to send to node 4. In order to stabilize the network, node 1 may then have to forgo any cooperative gain and use the single route through node 3. For such models, we are interested in developing a MDB-like policy which is throughput optimal.

We focus on "decode and forward" cooperative techniques, in which all cooperative nodes must decode a copy of a packet before forwarding it. With these schemes, a potential trade-

[1]For simplicity, we assume that node 1 cannot directly transmit to node 4, e.g. the direct link may be of too poor a quality to be feasible.

off emerges: exploiting cooperative gains requires that the congestion in the network must first increase due to the duplication of a packet in the network. This increase in traffic can be somewhat ameliorated by exploiting the broadcast nature of the wireless medium, e.g., in Fig. 1, node 1 can simultaneously transmit a packet to both nodes 2 and 3. In addition to decode and forward, a variety of other cooperative strategies have been considered, such as the "amplify and forward" technique (e.g. [9]). We do address such schemes here; since they deal with analog information, it is not obvious how to incorporate them into the queueing models considered here. In addition to improving throughput, cooperative relaying can also increase diversity in a fading environment (e.g. [8], [9]). Here, we assume that there is no fading, and so do not address these diversity gains.

The MDB policy in [4], [5] makes *myopic* decisions based on "backpressure weights" given by the differences in queue backlogs between two adjacent nodes on each link in a network. In adapting such a policy to a cooperative network, several new issues arise. First, in cooperative networks packets are not forwarded only over point-to-point links as in [4], [5]; hence, the underlying network model in [4], [5] must be generalized. Second, the notion of a backpressure weight must be generalized to account for the fact that a packet may be stored at multiple nodes. Third, since an MDB-like policy is myopic, it is not obvious if it would allow for the temporary increase in the congestion needed to exploit cooperative gains. We show that these issues can be addressed in a network with "two-hop" cooperative relaying. This model accommodates the example in Fig. 1 as well as a number of other important cooperative scenarios.

II. General Network Model

We study a generalization of the model in [5] to include cooperative relaying. A network $\mathcal{G}$ consists of a set of nodes $\mathcal{V}$, and a set $\mathcal{L}$ of *non-cooperative* or *direct links*, given by ordered pairs (u, v) for $u, v \in \mathcal{V}$. These represent point-to-point links.[2] Additionally, $\mathcal{G}$ contains two other sets of "links." First, we define a set $\mathcal{S}$ of *cooperative links*. These are denoted by ordered pairs (S, v), where $S \subset \mathcal{V}$ is a subset of nodes which cooperate to forward a packet to a single destination $v \in \mathcal{V}$. Second, we define a set $\mathcal{T}$ of *broadcast links*, denoted by ordered pairs

[2] In principle, a "link" exists between every pair of nodes. However, we do not require that $\mathcal{L}$ include all such links, e.g. certain poor links may not be considered to reduce routing complexity.

(u, T), where $u \in \mathcal{V}$ is a node which may broadcast a packet to all of the nodes in $T \subset \mathcal{V}$. For example, let $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{S}, \mathcal{T})$ be a model for the network in Fig. 1. Here, $\mathcal{V} = \{1, 2, 3, 4\}$, and $\mathcal{E}$ consists of the four direct links, shown by the arrows in the figure. Setting $\mathcal{S} = \{(\{2, 3\}, 4)\}$ and $\mathcal{T} = \{(1, \{2, 3\})\}$ allows us to model the case where node 1 can broadcast to nodes 2 and 3, who can then cooperatively relay a packet to node 4.

We restrict our attention to *two-hop* cooperative relay networks in which (*i*) for each cooperative link (S, v) there is at least one broadcast link (u, T) with $T = S$ and (*ii*) the only traffic that can be sent over (S, v) is that which is received on such a broadcast link. These restrictions rule out several possibilities, including the case where one cooperative group forwards a packet to a second cooperative group, which then forwards it on, or the case where different copies of a packet arrive at a cooperative group over different paths. Such possibilities are not considered in part to simplify notation and in part because the implementation complexity quickly becomes intractable.

We assume the network operates in slotted time, where the length of each time-slot is normalized to 1. There is no fading or changes in the topology over the time-scale of interest.[3] Within time-slot t, let $\boldsymbol{R}(t) = (R_l(t))$ denote the vector of realized transmission rates for all $l \in \mathcal{L} \cup \mathcal{S} \cup \mathcal{T}$. This vector is constrained to lie in an *instantaneous link capacity region* $\mathcal{C}$, which is a bounded subset of $\mathbb{R}_+^{|\mathcal{L} \cup \mathcal{S} \cup \mathcal{T}|}$, i.e., $\mathcal{C}$ is the set of feasible rates in any time-slot, including the rates on all cooperative and broadcast links.

Next, we give several examples of $\mathcal{C}$ where the channel between each pair of nodes i, j is given by an additive Gaussian noise channel with gain $\sqrt{h_{ij}}$, unit variance noise, and bandwidth $W = 1$ Hz. Each transmitter is assumed to have a power constraint of P during each time-slot. If link (i, j) is the only link activated, then the feasible transmission rate is given by $R_{ij} = \log(1 + h_{ij} P)$, i.e., the Shannon capacity of this channel.[4] The results in Section III are not restricted to this case, but apply to any model for $\mathcal{C}$ that gives a bounded subset of $\mathbb{R}_+^{|\mathcal{L} \cup \mathcal{S} \cup \mathcal{T}|}$.

Example 1: Let $\boldsymbol{R} = (R_{1S}, R_{12}, R_{13}, R_{S4}, R_{24}, R_{34})$ be a vector of transmission rates for the 6 links in the network in Fig. 1, where $S = \{2, 3\}$. Suppose that only one of the following two sets of transmitters may be simultaneously active: $\mathcal{A}_1 = \{1\}$ or $\mathcal{A}_2 = \{2, 3\}$. This enforces a

[3]Such effects can be incorporated in our analysis at the expense of more complicated notation.

[4]This is reasonable provided that each time-slot has sufficiently many degrees of freedom to allow for sophisticated coding.

half-duplexing constraint at nodes 2 and 3.[5] It follows that $\mathcal{C} = \mathrm{conv}(\mathcal{C}_1 \cup \mathcal{C}_2)$, where $\mathcal{C}_i$ is the set of feasible rates corresponding to $\mathcal{A}_i$. Here, $\mathrm{conv}(X)$ indicates the convex hull of the set X. This is included to model the possibility of time-sharing between the two activation sets within a time-slot.

When $\mathcal{A}_1$ is active, the network can be viewed as a Gaussian broadcast channel, where the traffic sent over link $(1, \{2, 3\})$ represents *common information*. Without loss of generality, assume that $h_{12} \leq h_{13}$. The feasible rates must then satisfy $(R_{12} + R_{1S}, R_{13}) \in \mathcal{C}_{BC}$, where $\mathcal{C}_{BC}$ is the capacity region of the Gaussian Broadcast channel. Therefore, we can define $\mathcal{C}_1$ as the set of all $(R_{1S}, R_{12}, R_{13}, 0, 0, 0)$ such that $(R_{12} + R_{1S}, R_{13}) \in \mathcal{C}_{BC}$.

When $\mathcal{A}_2$ is active, nodes 2 and 3 transmit over a Gaussian multiaccess channel. When these nodes send only direct traffic ($R_{S4} = 0$), the transmission rates (R_{24}, R_{34}) must lie in the corresponding multiaccess capacity region $\mathcal{C}_{MAC}$, i.e. they must satisfy

$$\sum_{i \in \mathcal{V}} R_{i4} \leq \log\big(1 + \sum_{i \in \mathcal{V}} h_{i4} P\big),\ \forall \mathcal{V} \subseteq \{2, 3\}. \tag{1}$$

Suppose that the nodes cooperate by beamforming so that if they send only cooperative traffic[6]

$$R_{S4} = \log(1 + (\sqrt{h_{24}} + \sqrt{h_{34}})^2 P). \tag{2}$$

In addition, we can allow the nodes to transmit both cooperative and direct traffic simultaneously. This can be modeled as a variation of a three-user multiaccess channel where two users correspond to the direct traffic from nodes 2 and 3, and a third user corresponds to the cooperative traffic.[7] The difference here is that the power constraints of the "users" are coupled. We assume that if both users 2 and 3 devote a fraction $\alpha \in [0, 1]$ of their power to cooperative traffic, then they can achieve any rates $(0, 0, 0, R_{S4}, R_{24}, R_{34}) \equiv (0, 0, 0, R_4, R_5, R_6) \in \mathbb{R}^6_+$ satisfying

$$\sum_{i \in \mathcal{V}} R_i \leq \log\big(1 + \sum_{i \in \mathcal{V}} P_i(\alpha)\big), \quad \forall \mathcal{V} \subseteq \{4, 5, 6\}, \tag{3}$$

[5]This example does not allow some schedules that do not violate the half-duplexing constraint, such as node 1 transmitting to node 2, while node 3 transmits to node 4. This can easily be accommodated in the general model; here we omit them to simplify the discussion.

[6]This requires that the two transmitters have perfect synchronization and so can coherently combine their signals at the receiver. Models for distributed beamforming that relax this assumption can also be found, e.g. [11]; these can be incorporated into the model by simply re-defining R_{S4}.

[7]A key assumption here is that the encoding of the traffic by these three "users" depends only on their own message and that the messages are independent.

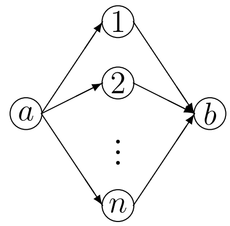

Fig. 2. A $n+2$ node parallel relay network model.

where $P_4(\alpha) = (\sqrt{h_{24}} + \sqrt{h_{34}})^2 \alpha P$, $P_5(\alpha) = h_{24}(1-\alpha)P$, and $P_6(\alpha) = h_{34}(1-\alpha)P$. Let $\mathcal{C}_{CMAC}(\alpha)$ be the set of rates which satisfy (3) for a particular value of α. We then set $\mathcal{C}_2 = \bigcup_{\alpha \in [0,1]} \mathcal{C}_{CMAC}(\alpha)$. It can be verified that the resulting region is convex.[8]

Example 2: The network in Fig. 2 is a generalization of Example 1 to the case where there are n relay nodes between a node a and node b. Here, all n nodes may form a cooperative link $(\{1, \ldots, n\}, b)$. Additionally, any subset of these n nodes can also form a cooperative link. Allowing all such possibilities, there are potentially $2^n - 1 - n$ different cooperative links between a and b in this network, each with its own corresponding broadcast link. In this case, the instantaneous link capacity region would have a dimension of $2(2^n - 1 - n) + 2n$, which can be modeled similarly as in Example 1.

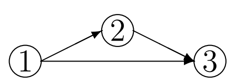

Fig. 3. A three node simple relay network model.

Example 3: The network in Fig. 3 is based on the classical relay channel [13]. Assume that $h_{12} > h_{13}$. We discuss two ways in which packets from node 1 can be cooperatively relayed to node 3. First, consider a cooperative link $(\{1, 2\}, 3)$, in which nodes 1 and 2 cooperatively forward a packet to node 3 again using cooperative beamforming. To do this, node 1 must first send a packet to node 2 and save a copy of the packet for itself. To incorporate such a scheme

[8]Note that here we require both nodes 2 and 3 to devote the same fraction of their power to the cooperative traffic. More generally, one can consider a model where each may devote a different fraction.

into our model, we view the first transmission as occurring over a broadcast link $(1, \{1, 2\})$, i.e. a link in which the source is also one of the destination nodes. The transmission rate on this link will be the same as the direct rate from node 1 to 2. Rate vectors in $\mathcal{C}$ will then have the form $(R_{1S}, R_{12}, R_{13}, R_{S3}, R_{23})$. Assuming a duplexing constraint at node 2, $\mathcal{C}$ can again be decomposed into two sets $\mathcal{C}_1$ and $\mathcal{C}_2$, where $\mathcal{C}_1$ ($\mathcal{C}_2$) is the set of feasible rates given that node 2 is receiving (transmitting), which can be modeled as a broadcast (multiaccess) channel.

A second cooperative scenario is for node 1 to first transmit a packet to node 2, but for node 3 to also store the received signal from this transmission (even though it cannot decode it). Then when node 2 forwards the packet to node 3, node 3 can use the information from both transmissions to decode the packet. This case is modeled by a broadcast link $(1, \{2, 3\})$ and a cooperative link $(\{2, 3\}, 3)$. The rate for the broadcast link $(1, \{2, 3\})$ is again the rate at which node 1 can transmit to node 2 (since node 3 is not decoding). The corresponding rate on the cooperative link $(\{2, 3\}, 3)$ is[9]

$$\overline{R}_{\{2,3\}3} = \log(1 + h_{23}P) + \log(1 + h_{13}P). \tag{4}$$

Here, the first term reflects the mutual information received from node 2's transmission and the second term is the mutual information received from node 1's original transmission to node 2. In this case, one can again define $\mathcal{C}$ for given duplexing constraints. We could also define $\mathcal{C}$ to includes both types of cooperative links; it would then contain vectors of dimension 7 corresponding to the three direct rates, two cooperative rates, and two broadcast rates.

We have focused on relatively simple network topologies to illustrate some possibilities for cooperation. In a general network, several of these scenarios, as well as others, could exist at different locations in the network. Moreover, we emphasize that while we restrict our attention to two-hop cooperative transmissions, we do not require that the overall network has a two-hop topology.

A. Traffic and queueing dynamics

Following [5], all traffic that enters the network is classified as a particular "commodity," which specifies its desired destination. Let $\mathcal{K} \subset \mathcal{V}$ be the set of commodities, where commodity

[9]For this model, we require that node 1 transmit on link $(1, \{2, 3\})$ with full power P. Otherwise, the corresponding rate on the cooperative link would depend on the power used in the previous time-slots.

k has destination node k. Exogenous traffic of each commodity $k \in \mathcal{K}$ arrives at node $i \in \mathcal{V} \setminus k$ according to an ergodic process $B_i^k(t)$, where $B_i^k(t)$ is the number of exogenous bit arrivals to node i in time-slot t. All arriving traffic is buffered until it is transmitted.

Let $U_i^k(t)$ be the unfinished work (in bits) of commodity k at node i, which is to be sent over a direct or broadcast link (we refer to this as *direct traffic*). For each cooperative link $(S, u) \in \mathcal{S}$, let $U_S^k(t)$ be the unfinished work of commodity k traffic that is to be forwarded cooperatively by the nodes in S. Each node keeps separate queues for each commodity of the direct traffic as well as each commodity of traffic for each cooperative set S to which it belongs.[10]

Let $(R_l^k(t))_{l \in \mathcal{L} \cup \mathcal{S} \cup \mathcal{T}, k \in \mathcal{K}}$ denote a joint rate allocation/routing assignment at time t, where $R_l^k(t)$ denotes the rate allocated to commodity k over link l. For feasibility, we must have

$$\sum_{k \in \mathcal{K}} R_l^k(t) \leq R_l(t) \ \ \forall l, \text{ and } (R_l(t))_{l \in \mathcal{L} \cup \mathcal{S} \cup \mathcal{T}} \in \mathcal{C}, \tag{5}$$

where $R_l(t)$ is the aggregate rate allocated to link l at time t. Given such a feasible rate allocation/routing assignment, the dynamics of the direct queue backlogs $U_i^k(t)$, for all i, k, are given by

$$\begin{aligned} U_i^k(t+1) \leq \Bigg[& U_i^k(t) - \sum_{T \in \mathcal{T}_i} R_{iT}^k(t) - \sum_{j \in \mathcal{O}_i} R_{ij}^k(t) \\ & + \sum_{S \in \mathcal{S}_i} R_{Si}^k(t) + \sum_{m \in \mathcal{I}_i} R_{mi}^k(t) + B_i^k(t) \Bigg]^+ . \end{aligned} \tag{6}$$

Here, $\mathcal{O}_i \equiv \{j \in \mathcal{V} | (i, j) \in \mathcal{L}\}$, $\mathcal{T}_i \equiv \{T \subseteq \mathcal{V} | (i, T) \in \mathcal{T}\}$, $\mathcal{I}_i \equiv \{m \in \mathcal{V} | (m, i) \in \mathcal{L}\}$, $\mathcal{S}_i \equiv \{S \subseteq \mathcal{V} | (S, i) \in \mathcal{S}\}$, and $[x]^+$ denotes $\max(x, 0)$. Similarly, the dynamics for each cooperative queue satisfy:

$$U_S^k(t+1) \leq \left[U_S^k(t) - \sum_{j \in \mathcal{O}_S} R_{Sj}^k(t) + \sum_{m \in \mathcal{I}_S} R_{mS}^k(t) \right]^+ . \tag{7}$$

Here, $\mathcal{O}_S \equiv \{j \in \mathcal{V} | (S, j) \in \mathcal{S}\}$ and $\mathcal{I}_S \equiv \{m \in \mathcal{V} | (m, S) \in \mathcal{T}\}$.

All traffic for cooperative queues arrives via broadcast links. In particular, there are no exogenous arrivals. This means that all the nodes in a cooperative set will always have the same queue backlog in the corresponding cooperative queues. One important caveat to this statement is in the second cooperative model in Example 3. In that case, the cooperative link

[10]If a node is part of several cooperative links involving the same cooperative set S, all the traffic of a given commodity for each of these links can be stored in one queue.

given by $(\{2,3\},3)$ corresponds to the case where node 3 cannot decode node 1's transmission to node 2, but stores some information about the received signal to aid it in decoding node 2's transmission. Thus, the cooperative queue backlog is not the actual amount of information stored at node 3. If the amount of data stored by node 3 is no greater than some bounded multiple of the actual number of bits transmitted, then stability of $U_S^k(t)$ still implies the stability of the cooperative queue at node 3.

III. Throughput Optimal Rate Allocation

Next we characterize the network stability region and give a throughput optimal joint rate allocation/routing policy. Although the results we obtain here may be reminiscent of results for conventional networks [4], [5], the cooperative nature of the relay network introduces some subtle differences.

A. Stability Region

Let $\rho_i^k = \lim_{t\to\infty} \frac{1}{t}\sum_{\tau=0}^{t} B_i^k(\tau)$ be the exogenous bit arrival rate to the direct queue at node i for commodity k. We say that this queue is *stable* if $\limsup_{t\to\infty} \frac{1}{t}\sum_{\tau=1}^{t} 1_{[U_i^k(\tau)>\xi]} d\tau \to 0$ as $\xi \to \infty$, where $1_{\{\cdot\}}$ is the indicator function. Stability for the cooperative queues is defined in the same manner.

The *network stability region* Λ is defined as the closure of the set of all $(\rho_i^k)_{i\in\mathcal{V},k\in\mathcal{K}} \in \mathbb{R}_+^{|\mathcal{K}||\mathcal{V}|}$ for which there exists some feasible joint rate allocation and routing policy $\mathcal{R}(\boldsymbol{u})$ which can guarantee that all queues are stable. This includes all policies which dynamically make rate allocation and routing decisions given (possibly non-causal) knowledge of the joint queue backlogs, $\boldsymbol{u}(t) = ((u_i^k(t))_{i\in\mathcal{V}}, (u_S^k(t))_{S\in\mathcal{U}})_{k\in\mathcal{K}}$. By feasible, we mean that at each time t, the policy specifies a rate vector $(R_l^k(t))_{l\in\mathcal{L}\cup\mathcal{S}\cup\mathcal{T},k\in\mathcal{K}}$ satisfying (5). The following result characterizes the stability region for a cooperative relay network. The proof is a direct generalization of the arguments in [4], [5], and so is omitted.

Theorem 1: The stability region Λ of a network $\mathcal{G} = (\mathcal{V},\mathcal{L},\mathcal{S},\mathcal{T})$ with two-hop cooperative forwarding is the set of all $(\rho_i^k)_{i\in\mathcal{V},k\in\mathcal{K}} \in \mathbb{R}_+^{|\mathcal{K}||\mathcal{V}|}$ for which there exist non-negative flow variables $((f_{ij}^k)_{(i,j)\in\mathcal{L}}, (f_{iT}^k)_{T\in\mathcal{T}}, (f_{Si}^k)_{S\in\mathcal{S}})_{k\in\mathcal{K}} \in \mathrm{conv}(\mathcal{C})$ that satisfy the following flow conservation relations:

$$\rho_i^k = \sum_{j\in\mathcal{O}_i} f_{ij}^k + \sum_{T\in\mathcal{T}_i} f_{iT}^k - \sum_{m\in\mathcal{I}_i} f_{mi}^k - \sum_{S\in\mathcal{S}_i} f_{Si}^k,$$

for all $k \in \mathcal{K}$ and all $i \in \mathcal{V} \setminus k$;

$$0 = \sum_{j\in\mathcal{O}_S} f_{Sj}^k - \sum_{m\in\mathcal{I}_S} f_{mS}^k,$$

for all $k \in \mathcal{K}$ and all cooperative sets S; and

$$\sum_{i\in\mathcal{V}} \rho_i^k = \sum_{i\in\mathcal{I}_k} f_{ik}^k + \sum_{(S,k)\in\mathcal{S}} f_{Sk}^k,$$

for all $k \in \mathcal{K}$.

B. Throughput Optimal Rate Allocation and Routing

Theorem 1 states that if $\boldsymbol{\rho} = (\rho_i^k)_{i\in\mathcal{V},k\in\mathcal{K}} \in \mathrm{int}(\Lambda)$, then the queues can be stabilized. In general, however, this may require knowing the value of $\boldsymbol{\rho}$. In reality, $\boldsymbol{\rho}$ can be learned only over time, and may be variable. One would prefer to find *adaptive* rate allocation/routing policies which can stabilize the network *without* knowing $\boldsymbol{\rho}$, as long as $\boldsymbol{\rho} \in \mathrm{int}(\Lambda)$. As pointed out previously in [5], a throughput optimal resource allocation policy for stochastic networks with physical-layer capacity regions turns out to be a generalization of the *maximum differential backlog* (MDB) policy first proposed by Tassiulas [4]. Due to cooperative transmissions, however, the general relay networks considered here are somewhat different from the networks considered in [5]. Nevertheless, we show that the MDB policy can be adapted to produce a throughput optimal rate allocation/routing policy for a cooperative relay network.

Let $\boldsymbol{B}(t) = (B_i^k(t))_{i\in\mathcal{V},k\in\mathcal{K}}$ be the vector of bit arrivals in the tth time slot. In this section, to simplify our arguments, we restrict attention to the case where $\{\boldsymbol{B}(t) : t \in \mathbb{Z}_+\}$ are i.i.d. according to distribution $\pi_{\boldsymbol{B}}$ with finite mean $\mathsf{E}[\boldsymbol{B}] = \boldsymbol{\rho}$, where $\boldsymbol{\rho} = (\rho_i^k)_{i\in\mathcal{V},k=\in\mathcal{K}}$ is the vector of exogenous bit arrival rates. Furthermore, assume that $\mathsf{E}[(B_i^k)^2] < \infty$ for each i and each k, and $\Pr\left(\cap_{i\in\mathcal{V}} \cap_{k\in\mathcal{K}} \{B_i^k = 0\}\right) > 0$. These assumptions on the arrival process clearly hold, for example, for independent homogeneous Poisson arrival processes. Following similar arguments as in [6], the above assumptions can be relaxed to the Markov modulated case.

Theorem 2: A throughput optimal rate allocation/routing policy $\mathcal{R}^*(\boldsymbol{u}(t))$ for a network with two-hop cooperative forwarding is given by first finding a rate allocation $\boldsymbol{R}^*(t)$ which is a solution to the following optimization problem:

$$\max_{\boldsymbol{R}(t)\in\mathcal{C}} \sum_{(i,j)\in\mathcal{L}} b^*_{ij} R_{ij}(t) + \sum_{(i,T)\in\mathcal{T}} b^*_{iT} R_{iT}(t) + \sum_{(S,i)\in\mathcal{S}} b^*_{Si} R_{Si}(t) \tag{8}$$

where

$$b^*_{ij} \equiv \max_{k\in\mathcal{K}} u^k_i(t) - u^k_j(t), \tag{9}$$

$$b^*_{iT} \equiv \max_{k\in\mathcal{K}} u^k_i(t) - |T| u^k_T(t), \tag{10}$$

$$b^*_{Si} \equiv \max_{k\in\mathcal{K}} |S| u^k_S(t) - u^k_i(t). \tag{11}$$

The corresponding routing policy is then implemented by sending only bits from traffic class k^* which attains the maximum in (9) ((10) and (11), respectively) at rate $R^*_{ij}(t)$ ($R^*_{iT}(t)$ and $R^*_{Si}(t)$, respectively) for all $(i,j)\in\mathcal{L}$ ($(i,T)\in\mathcal{T}$ and $(S,i)\in\mathcal{S}$, respectively). That is, over link $l\in\mathcal{L}\cup\mathcal{S}\cup\mathcal{T}$, $R^k_l(t) = R^*_l(t)$ for $k = k^*$ and $R^k_l(t) = 0$ otherwise.

Note that the policy in (8) is the not the same as the conventional MDB policy of [4], [5]. In particular, the terms $u^k_i - |T| u^k_T$ and $|S| u^k_S - u^k_i$ reflect the *queue coupling* effect induced by the cooperative transmission structure.[11] We refer to the policy of (8) as the *Cooperative Maximum Differential Backlog* (CMDB) policy.

Proof of Theorem 2: We give the outline of the proof here. For details, see [14]. To show that the CMDB policy stabilizes this network for any $\boldsymbol{\rho} = (\rho^k_i)_{i\in\mathcal{V},k\in\mathcal{K}} \in \mathrm{int}(\Lambda)$, it is convenient to consider a "fictitious network" $\mathcal{G}_f$ that is the same as the network $\mathcal{G}$, except that arrivals are allowed to enter the cooperative queues. Let $\mathcal{U}$ be the set of all cooperation sets. In the fictitious network, for each $S\in\mathcal{U}$, $i\in S$, and $k\in\mathcal{K}$ let ρ^k_{iS} denote the exogenous bit arrival rate to the queue at node i for cooperative set S and commodity k. We assume that the same arrivals occur simultaneously at each $i\in S$, so that $\rho^k_{iS} = \rho^k_S$ for all $i\in S$. Let Λ_f be the stability region of $\mathcal{G}_f$. [12] It is clear that if the CMDB policy stabilizes $\mathcal{G}_f$ for all $((\rho^k_i),(\rho^k_{iS})_{S\in\mathcal{U}})_{i\in\mathcal{V},k\in\mathcal{K}} \in \mathrm{int}(\Lambda_f)$

[11] The exact value of these terms is due to our choice of the Lyapunov function used in the proof of Theorem 2, which is a natural generalization of the Lyapunov function used in [4], [5]. Other choices of Lyapunov functions can be used to derive other throughput optimal policies.

[12] This can be characterized as in Theorem 1, except the second flow conservation equation will now have ρ^k_S on the left-hand side.

such that $\rho_{iS}^k = 0$ for $S \ni i$, all $i \in \mathcal{V}$ and all $k \in \mathcal{K}$, then CMDB also stabilizes $\mathcal{G}$ for all $\boldsymbol{\rho} = (\rho_i^k)_{i\in\mathcal{V},k\in\mathcal{K}} \in \mathrm{int}(\Lambda)$. Therefore, from now on, we concentrate on the artificial network $\mathcal{G}_f$.

To show that the CMDB policy stabilizes $\mathcal{G}_f$ for all $((\rho_i^k),(\rho_{iS}^k)_{S\in\mathcal{U}})_{i\in\mathcal{V},k\in\mathcal{K}}$ such that $\rho_{iS}^k = 0$ for $S \ni i$, all $i \in \mathcal{V}$ and all $k \in \mathcal{K}$, we use an extension of Foster's Criterion for the convergence of Markov chains [2], [3], [5]. Consider the Lyapunov function

$$\begin{aligned} V(\boldsymbol{u}) &\equiv \sum_{k\in\mathcal{K},i\in\mathcal{V}} \left[(u_i^k)^2 + \sum_{S\ni i}(u_S^k)^2\right] \\ &= \sum_{k\in\mathcal{K}} \left[\sum_{i\in\mathcal{V}}(u_i^k)^2 + \sum_{S\in\mathcal{U}} |S|(u_S^k)^2\right]. \end{aligned} \tag{12}$$

We will show that there exists a compact subset $\Gamma \subset \mathbb{R}_+^{|\mathcal{K}|(|\mathcal{V}|+|\mathcal{U}|)}$ such that under the CMDB policy,

$$\mathsf{E}[V(\boldsymbol{U}(t+1)) - V(\boldsymbol{U}(t))|\boldsymbol{U}(t) = \boldsymbol{u}] < -\epsilon,$$

for all $\boldsymbol{u} \notin \Gamma$, where $\epsilon > 0$. This, along with some other technical conditions [5], implies the existence of a steady state distribution for $\boldsymbol{U}$.

From (6), we can show

$$\begin{aligned} &(U_i^k(t+1))^2 \\ &\le (U_i^k(t))^2 - 2U_i^k(t)\Bigg(\sum_{T\in\mathcal{T}_i} R_{iT}^k(t) + \sum_{j\in\mathcal{O}_i} R_{ij}^k(t) - \sum_{S\in\mathcal{S}_i} R_{Si}^k(t) \\ &\quad + \sum_{m\in\mathcal{I}_i} R_{mi}^k(t) - B_i^k(t)\Bigg) + (B_i^k(t))^2 + 2B_i^k(t)\Bigg(\sum_{S\in\mathcal{S}_i} R_{Si}^k(t) \\ &\quad + \sum_{m\in\mathcal{I}_i} R_{mi}^k(t)\Bigg) + \Bigg(\sum_{T\in\mathcal{T}_i} R_{iT}^k(t) - \sum_{j\in\mathcal{O}_i} R_{ij}^k(t)\Bigg)^2 \\ &\quad + \Bigg(\sum_{S\in\mathcal{S}_i} R_{Si}^k(t) + \sum_{m\in\mathcal{I}_i} R_{mi}^k(t)\Bigg)^2 \end{aligned} \tag{13}$$

Similarly, using (7) (since $\rho_{iS}^k = 0$ for all $i \in S$, $S \in \mathcal{U}$ and $k \in \mathcal{K}$), we can show

$$\begin{aligned} (U_S^k(t+1))^2 \le (U_S^k(t))^2 - 2U_S^k(t)\Bigg(\sum_{j\in\mathcal{O}_S} R_{Sj}^k(t) - \sum_{m\in\mathcal{I}_S} R_{mS}^k(t)\Bigg) \\ + \Bigg(\sum_{j\in\mathcal{O}_S} R_{Sj}^k(t)\Bigg)^2 + \Bigg(\sum_{m\in\mathcal{I}_S} R_{mS}^k(t)\Bigg)^2. \end{aligned} \tag{14}$$

Taking conditional expected value of both sides of inequalities (13)-(14) given the event $\boldsymbol{U}(t)=\boldsymbol{u}$, and re-arranging, we have

$$\begin{aligned}
&\mathsf{E}[V(\boldsymbol{U}(t+1))-V(\boldsymbol{U}(t))|\boldsymbol{U}(t)=\boldsymbol{u}]\\
&\leq \sum_{k\in\mathcal{K}}\Bigg\{\sum_{i\in\mathcal{V}}-2u_i^k\mathsf{E}\Bigg[\sum_{T\in\mathcal{T}_i}R_{iT}^k(t)+\sum_{j\in\mathcal{O}_i}R_{ij}^k(t)-\sum_{S\in\mathcal{S}_i}R_{Si}^k(t)\\
&\quad-\sum_{m\in\mathcal{I}_i}R_{mi}^k(t)-B_i^k(t)|\boldsymbol{U}(t)=\boldsymbol{u}\Bigg]\\
&\quad+\sum_{S\in\mathcal{U}}-2u_S^k|S|\mathsf{E}\left[\sum_{j\in\mathcal{O}_S}R_{Sj}^k(t)-\sum_{m\in\mathcal{I}_S}R_{mS}^k(t)|\boldsymbol{U}(t)=\boldsymbol{u}\right]\Bigg\}\\
&\quad+\beta \qquad (15)
\end{aligned}$$

where $\beta>0$ is an upper bound on a sum of terms involving the second moments of the bit arrivals in the tth slot (which are bounded since the second moments of the packet arrivals and the packet sizes are bounded), and powers of transmission rates (which are bounded since $\mathcal{C}$ is bounded).

Let $\mathsf{E}_{\boldsymbol{u}}[X]$ denote $\mathsf{E}[X|\boldsymbol{U}(t)=\boldsymbol{u}]$. Note that

$$\begin{aligned}
&\sum_{k\in\mathcal{K}}\Bigg\{\sum_{i\in\mathcal{V}}u_i^k\mathsf{E}_{\boldsymbol{u}}\Bigg[\sum_{T\in\mathcal{T}_i}R_{iT}^k(t)+\sum_{j\in\mathcal{O}_i}R_{ij}^k(t)-\sum_{S\in\mathcal{S}_i}R_{Si}^k(t)\\
&\quad-\sum_{m\in\mathcal{I}_i}R_{mi}^k(t)\Bigg]+\sum_{S\in\mathcal{U}}u_S^k|S|\mathsf{E}_{\boldsymbol{u}}\Bigg[\sum_{j\in\mathcal{O}_S}R_{Sj}^k(t)-\sum_{m\in\mathcal{I}_S}R_{mS}^k(t)\Bigg]\Bigg\}\\
&=\sum_{k\in\mathcal{K}}\Bigg\{\sum_{(i,j)\in\mathcal{L}}\mathsf{E}_{\boldsymbol{u}}[R_{ij}^k(t)](u_i^k-u_j^k)+\sum_{(i,T)\in\mathcal{T}}\Bigg[\mathsf{E}_{\boldsymbol{u}}[R_{iT}^k(t)]\\
&\quad\times(u_i^k-|T|u_T^k)\Bigg]+\sum_{(S,i)\in\mathcal{S}}\mathsf{E}_{\boldsymbol{u}}[R_{Si}^k(t)](|S|u_S^k-u_i^k)\Bigg\}. \qquad (16)
\end{aligned}$$

For any $((\rho_i^k),(\rho_{iS}^k)_{S\in\mathcal{U}})_{i\in\mathcal{V},k\in\mathcal{K}}\in\operatorname{int}(\Lambda_f)$ such that $\rho_{iS}^k=0$ for $S\ni i$, all $i\in\mathcal{V}$ and all $k\in\mathcal{K}$, there exists $\delta>0$ such that $((\rho_i^k+\delta),(\rho_{iS}^k)_{S\in\mathcal{U}})_{i\in\mathcal{V},k\in\mathcal{K}}\in\Lambda_f$ such that $\rho_{iS}^k=\delta$ for $S\ni i$, all $i\in\mathcal{V}$ and all $k\in\mathcal{K}$. Therefore, an application of Theorem 1 to $\mathcal{G}_f$ shows that there exist non-negative flow variables $((f_{ij}^k)_{(i,j)\in\mathcal{L}},(f_{iT}^k)_{T\in\mathcal{T}},(f_{Si}^k)_{S\in\mathcal{S}})_{k\in\mathcal{K}}\in\operatorname{conv}(\mathcal{C})$ such that

$$\begin{aligned}
\rho_i^k+\delta&=\sum_{j\in\mathcal{O}_i}f_{ij}^k+\sum_{T\in\mathcal{T}_i}f_{iT}^k-\sum_{m\in\mathcal{I}_i}f_{mj}^k-\sum_{S\in\mathcal{S}_i}f_{Si}^k, i\in\mathcal{V},k\in\mathcal{K}.\\
\delta&=\sum_{j\in\mathcal{O}_S}f_{Sj}^k-\sum_{m\in\mathcal{I}_S}f_{mS}^k,\quad S\in\mathcal{U}.
\end{aligned}$$

We therefore have

$$\sum_{k\in\mathcal{K}}\left\{\sum_{i\in\mathcal{V}} u_i^k(\rho_i^k+\delta)+\sum_{S\in\mathcal{U}}|S|u_S^k\delta\right\}$$
$$=\sum_{k\in\mathcal{K}}\left\{\sum_{(i,j)\in\mathcal{L}} f_{ij}^k(u_i^k-u_j^k)+\sum_{(i,T)\in\mathcal{T}} f_{iT}^k(u_i^k-|T|u_T^k)\right.$$
$$\left.+\sum_{(S,i)\in\mathcal{S}} f_{Si}^k(|S|u_S^k-u_i^k)\right\}.$$

Let $((R_{ij}^k)_{(i,j)\in\mathcal{L}},(R_{iT}^k)_{T\in\mathcal{T}},(R_{Si}^k)_{S\in\mathcal{S}})_{k\in\mathcal{K}}$ be chosen according to the CMDB rule described in (8). Then, since $((f_{ij}^k)_{(i,j)\in\mathcal{L}},(f_{iT}^k)_{T\in\mathcal{T}},(f_{Si}^k)_{S\in\mathcal{S}})_{k\in\mathcal{K}}\in\mathrm{conv}(\mathcal{C})$, $\sum_{k\in\mathcal{K}}\{\sum_{i\in\mathcal{V}}u_i^k(\rho_i^k+\delta)+\sum_{S\in\mathcal{U}}|S|u_S^k\delta\}$ is less than or equal to the RHS of (16). Combining this fact with a rearrangement of the RHS of (15), and noting $\mathsf{E}[B_i^k(t)]=\rho_i^k$, we have

$$\mathsf{E}[V(\boldsymbol{U}(t+1))-V(\boldsymbol{U}(t))|\boldsymbol{U}(t)=\boldsymbol{u}]$$
$$\leq\beta-2\delta\left(\sum_{k\in\mathcal{K}}\left\{\sum_{i\in\mathcal{V}}u_i^k+\sum_{S\in\mathcal{U}}|S|u_S^k\right\}\right)$$

Let $\Gamma=\{\boldsymbol{u}:\sum_{k\in\mathcal{K}}[\sum_{i\in\mathcal{V}}u_i^k+\sum_{S\in\mathcal{U}}|S|u_S^k]\leq\frac{\beta+\epsilon}{2\delta}\}$. Then, for any $\epsilon>0$, and any $\boldsymbol{u}\notin\Gamma$, $\mathsf{E}[V(\boldsymbol{U}(t+1))-V(\boldsymbol{U}(t))|\boldsymbol{U}(t)=\boldsymbol{u}]<-\epsilon$.

We have shown that the CMDB policy stabilizes $\mathcal{G}_f$ for all $((\rho_i^k),(\rho_{iS}^k)_{S\in\mathcal{U}})_{i\in\mathcal{V},k\in\mathcal{K}}\in\mathrm{int}(\Lambda_f)$ such that $\rho_{iS}^k=0$ for $S\ni i$, all $i\in\mathcal{V}$ and all $k\in\mathcal{K}$. Thus, we have also shown that the CMDB policy stabilizes $\mathcal{G}$ for all $\boldsymbol{\rho}=(\rho_i^k)_{i\in\mathcal{V},k\in\mathcal{K}}\in\mathrm{int}(\Lambda)$. □

IV. CALCULATING THE CMBD POLICY

Implementing the CMBD policy requires solving (8. In this section, we examine the solution to this problem for the $n+2$ node parallel relay network introduced in Example 2 in Section II. As in Example 1, we assume there are two activation sets so that the link capacity region is again given by $\mathcal{C}=\mathrm{conv}(\mathcal{C}_1\cup\mathcal{C}_2)$, where $\mathcal{C}_1$ corresponds to activation set $\mathcal{A}_1=\{1\}$ and $\mathcal{C}_2$ corresponds to activation set $\{1,\ldots,n\}$. As mentioned in Example 2, any subset of the n relays can potentially form a cooperative link. Let $\mathcal{U}\subseteq\{1,\ldots,n\}$ be the set of all potential cooperative subsets. For simplicity of notation, we also include all direct links, i.e. singleton subsets, in $\mathcal{U}$.

Without loss of generality, assume $h_{a1} \leq \cdots \leq h_{an}$. Let $\mathcal{C}_{BC}$ be the capacity region of the n-user Gaussian broadcast channel corresponding to the model. It follows that the rate vector $((R_{aS})_{S\in\mathcal{U}}, \mathbf{0}) \in \mathbb{R}_+^{2|\mathcal{U}|}$ lies in $\mathcal{C}_1$ if and only if $(R_{aS})_{S\in\mathcal{U}}$ satisfies

$$R_i = \sum_{S\in\mathcal{U}_i} R_{aS}, i = 1, \ldots, n \quad \text{and} \quad (R_1, \ldots, R_n) \in \mathcal{C}_{BC} \tag{17}$$

where $\mathcal{U}_i = \{S \in \mathcal{U} : i = \min S\}$. For a symmetric network ($h_{a1} = \cdots = h_{an}$), $\mathcal{C}_1$ reduces to the set of all $((R_{aS})_{S\in\mathcal{U}}, \mathbf{0}) \in \mathbb{R}_+^{2|\mathcal{U}|}$ satisfying the simplex constraint

$$\sum_{S\in\mathcal{U}} R_{aS} \leq \log(1 + h_{a1}P). \tag{18}$$

.

For the multiaccess side of the parallel relay network, let $\boldsymbol{\alpha} = (\alpha_S^i)_{S\ni i, i=1,\ldots,n}$ be the vector of power splitting parameters, where $\alpha_S^i P$ is the power allocated by node i to cooperation set $S \ni i$. The multiaccess capacity region $\mathcal{C}_{CMAC}(\boldsymbol{\alpha})$ for a given $\boldsymbol{\alpha}$ is the set of all $(\mathbf{0}, (R_{Sb}))_{S\in\mathcal{U}} \in \mathbb{R}_+^{2|\mathcal{U}|}$ such that

$$\sum_{S\in\mathcal{V}} R_{Sb} \leq \log\left(1 + \sum_{S\in\mathcal{V}} P_S(\boldsymbol{\alpha})\right) \quad \forall \mathcal{V} \subseteq \mathcal{U}. \tag{19}$$

Note that $\mathcal{C}_{CMAC}(\boldsymbol{\alpha})$ is defined by as many as $2^{2^n-1} - 1$ constraints! Finally, the overall multiaccess region $\mathcal{C}_2 = \cup_{\boldsymbol{\alpha}} \mathcal{C}_{CMAC}(\boldsymbol{\alpha})$.

Let u_a be the queue backlog at node a and u_S be the queue backlog corresponding to cooperative set S. The CMDB policy can now be expressed as

$$\max_{\boldsymbol{R}\in\mathcal{C}} \sum_{S\in\mathcal{U}} (u_a - |S|u_S)R_{aS} + |S|u_S R_{Sb}. \tag{20}$$

Note that the solution $\boldsymbol{R}^*$ to (20) lies in $\mathrm{conv}(\mathcal{C}_1, \mathcal{C}_{CMAC}(\boldsymbol{\alpha}^*))$ for some $\boldsymbol{\alpha}^*$. Since $\mathcal{C}_1$ and $\mathcal{C}_{CMAC}(\boldsymbol{\alpha}^*)$ are orthogonal and the objective is linear, $\boldsymbol{R}^*$ lies either in $\mathcal{C}_1$ or in $\mathcal{C}_{CMAC}(\boldsymbol{\alpha}^*)$. We consider two cases.

Case 1: $\boldsymbol{R}^*$ lies in $\mathcal{C}_1$. In the symmetric case, $\boldsymbol{R}^*$ takes the form $R^*_{aS^*} = \log(1 + h_{a1}P)$ and $R^*_{aS} = 0$ otherwise, where $S^* = \arg\max_{S\in\mathcal{U}}[u_a - |S|u_S]$. Thus, at any time, only one cooperative set (or direct link) is active. In the asymmetric case, due to the linear constraint in (17), (20) reduces to

$$\max_{(R_1,\ldots,R_n)\in\mathcal{C}_{BC}} \sum_{i=1}^{n} \left(\max_{S\in\mathcal{U}_i}[u_a - |S|u_S]\right) \cdot R_i. \tag{21}$$

Let $(R_1^*, \ldots, R_n^*)$ be the solution to (21). Then the optimal solution to (20) has the form $R_{aS^*}^* = R_i^*$ for $S^* = \arg\max_{S \in \mathcal{U}_i}[u_a - |S|u_S]$ and $R_{aS}^* = 0$ for all other $S \in \mathcal{U}_i$, $i = 1, \ldots, n$. That is, at any time, multiple cooperative sets (or direct links) can be active, but *each relay node i participates in only one cooperative set (or direct link)*, namely the cooperative set (or direct link) in $\mathcal{U}_i$ with the largest differential backlog $u_a - |S|u_S$. For a general broadcast region $\mathcal{C}_{BC}$, the optimization in (21) can be solved using the greedy technique from [16], [17]. Note that even though the number of variables $(R_{aS})_{S \in \mathcal{U}}$ in the original optimization (20) can be exponentially large in n, the actual resulting optimization problem in (21) is only n-dimensional.

Case 2: $\boldsymbol{R}^*$ lies in $\mathcal{C}_{CMAC}(\boldsymbol{\alpha}^*)$. In this case, the optimization in (20) reduces to

$$\max_{(\mathbf{0}, R_{Sb})_{S \in \mathcal{U}} \in \mathcal{C}_{CMAC}(\boldsymbol{\alpha}^*)} \sum_{S \in \mathcal{U}} |S| u_S R_{Sb} \tag{22}$$

As mentioned above, $\mathcal{C}_{CMAC}(\boldsymbol{\alpha}^*)$ is potentially defined by a doubly exponential number of constraints. However, since $\mathcal{C}_{CMAC}(\boldsymbol{\alpha}^*)$ is a *polymatroid* [15], the maximization in (22) merely involves a *sorting* of the coefficients $|S|u_S$. The solution to (22) is then given by successively decoding the cooperative sets (or direct links) in increasing order of the coefficients $|S|u_S$ [15]. Since there are at most $2^n - 1$ coefficients $|S|u_S$, the maximization in (22) can be solved in $O(n)$ (linear) time.

V. Conclusions

We considered throughput optimal control of a wireless networks with cooperative relaying. Our model applies to a general network topology and several different types of cooperative scenarios. We established the network stability region and gave a variation of the Maximum Differential Backlog policy, which we proved to be throughput optimal. We focused on a centralized implementation and showed how the structure of the underlying capacity regions can aid in implementing this policy. In practice, a distributed solution is more desirable, particularly for managing the complexity of a cooperative network. Moreover, in a large network, there may be many potential cooperative sets. A useful direction for future work would be to develop a means for determining the most "useful" of these sets.

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